

# Paul Terwilliger VITA

Paul M. Terwilliger  
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Birthdate 6/24/55

## Education:

Ph.D. in Mathematics, University of Illinois, Urbana-Champaign, 1982  
B.S. in Math and Physics, University of Michigan, 1977

## Professional Experience:

Professor, University of Wisconsin-Madison, Sept. 1994–  
Associate Professor, University of Wisconsin-Madison, 1989–1994  
Assistant Professor, University of Wisconsin-Madison, 1985–1989  
Instructor, Ohio State University, 1982–1985  
Teaching Assistant, University of Illinois, 1978–1982

**Research Areas:** (i) Algebraic graph theory: distance-regular graphs, the subconstituent algebra, the  $Q$ -polynomial property, association schemes; (ii) Linear algebra: Leonard pairs, tridiagonal pairs, billiard arrays, lowering-raising triples; (iii) Special functions: a linear algebraic approach to the orthogonal polynomials of the Askey scheme, the universal Askey-Wilson algebra; (iv) Lie theory: the tetrahedron algebra, the Onsager algebra and its alternating central extension; (iv) Quantum groups: the equitable presentation of  $U_q(sl_2)$ , the algebra  $U_q(\widehat{\mathfrak{sl}}_2)$  and its alternating central extension, the  $q$ -tetrahedron algebra, the  $q$ -Onsager algebra and its alternating central extension.

**AMS Author citations as of August 2023:** Cited 3219 times by 389 authors.

## Notable Papers (number of citations as of August 2023):

The subconstituent algebra of an association scheme (191); Two linear transformations each tridiagonal with respect to an eigenbasis of the other (180); Some algebra related to  $P$ - and  $Q$ -polynomial association schemes (129); Two relations that generalize the  $q$ -Serre relations and Dolan-Grady relations (93); An algebraic approach to the Askey scheme of orthogonal polynomials (98); The quantum algebra  $U_q(sl_2)$  and its equitable presentation (57); The tetrahedron algebra, the Onsager algebra, and the  $sl_2$  loop algebra (54); The  $q$ -tetrahedron algebra and its finite-dimensional irreducible modules (50); A classification of sharp tridiagonal pairs (24); The universal Askey-Wilson algebra (52); Billiard arrays and finite-dimensional irreducible  $U_q(sl_2)$ -modules (8); Lowering-raising triples and  $U_q(sl_2)$  (5); Using Catalan words and a  $q$ -shuffle algebra to describe a PBW basis for the positive part of  $U_q(\widehat{\mathfrak{sl}}_2)$  (4); The alternating central extension of the  $q$ -Onsager algebra (3).

## Research Grants:

J.S.P.S Grant L-07512, 2007–2008  
N.S.F Grant DMS-9200792, 1992–1994  
N.S.F Grant DMS-8800764, 1988–1991  
N.S.F Grant DMS-8600882, 1986–1988  
University of Wisconsin Graduate School research grant, 1986–1987  
N.S.F Grant DMS-8504055, 1985–1986

## Honors and Awards:

CRM-Simons Professor, July 25 to August 19, 2022.  
University of Illinois Graduate Fellowship, 1980–1981  
Honors degree in Mathematics, University of Michigan, 1977

## Graduate Courses Taught:

744 Algebraic Graph Theory, Fall 1985  
744 Algebraic Graph Theory, Fall 1988  
875 Theory of Buildings, Spring 1990  
744 Algebraic Graph Theory, Spring 1991  
875 Combinatorics of Finite Geometries, Fall 1991

841 Algebraic Groups I, Spring 1992  
841 Algebraic Groups II, Fall 1992  
875 Algebraic Graph Theory, Spring 1993  
875 Partially Ordered Sets, Fall 1993  
744 Algebraic Graph Theory, Spring 1994

875 Algebraic Graph Theory II, Fall 1994  
875 Topics in Algebraic Combinatorics: Matroids, Quantum Matroids, and Association Schemes, Spring 1995  
875 The Subconstituent algebra of an association scheme, Spring 1996  
843 The Algebra and Combinatorics of the Askey-Wilson Polynomials, Fall 1996  
843 Introduction to Lie algebras, Spring 1997

875 Algebraic Combinatorics, Fall 1997  
841 The Representation Theory of the Symmetric group, Spring 1998  
875 The Askey-Wilson algebra, Spring 1999  
875 Harmonic analysis for algebraists and combinatorialists, Fall 1999  
875 Alternating sign matrices, Spring 2000

875 Leonard pairs, orthogonal polynomials, and graphs, Fall 2000  
875 Geometric algebra and the classical groups, Spring 2002  
843 Infinite dimensional Lie algebras, Spring 2004  
847 Introduction to quantum groups, Fall 2004

