WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

Math 846: Algebraic Combinatorics: Association Schemes
MWF 8:50–9:40 AM, Van Vleck B231
Syllabus for Semester II, 2022/2023 (term 1234)

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Text: Bannai, Bannai, Ito, Tanaka. Algebraic Combinatorics. De Gruyter Series in Discrete Mathematics and Applications 5 (2021).

On MWF I will post my lecture notes near the top of my Terwilliger/Teaching website.

Prerequisites: Good understanding of linear algebra.

Course Content: This course is about a combinatorial concept called an association scheme. This concept provides an attractive conceptual framework for the study of algebraic graph theory and finite group theory. As we will see in the course, any finite group gives an example of an association scheme. Perhaps you have seen the character table of a finite group. Roughly speaking, an association scheme is a generalization of a finite group, that retains enough of the group structure so that we may still speak of the character table. In the course, we will examine association schemes from both a combinatorial and algebraic point of view. Our goal is to highlight the fascinating interaction between these two points of view. We will work through the above text, starting with chapter 2.

The lectures will be self contained and no prior knowledge of the subject is assumed. The only assumption is a good understanding of undergraduate linear algebra, such as eigenvalues, eigenspaces, bilinear forms, and tensor products. The course is recommended for anyone interested in algebraic combinatorics, algebraic graph theory, group theory, special functions, Lie theory, and quantum groups.

Course Credits: 3. Each week there will be three 50 minute lectures.

Evaluation: There are no exams. Near the end of the semester each non-dissertator student is expected to give one lecture, on a topic of your choice that is related to the course. As the time approaches I will suggest topics and organize the speaking schedule.

Course goals/Learning outcomes: Master the material presented in lecture. For this I recommend the following study strategy: (i) Get your hands dirty as you play with the examples. (ii) If I prove in lecture that something is true for all n, then on your own, verify the thing by brute force if necessary, for some small values of n. (iii) In lecture I will give careful proofs for the main results. For each result, try to write your own proof starting from first principles and without looking at your notes. It is not important if your proof matches mine or not. Done properly this strategy is easy to carry out, since every result in the course builds naturally on what came before. (iv) As you study your lecture notes, try to guess what comes next. Make conjectures and try to prove them.