Math 240, Fall Semester 2001-02   NAME:

(Prof.) R.A. Brualdi

Exam 2: October 12, 2001,   Total Points:

(**[points in brackets], calculations of factorial, binomial coeffi-cients can be omitted.**

1. [24 points] A **ternary string** is a string (sequence) whose terms are 0's, 1's, and 2's. **Give**

(a) the number of ternary strings of length n:

(b) the number with no 0's:

(c) the number with exactly one 0:

(d) the number with exactly three 0's:

(e) the number with at least two 0's:

(f) the number with exactly one 0 and exactly one 1:

1

2. [5 points] The coefficient of $x^4 y^9$ in the expansion of $(2x - 3y)^{13}$ is:

3. [10 points] A case of sodapop contains 24 cans (order unimportant) taken from Pepsi, Coke, 7-Up, and Dr. Pepper. The number of different cases of sodapop possible is:

4. [15 points] Consider an ordinary deck of cards consisting of 13 ranks and 4 suits. A **pair** consists of two cards of the same rank. A **hand** is an (unordered) set of 6 cards. The number of hands with

(a) exactly one pair is:

(b) exactly two pairs is:

(c) exactly three pairs is:

5. [12 points] Find but **do not solve** a recurrence relation for the number a_n of bit strings of length n that do not contain the pattern 001 (i.e. two 0's followed by a 1):

6. [8 points] Let R be the relation among points in the plane defined by: $X R Y$ (i.e. $(X, Y) \in R$) provided the distance between X and Y is at most 3.

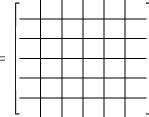Is R an equivalence relation? Why or why not?

Describe the relation R^2, i.e.
$X R^2 Y$ if and only if:

7. [12 points] The number of permutations of the 10 letters B,R,A,D,S,U,E,T,I,M that do not contain any of the names BRAD, SUE, or TIM equals:
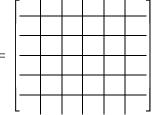
8. [12 points] Use **Warshall's algorithm** to determine the transitive closure of the relation R on 6 elements whose Boolean matrix is

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

You need not put in the 0's below; blank will be interpreted as 0.

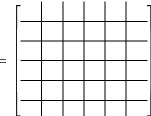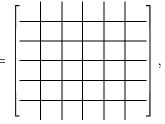$$W_0 = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}, W_1 = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}, W_2 = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix},$$

$$W_3 = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}, W_4 = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}, W_5 = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix},$$

$$W_6 = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$