843 Infinite dimensional Lie algebras II, Spring 2005

843 Representation theory of Kac-Moody algebras, Fall 2005

843 Representation theory, Spring 2006

847 Introduction to Kac-Moody algebras, Fall 2006

847 Quantum groups, Fall 2008

846 Algebraic graph theory, Spring 2009

747 Introduction to Lie algebras, Fall 2009

846 Representation theory of Lie algebras, Spring 2010

805 Special functions, Fall 2010

846 The Double affine Hecke algebra, Fall 2011

846 Representation theory of Lie algebras, Spring 2013

846 Tridiagonal pairs and related topics, Fall 2013

747 Introduction to Lie algebras, Fall 2014

846 Quantum groups and Hopf algebras, Fall 2015

846 Hopf algebras in Combinatorics, Fall 2016

846 Introduction to Lie algebras, Spring 2018

846 Introduction to quantum groups, Spring 2019

846 Crystal bases in Algebraic Combinatorics, Fall 2019

846 Algebraic graph theory, Spring 2022

846 Association schemes, Spring 2023

### Conference Organizing:

- June 18–24, 2023. Coorganizer (with Jae-ho Lee and Stefko Miklavic) of a minisymposium on Association Schemes and Related Algebras, included in the 10th Slovenian Conference on Graph Theory. Kranjska Gora, Slovenia.
- July 25–August 19, 2022. Coorganizer (with Ada Chan, Gabriel Coutinho, Christino Tamon, and Luiz Valazquez) of thematic month on Graph Theory, Algebraic Combinatorics and Mathematical Physics, included in SYMMETRIES: ALGEBRAS AND PHYSICS; A THEMATIC PROGRAM in the SUMMER/FALL 2022 at U. Montreal, Canada.
- Sept 14–15, 2019. Coorganizer (with Sarah Post), AMS special session on Special Functions and Orthogonal Polynomials. Madison WI.
- Dec. 6–7, 2013. Coorganizer (with Tom Koornwinder, Dennis Stanton, Ole Warnaar), Askey80 birthday conference for Richard Askey, Madison WI.
- Oct. 5–6, 2007. Coorganizer (with Sung-Yell Song), AMS special session on Algebraic Combinatorics. Chicago, Illinois.

- Oct. 21–23, 2005. Coorganizer (with Sung-Yell Song), AMS special session on Association Schemes and Related Topics. Lincoln, Nebraska.
- Oct. 12–13, 2002. Coorganizer (with Richard Askey), AMS special session on Combinatorics and Special Functions. Madison WI.
- October 22–23, 1993. Coorganizer (with Sung-Yell Song), AMS special session on Algebraic Combinatorics. College Station, TX.

### Invited Addresses 22/23:

- *A  $Q$ -polynomial structure associated with the projective geometry  $L_N(q)$ .* 20 minute talk, AMS special session on Algebraic Combinatorics and Applications in Harmonic Analysis. Salt Lake City, October 22–23, 2023.
- *The  $q$ -shuffle algebra, the alternating elements, and the positive part of  $U_q(\widehat{\mathfrak{sl}}_2)$ .* 50 minute talk, Algebra seminar, UW-Milwaukee. March 14, 2023.
- *The  $q$ -shuffle algebra, the alternating elements, and the positive part of  $U_q(\widehat{\mathfrak{sl}}_2)$ .* 50 minute talk, Discrete Math seminar, University of Central Florida. April 5, 2023.
- *The  $\mathbb{Z}_3$ -Symmetric Down-Up Algebra.* 20 minute talk, minisymposium on Association Schemes and Related Algebras, Kranjska Gora, Slovenia, June 18–24, 2023.

### Invited Addresses 21/22:

- *The alternating central extension of the  $q$ -Onsager algebra.* 50 minute on-line talk, Workshop on Algebraic Combinatorics, Institute of Mathematics, Academia Sinica, January 24–26, 2022.
- *The alternating central extension of the  $q$ -Onsager algebra.* 50 minute on-line talk, Seminar on Mathematical Physics, CRM Canada, February 8, 2022.
- *$Q$ -polynomial graphs and the positive part  $U_q^+$  of  $U_q(\widehat{\mathfrak{sl}}_2)$ .* 20 minute on-line talk, AMS special session on Combinatorics and Representations of Noncommutative Algebras, Purdue U., March 26–27, 2022.
- *Tridiagonal pairs and  $U_q(\widehat{\mathfrak{sl}}_2)$ .* 45 minute on-line talk, Workshop on Noncommutative Algebras, Representation Theory and Special Functions, CRM Canada, May 23-June 10, 2022
- *Compatibility and companions for Leonard pairs.* 30 minute on-line talk, OPSFA 16, CRM Canada, June 13–17, 2022.
- *Distance-regular graphs, the subconstituent algebra, and the  $Q$ -polynomial property.* Minicourse consisting of five 60 minute talks, Workshop on Graph Theory, Algebraic Combinatorics and Mathematical Physics, CRM Canada, July 25 to August 19, 2022.

