Math 747: Introduction to Lie algebras
Lecture 001, MWF 11:00–11:50 AM, Soc Sci 6240
Syllabus for Semester II, 2023/2024 (term 1244)

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Text: Roger Carter. Lie algebras of finite and affine type. Cambridge U. Press, 2005.

Supplemental texts (on reserve in the College Library):
J. Humphreys. Introduction to Lie algebras and representation theory.
W. Fulton and J. Harris. Representation theory, a first course.
V. Kac. Infinite dimensional Lie algebras.
L.C. Grove and C.T. Benson. Finite reflection groups.
J. Humphreys. Reflection groups and Coxeter groups.
H. Hiller. Geometry of Coxeter groups.
A. Bjorner and F. Brenti. Combinatorics of Coxeter groups.

Prerequisites: Good understanding of linear algebra, such as eigenvalues, eigenspaces, bilinear forms, and tensor products.

Course Content: This course is an introduction to Lie algebras, with a focus on finite-dimensional semisimple Lie algebras over the complex numbers. Topics will include: solvable and nilpotent Lie algebras, Cartan subalgebras, the Cartan decomposition, the semisimple case, the Killing form, the root system, the Weyl group, Dynkin diagrams, and Cartan matrices. Our main goal is to classify up to isomorphism the finite-dimensional semisimple Lie algebras over the complex numbers. This remarkable classification leaves most people dazed and amazed.

The lectures will be self contained, and no prior knowledge of the subject is assumed. I will follow the Carter text more or less, covering roughly Chapters 1–8. This course should be valuable to anyone interested in Lie theory, quantum groups, algebraic combinatorics, number theory, and special functions. The Carter text is valued for its clarity. However, this text does not have exercises. Many good exercises can be found in the supplemental texts.

Course Credits: 3. Each week there will be three 50 minute lectures.

Evaluation: Each non-dissertator can choose one of the following options: (i) Solve at least 20 problems/exercises found in the supplemental textbooks. You can pick whatever problems interest you the most. Hand in your solutions by the last day of class. (ii) Near the end of the semester, give one lecture in class, on a topic either from the text or a related topic of your choice. As the time approaches I will organize the speaking schedule and suggest topics. One popular topic is a section from Chapter 8. Let me know which option you prefer, by April 1. There are no formal exams for this class.

Course goals/Learning outcomes: Master the material in Chapters 1–8 of the text. For this I recommend the following study strategy. After each lecture do the following: for each stated definition write out numerous examples and non examples. For each stated result, write your own proof starting from first principles and without looking at your notes. It is not important if your proof matches mine or not. Done properly this strategy is easy to carry out, since every result in the course builds naturally on what came before. In addition to the above activity, work out many problems from the supplemental texts, especially Humphreys and Fulton/Harris. The more problems solved, the better.