Math 475, Exam I Name:
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1. (10 points) It's your turn in a game of NIM. In front of you, you see four piles, with 15, 13, 10, and 7 sticks, respectively. You want to win the game. **What's your move?**

2. (10 points) How many ways are there to place 8 non-attacking rooks – 4 identical red rooks and 4 identical blue rooks – on a 8 by 8 chessboard, if the square in the upper left corner of the board cannot contain a rook?

3. (5 points each) Answer the following questions:

a) The number of circular permutations of 5 X's, 3 Y's and 1 Z equals:

b) The permutation of $\{1, 2, 3, 4, 5, 6, 7\}$ with inversion sequence $3, 4, 3, 1, 1, 1, 0$ is:

c) The combination of $\{x_6, x_5, x_4, x_3, x_2, x_1, x_0\}$ that comes immediately **before** $\{x_6, x_3\}$ in the binary arithmetic generating scheme is:

d) The next 5-combination of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ that comes immediately **after** $\{2, 4, 5, 6, 9\}$ in the lexicographic ordering of 5-combinations is:

e) The 5-tuple that comes immediately **after** 0 1 0 0 0 in reflected Gray code ordering of the binary 5-tuples is:

f) The coefficient of $a^3b^2c^4d$ in the expansion of $(a + b + c + d)^{10}$ is:

g) There are N people in a room with ages between 21 and 40, inclusively. The smallest value of N that guarantees there are at least five people of the same age is:

h) $K_n \to K_r, K_b, K_g$ means that:

i) A partition of the partially ordered set $\mathcal{P}(X), \subseteq$ of combinations of $X = \{1, 2, 3, 4\}$ into symmetric chains is:

j) The reason $\mathcal{P}(X)$ cannot be partitioned into fewer chains (not necessarily symmetric chains) than you found above is:

k) The relation R defined on the real numbers by $a\ Rb$ if and only if $|a - b| \leq 1$ is a (choose one) a **partial order** **equivalence relation** **neither** because:

l) The diagram for the partially ordered set of integers $\{1, 2, \ldots, 12\}$ whose partial order is $|$ (is a factor of) is:

m) The number of even combinations of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ is:
because:

5. (15 points) Determine the number of 10-combinations of the multiset $\{4 \cdot A, 4 \cdot B, 6 \cdot C, 3 \cdot D\}$.