### Invited Addresses 20/21:

- *Leonard pairs, spin models, and distance-regular graphs.* 60 minute on-line talk, Algebraic Graph Theory Seminar, U. Waterloo Canada, Sept. 21, 2020.
- *$Q$ -polynomial graphs and the positive part of  $U_q(\widehat{\mathfrak{sl}}_2)$ .* 45 minute on-line talk, Algebraic Graph Theory and Quantum Information Workshop, Fields Institute, Canada, August 23–27, 2021.

### Invited Addresses 19/20:

- *The alternating PBW basis for the positive part of the quantum affine  $\mathfrak{sl}_2$  algebra.* 20 minute talk, AMS special session on Special Functions and Orthogonal Polynomials. Madison WI, Sept. 14-15, 2019.
- *The alternating central extension for the positive part of the quantum affine  $\mathfrak{sl}_2$  algebra.* 20 minute talk, AMS special session on Aspects and Applications of Algebraic Combinatorics. Denver CO, Jan. 15–18, 2020.

### Invited Addresses 18/19:

- *Catalan words and  $q$ -shuffle algebras.* Colloquium talk. Anhui University Math Dept., Hefei China, Nov. 19, 2018.
- *The Lusztig automorphism of the  $q$ -Onsager algebra.* Colloquium talk. Anhui University Math Dept., Hefei China, Nov. 20, 2018.
- *An infinite-dimensional  $\square_q$ -module obtained from the  $q$ -shuffle algebra for affine  $\mathfrak{sl}_2$ .* Plenary talk. International Workshop on Algebraic Combinatorics. Anhui University Math Dept., Hefei China, Nov. 23, 2018.
- *The alternating PBW basis for the positive part of the quantum affine  $\mathfrak{sl}_2$  algebra.* Colloquium talk. Institut Denis Poisson CNRS. Tours, France. May 22, 2019.
- *Leonard pairs, spin models, and distance-regular graphs.* 20 minute talk. 9th Slovenian International Conference on Graph Theory. Bled, Slovenia. June 26, 2019.

### Invited Addresses 17/18:

- *Totally bipartite tridiagonal pairs.* Colloquium talk. Anhui University Math Dept., Hefei China, Nov. 22, 2017.
- *Leonard triples of  $q$ -Racah type and their pseudo intertwiners.* Plenary talk. International workshop on Bannai-Ito theory, Zhejiang U., Hangzhou China, Nov 23–26, 2017.
- *The universal Askey-Wilson algebra.* Colloquium talk. U. Hawaii at Manoa, Jan 12, 2018.

- *Totally bipartite tridiagonal pairs.* 20 minute talk. AMS Special Session on Algebraic Combinatorics, Ohio State U., March 17, 2018.
- *An infinite-dimensional  $\square_q$ -module obtained from the  $q$ -shuffle algebra for affine  $\mathfrak{sl}_2$ .* 30 minute talk. 8th Summer School in Discrete Math, Rogla, Slovenia July 1–7, 2018.
- *An infinite-dimensional  $\square_q$ -module obtained from the  $q$ -shuffle algebra for affine  $\mathfrak{sl}_2$ .* 40 minute talk. Algebraic methods in Mathematical Physics, Montreal, Canada, July 16–20, 2018.

#### Invited Addresses 16/17:

- *Tridiagonal pairs of  $q$ -Racah type, the Bockling operator  $\psi$ , and  $L$ -operators for  $U_q(\mathfrak{sl}_2)$ .* 20 minute talk. AMS Special Session on Algebraic Combinatorics. U. Denver, Colorado. Oct. 8–9, 2016.
- *Leonard triples of  $q$ -Racah type and their pseudo intertwiners.* 25 minute talk. International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA-14). University of Kent, Canterbury, UK. July 3–7, 2017.
- *Leonard triples of  $q$ -Racah type and their pseudo intertwiners.* 25 minute talk. 2017 Meeting of the International Linear Algebra Society (ILAS). Iowa State U. Ames Iowa, July 24–28, 2017.

#### Invited Addresses 15/16:

- *Lowering-raising triples of linear transformations.* 20 minute talk. AMS Special Session on Algebraic Methods Common to Association Schemes, Hopf algebras, and Tensor Categories. Loyola U. Chicago. Oct. 2–4. 2015.
- *Billiard Arrays and finite-dimensional irreducible  $U_q(\mathfrak{sl}_2)$ -modules (40 min); The universal Askey-Wilson algebra (60 min); Lowering-raising triples and  $U_q(\mathfrak{sl}_2)$  (40 min).* Three talks at a Workshop on Mathematical Physics. U. Tours, France. November 25–27, 2015.
- *Lowering-raising triples and  $U_q(\mathfrak{sl}_2)$ .* Colloquium talk. De Paul University Chicago IL. May 20, 2016.

#### Invited Addresses 14/15:

- *Billiard Arrays and finite-dimensional irreducible  $U_q(\mathfrak{sl}_2)$ -modules.* 20 minute talk. AMS Special Session on Lie theory and Representation Theory. Eau Claire WI. Sept. 20–21. 2014.
- *A classification of the Lowering-Raising triples.* Plenary speaker. Algebraic Combinatorics Workshop at USTC. University of Science and Technology of China. Anhui, China. November 26–29, 2014.

- *Lowering-raising triples of linear transformations.* 30 minute talk. International Conference on Orthogonal Polynomials and  $q$ -Series. University of Central Florida. Orlando, FL. May 10–12, 2015.
- *A classification of the Lowering-Raising triples.* 25 minute talk. OPSFA 13. National Institute of Standards and Technology. Gaithersburg Maryland. June 1–5, 2015.
- *$Q$ -polynomial distance-regular graphs and the  $q$ -Onsager algebra.* 25 minute talk. Systems of lines: Applications of Algebraic Combinatorics. Worcester MA. August 10–14, 2015.

#### Invited Addresses 13/14:

- *Billiard Arrays and finite-dimensional irreducible  $U_q(sl_2)$ -modules.* 50 minute talk. Workshop on Algebraic Combinatorics. Hebei Normal University, China. November 17–21, 2013.
- *Leonard pairs and the  $q$ -tetrahedron algebra.* 30 minute talk. Askey80. U. Wisconsin. Dec 6-7, 2013.
- *Leonard pairs and the  $q$ -tetrahedron algebra.* 20 minute talk. AMS special session on Algebraic Methods in Graph Theory and Combinatorics. Knoxville, TN. March 21–23, 2014.
- *Tridiagonal pairs in Algebraic Graph Theory.* 60 minute talk. Japan Conference on Graph Theory and Combinatorics. Nihon University, Japan. May 17–21, 2014.
- *Billiard Arrays and finite-dimensional irreducible  $U_q(sl_2)$ -modules.* 20 minute talk. Modern Trends in Algebraic Graph Theory. Villanova U., PA. June 2–5, 2014.
- *Billiard Arrays and finite-dimensional irreducible  $U_q(sl_2)$ -modules.* 30 minute talk. Symmetries of Graphs and Networks IV. Rogla, Slovenia. June 29–July 5, 2014.
- *Billiard Arrays and finite-dimensional irreducible  $U_q(sl_2)$ -modules.* 30 minute talk. Exact Solvability and Symmetry Avatars. CRM, Montreal Canada. August 25–29, 2014.

#### Invited Addresses 12/13:

- *Finite-dimensional irreducible  $U_q(sl_2)$ -modules from the equitable point of view.* 40 minute talk. Algebraic Graph Theory, Spectral Graph Theory, and Related Topics. Nagoya Japan, January 5–6, 2013.
- *The universal Askey-Wilson algebra.* 20 minute talk. AMS special session on Algebraic and Geometric Combinatorics. Iowa State U. April 26–28, 2013.
- *Leonard pairs and the  $q$ -tetrahedron algebra.* 30 minute talk. ILAS annual meeting, Providence RI. June 3–7, 2013.

- *Leonard pairs and the  $q$ -tetrahedron algebra.* 30 minute talk. PhD summer school in Discrete Mathematics. Rogla, Slovenia, June 16–21, 2013.

#### Invited Addresses 11/12:

- *The universal Askey-Wilson algebra.* 20 minute talk. AMS special session on Algebraic Combinatorics. Lincoln, Nebraska. October 14–16, 2011.
- *The universal Askey-Wilson algebra.* 30 minute talk. 3rd SYGN Workshop and PhD Summer School in Discrete Mathematics. Rogla, Slovenia. June 25–29, 2012.
- *The universal Askey-Wilson algebra.* 50 minute talk. Shanghai Conference on Algebraic Combinatorics. Shanghai, China. August 17–22, 2012.

#### Invited Addresses 10/11:

- *The Rahman polynomials and the Lie algebra  $sl_3$ .* 30 minute talk. Conference on New Perspectives in Univariate and Multivariate Orthogonal Polynomials. BIRS, Banff Canada. Oct 10–15, 2010.
- *Tridiagonal pairs and distance-regular graphs.* 40 minute talk. Conference on Linear Algebraic Techniques in Combinatorics and Graph theory. BIRS, Banff Canada. Jan 30–Feb 4, 2011.
- *The Rahman polynomials and the Lie algebra  $sl_3$ .* 25 minute talk. AMS special session on Algebraic Combinatorics. Iowa City Iowa. March 18–20, 2011.
- *Tridiagonal pairs and distance-regular graphs.* 30 minute talk. Conference on Graph Theory. Bled, Slovenia. June 19–25, 2011.
- *The Universal Askey-Wilson Algebra.* 30 minute talk. Conference on Geometric and Algebraic Combinatorics. Oisterwijk, the Netherlands. August 14–19, 2011.

#### Invited Addresses 09/10:

- *The classification of sharp tridiagonal pairs.* Seminar talk, Georgia Tech. Atlanta GA. March 31, 2010.
- *The classification of tridiagonal pairs.* 20 minute talk, AMS special session on cohomology and representation theory of algebraic groups and related structures. Macalester College. St. Paul MN April 10, 2010.
- *A classification of sharp tridiagonal pairs.* Plenary talk, Annual Convention of the Philippine Math. Soc. Cebu City, Phils. May 20, 2010.
- *$Q$ -polynomial distance-regular graphs and their connections to tridiagonal pairs.* A minicourse of six 90 minute lectures at De La Salle U. Manilla Philippines. May 24–June 4, 2010.



- *A classification of sharp tridiagonal pairs.* 50 minute talk. Conference on Algebraic and Geometric Combinatorics. Gyeongju, South Korea. July 12–16, 2010.
- *A classification of sharp tridiagonal pairs.* 40 minute talk. Symmetries of Graphs and Networks. Rogla, Slovenia. August 1–6, 2010.

#### Invited Addresses 08/09:

- *Tridiagonal pairs of  $q$ -Racah type.* 20 minute talk, AMS special session on Special Functions and Orthogonal Polynomials, Vancouver, B.C., October 4, 2008.
- *Towards a classification of the tridiagonal pairs.* 20 minute talk, AMS special session on Algebraic Combinatorics, Worcester MA, April 25–26, 2009.
- *The classification of tridiagonal pairs of  $q$ -Racah type.* 30 minute talk, OPSFA10: Orthogonal Polynomials, Special Functions and Applications, Katholieke Universiteit Leuven (Belgium), July 20–25, 2009.

#### Invited Addresses 07/08:

- *Tridiagonal pairs in Lie theory, quantum groups, and orthogonal polynomials.* Colloquium talk, Kanazawa University, Japan. August 21, 2007.
- *Distance-regular graphs of  $q$ -Racah type and the  $q$ -tetrahedron algebra.* 45 minute talk, Japan-Korea Workshop on Algebra and Combinatorics. Kyushu University, Japan. September 14–15, 2007.
- *Tridiagonal pairs in Lie theory, quantum groups, and orthogonal polynomials.* Plenary talk, Annual meeting of the Japanese Math Society. Sendai, Japan. September 21–24, 2007.
- *Evaluation modules for the three-point  $\mathfrak{sl}_2$  loop algebra.* Hour talk, Workshop on Algebras, Groups, and Geometries. Tambara Institute, Numata, Japan. October 14–18, 2007.
- *Recent results on tridiagonal pairs.* 50 minute talk, RIMS Workshop on Finite Groups and Algebraic Combinatorics. Kyoto, Japan. December 17–20, 2007.
- *The Drinfel'd polynomial for tridiagonal pairs.* 40 minute talk, Fourth Korea-Japan Workshop on Algebra and Combinatorics. Pohang University, Korea. February 1–2, 2008.
- *Tridiagonal pairs and the quantum affine algebra  $U_q(\widehat{\mathfrak{sl}}_2)$ .* 30 minute talk, International Joint Symposium on Computational Science. Kanazawa University, Japan. March 3–7, 2008.
- *The Drinfel'd polynomial of a tridiagonal pair.* 50 minute seminar talk. *Tridiagonal pairs in Lie theory, quantum groups, and orthogonal polynomials.* Colloquium talk. National Chiao Tung University, Taiwan. March 10–11, 2007.

- *The Drinfel'd polynomial of a tridiagonal pair.* Colloquium talk, Okayama University, Japan. May 8, 2008.
- *Some open problems concerning distance-regular graphs and related algebra.* Three 90 minute talks, Pohang University, Korea. June 2–6, 2008.
- *The classification of tridiagonal pairs of  $q$ -Racah type.* 50 minute talk, The 26th PNU–POSTECH Algebraic Combinatorics Seminar. Pusan, Korea. June 7, 2008.
- *The classification of tridiagonal pairs of  $q$ -Racah type.* 50 minute talk, 25th Conference on Algebraic Combinatorics. Sapporo, Japan. June 23–25, 2008.
- *The classification of tridiagonal pairs of  $q$ -Racah type.* Colloquium talk, Kanazawa University, Japan. July 9, 2008.
- *The classification of tridiagonal pairs of  $q$ -Racah type.* 30 minute talk, Geometric and Algebraic Combinatorics 4, Oisterwijk, The Netherlands. August 17–22, 2008.

### Some Earlier Invited Addresses:

Colloquium, Tufts University, January 1985

Colloquium, Kent State University, May 1985

Special Session of Special Functions and Combinatorics, Summer meeting of the AMS, Laramie, August 1985

Colloquium, University of Illinois, April 1986

Special Session of Combinatorics, meeting of the AMS, Kent, Ohio, April 1987

Special Session on Alg. Graph Theory, Summer meeting of the Canadian Math. Soc. Regina, June 1988

Hour talk, Workshop on Design Theory, Inst. for Math and its Appl., U. of Minnesota, July 1988

Meeting on Finite Buildings, Related Geometries and Applications, Colorado State U., July 1988

Hour talk, Conference on Combinatorics and Algebra, U. Michigan, June 1989

Hour talk, LMS Durham Conference on Groups and Combinatorics, July 1990

Special Session on Combinatorics, Summer meeting of the AMS, Ohio State University, Aug. 1990

Colloquium, Kyushu University, Japan, December 1990

Hour talk, Conference on Algebraic Combinatorics, Vladimir, USSR, August 1991

40 minute talk, Waterloo 92, Section on Algebraic Combinatorics, Waterloo, Canada, June 1992

Special Session on Algebraic Combinatorics, Meeting of the AMS, College Station, TX, October 1993

Hour talk, Conference on Algebraic Combinatorics, Fukuoka, Japan, Dec. 1993

Hour talk, Meeting on Algebraic Combinatorics, Oberwolfach, January, 1994

Hour talk, Meeting on Algebraic Combinatorics, Kyoto, Japan, March 1995

3 hour talks, Rims conference on Algebraic Combinatorics, Kyoto, Japan, Dec 1996

30 minute talk, Algebra and its applications, Vanderbilt, June 1996

hour talk, Conference on distance-regular graphs, Montreal, Nov 1996  
 3 hour talks, Workshop on Algebraic Combinatorics, Kyoto, Dec 1996  
 Special Session on Algebraic Combinatorics, meeting of AMS, Chicago, Sept 1998  
 30 minute talk, International workshop on special functions, Hong Kong, June 1999  
 Colloquium talk, Academia Sinica, Taipei, June 1999  
 30 minute talk, Workshop on Algebraic Combinatorics, Oisterwijk, the Netherlands, Aug 1999  
 Hour talk, Workshop on Combinatorics and Physics, Nagoya, Aug 1999  
 30 minute talk, DIMACS conference on codes and Association schemes, Rutgers, Nov 1999  
 Hour talk, Special functions 2000, Tempe Arizona, June 2000  
 Hour talk, Com2Mac conference on Association schemes, codes and designs, June 2000  
 30 minute talk, Rome conference on special functions, June 2001.  
 30 minute talk, Edinburgh workshop on algebraic graph theory, July 2001.  
 Hour talk, Durham conference on groups and geometies, July 2001.  
 20 minute talk, AMS special session on association schemes, Portland, June 2002.  
 Hour talk, IMA workshop on special functions, Minn. Aug 2002.  
 30 minute talk, Oisterwijk conference on combinatorics, Aug 2002.  
 Hour talk, RIMS conference on algebraic combinatorics, Kyoto, December 2002.  
 20 minute talk, AMS special session on special functions, Baton Rouge, March 2003.  
 Poster, FPSAC, Vadstena, Sweden, June 2003.  
 25 minute talk, Orthogonal polynomials, special functions and applications, Copenhagen August 2003.  
 20 minute talk, AMS special session on character theory and algebraic combin. Binghampton NY, Oct 2003.  
 40 minute talk, Special functions, orthogonal polynomials, and quantum groups, Bexbach Germany, Oct 2003.  
 20 minute talk, AMS special session on association schemes and coding theory. Chapel Hill, Oct 2003.  
 20 minute talk, AMS special session on orthogonal polynomials and their applications. Pheonix, Jan 2004.  
 Six 90 minute lectures, Summer school on orthogonal polynomials, Madrid, July 2004.  
 50 minute talk, Association schemes, codes and designs, Busan Korea, July 2004.  
 20 minute talk, AMS special session on special functions, Northwestern, Oct 2004.  
 50 minute talk, RIMS conference on algebraic combinatorics, Kyoto, Dec 04.  
 30 minute talk, OPSFA8 Munich, July 2005.  
 30 minute talk, Geometric and algebraic combinatorics III, Oisterwijk, Aug 05.  
 20 minute talk, AMS special session on special functions, Bard College, Oct 2005.  
 20 minute talk, AMS special session on association schemes and related topics, Lincoln Nebraska, Oct 2005.  
 20 minute talk, AMS special session on noncommutative algebra, Eugene OR, Nov 2005.  
 20 minute talk, AMS special session on exactly solvable models, Notre Dame IN, April 2006.  
 hour talk, Algebraic Combinatorics; an international conference in honor of Eiichi Bannai's 60th birthday. Sendai Japan, June 2006.

### **Academic Service:**

I have organized the weekly Combinatorics Seminar for over 30 years. In addition I have served on various departmental committees such as Graduate Admissions, Undergraduate Advising, Salary and Post-Tenure Review, Algebra Caucus Contact, Budget and Planning, TA evaluation/CTAPP, Tutorial Oversight, Math Education liaison, Academic Staff review. I have been a senator for the Faculty Senate, and the course coordinator for Math 475: Introduction to Combinatorics.

### **Former PhD Students and current position:**

Garth Dickie 1995 (Software Engineer, Ab Initio)  
Chih-Wen Weng 1995 (Professor, National Chiao Tung University)  
Benjamin Collins 1996 (Technical Solutions Engineer, Epic)  
Brian Curtin 1996 (Associate Professor, University of South Florida)  
Heather Lewis 1997 (Professor, Nazareth College)  
John Caughman 1998 (Professor, Portland State University)  
Eric Egge 2000 (Professor and Associate Provost, Carleton College)  
Michael Lang 2001 (Associate Professor, Bradley University)  
Mark Maclean 2001 (Professor, Seattle University)  
Brian Hartwig 2006 (NSA)  
John Bowman 2007 (Marketing technology, Chicago)  
Darren Funk-Neubauer 2007 (Professor, Colorado State University at Pueblo)  
Ali Godjali 2011 (Math Education Department, Surya College of Education, Indonesia)  
Chalermpong Worawannotai 2012 (Assistant Professor, Silpakorn University, Thailand)  
George Brown 2012 (Epic)  
Edward Hanson 2013 (Mathematician, Derivita)  
Jae-Ho Lee 2013 (Associate Professor, University of North Florida)  
Sarah Bockting-Conrad 2014 (Associate Professor, DePaul University, Chicago)  
Alison Gordon Lynch 2015 (Associate Professor, California State University, Monterey Bay)  
Gabriel Pretel 2015 (Faculty, Santa Barbara City College)  
Yang Yang 2017 (Financial consultant, NYC).

**Note:** Garth Dickie currently owns over 100 patents. In 1996 he won an Academy Award for Technical Achievement, for the invention of shape-based warping and morphing while employed at Elastic Reality. The award recognizes that all shape-based warping and morphing systems descend from this innovation.

### **Visiting Professors and Honorary Fellows that I have hosted:**

Arnold Neumeier 1990, Tayuan Huang 1994, Arlene Pascasio 1996, Kenichiro Tanabe 1996, Masato Tomiyama 1997, 2016, Junie Go 1998, Jack Koolen 2001, Melvin Vidar 2006, Stefko Miklavic 2007, Joohyung Kim 2007, Dianne Cerzo 2009, Hajime Tanaka 2010, Hau-Wen Huang 2010, Bostjan Frelik 2011, Aiping Deng 2014, Yuta Watanabe 2015, Wen Liu 2015, Rafael Cantuba 2015, Supalak Sumalroj 2017, Siwaporn Mamart 2017.

**Current PhD Students:** Ian Seong, Owen Goff, Peter Ruan, Nathan Nicholson, Keru Zhou.

### Selected quotations concerning the work of Paul Terwilliger:

“This paper contains a very interesting characterization of association schemes whose representation diagram is a forest. As an application, a set of inequalities for  $P$ -polynomial schemes (distance regular graphs) is obtained which has equality precisely for  $P$ - and  $Q$ -polynomial schemes. Since equality implies geometric information about the graph, this seems to provide a big step towards the aim of classifying all such schemes.”

Arnold Neumaier, in a MathSciNet review of the paper: P. Terwilliger. A characterization of  $P$ - and  $Q$ -polynomial association schemes. *J. Combin. Theory Ser. A* 45 (1987).

“Dear Brian Curtin, your work is certainly of interest to me. I read your outline and I will make some comments below. I am more interested these days in spin models in the more general context of planar algebras where one looks for systems of matrices closed under general planar contraction schemes (the association scheme axioms take care of two kinds of contractions, the type III Reidemeister moves take care of another and I am looking at variants of the type III move that allow one to include examples more general than Higman-Sims, eg MacLauchlin). So I would be happy to be your host for an NSF postdoc. The complication is that I have reasonably firm plans to be on sabbatical in Geneva for '98-'99. This is not incompatible with the NSF postdoc. Ted Stanford took one year of his in Geneva in '93-'94 when I was last there. I will not know for sure whether I am going to Geneva or not until some time around February. If this could work for you, let me know.

A couple of comments on your plan: the idea of the Terwilliger algebra is a good one. I remember thinking about an algebra like that at one point. It is nice to see  $U_q(\mathfrak{sl}_2)$  coming up. On the other hand it is a very universal object. One would expect much more in specific cases. For instance it is possible to argue for the  $\mathfrak{sl}_2$  nature of the knot invariant corresponding to the 5-state non-Potts spin model but Jaeger's Higman-Sims model corresponds definitely to the symplectic group so you might look for  $U_q(\mathfrak{sp}_4)$  commutation relations in that case. Someone may even have already found them as there is a definite coincidence between small representations of Higman-Sims and reps of  $\mathfrak{sp}_4$ . I think Kuperberg may even have a paper on that. I actually suspect there are lots more spin models out there - they are just very elusive.

Anyway, whatever you decide about the NSF postdoc, good luck with your research and please keep me informed.

Best, Vaughan Jones”

Email letter from Fields Medalist Vaughan Jones to Brian Curtin, September 1997.

“Douglas Leonard established in 1982 that the first and second eigenvalues of  $P$ - and  $Q$ -polynomial association schemes can be represented by Askey-Wilson polynomials. The mere statement of this important result takes 10 pages of the Bannai-Ito monograph. By considering an algebraic axiomatization of the situation (Leonard systems) Terwilliger manages to clarify and simplify considerably this remarkable result.”

Patrick Solé, in a zbMath review of the paper: P. Terwilliger. Two linear transformations each tridiagonal with respect to an eigenbasis of the other. *Linear Algebra Appl.* 330 (2001).

“The fact that there are no more classical type polynomials beyond those mentioned in the previous paragraph follows from a theorem of Leonard (Leonard 1982). This theorem has been put into a very attractive setting by Terwilliger, some of whose work has been summarized in Chapter 20.”

Richard Askey, in the Forward to the book by Mourad E. H. Ismail: *Classical and Quantum Orthogonal Polynomials in One Variable*. Cambridge U. Press, 2005.

“This 76 page paper gives a beautiful description of the intimate connection between certain Leonard pairs and a special class of orthogonal polynomials (the ‘terminating’ branch of the Askey scheme:  $q$ -Racah,  $q$ -Hahn, dual  $q$ -Hahn,  $q$ -Krawtchouk, dual  $q$ -Krawtchouk, affine  $q$ -Krawtchouk, Racah, Hahn, dual Hahn, Krawtchouk, Bannai/Ito and orphan polynomials).”

Marcel de Bruin, in a zbMath review of the paper: P. Terwilliger. An algebraic approach to the Askey scheme of orthogonal polynomials. *Orthogonal polynomials and special functions*, 255–330, *Lecture Notes in Math.*, 1883, Springer, Berlin, 2006.

“In the paper under review the authors prove and derive some corollaries from the following amazing result:

Theorem: Let  $\mathbb{K}$  denote a field and  $q$  a nonzero scalar in  $\mathbb{K}$  such that  $q^2 \neq 1$ . Let further  $U_q(\mathfrak{sl}_2)$  be the unital associative  $\mathbb{K}$ -algebra, generated by  $k^{\pm 1}$ ,  $e$ , and  $f$ , subject to the relations  $kk^{-1} = k^{-1}k = 1$ ,  $ke = q^2ek$ ,  $kf = q^{-2}fk$ , and  $ef - fe = (k - k^{-1})/(q - q^{-1})$ . Then  $U_q(\mathfrak{sl}_2)$  is isomorphic to the unital associative  $\mathbb{K}$ -algebra, generated by  $x^{\pm 1}$ ,  $y$ , and  $z$ , and subject to the following relations:

$$xx^{-1} = x^{-1}x = 1, \quad \frac{qxy - q^{-1}yx}{q - q^{-1}} = 1, \quad \frac{qyz - q^{-1}zy}{q - q^{-1}} = 1, \quad \frac{qzx - q^{-1}xz}{q - q^{-1}} = 1.$$

Volodymyr Mazorchuk, in a zbMath review of the paper: T. Ito, P. Terwilliger, C.W. Weng. The quantum algebra  $U_q(\mathfrak{sl}_2)$  and its equitable presentation. *J. Algebra* 298 (2006).

“The Terwilliger (or subconstituent) algebra of an association scheme was introduced in [616]. Though it should be stressed that this algebra also plays an important role in the theory of general distance-regular graphs (cf. Section 6), it is particularly well-suited for  $Q$ -polynomial distance-regular graphs.”

Edwin van Dam, Jack Koolen, Hajime Tanaka. *Distance-regular graphs*. *Electron. J. Combin.* (2016).