

Consider problem 2 for K_n (expanded ver)

Consider

$$A A^* A^{-1} A^{*-1}$$

$$A^* A A^{*-1} A^{-1}$$

$$A^* A^{-1} A^{*-1} A$$

$$A^{-1} A^* A A^{*-1}$$

$$A^{-1} A^{*-1} A A^*$$

$$A^{*-1} A^{-1} A^* A$$

$$A^{*-1} A A^* A^{-1}$$

$$A A^{*-1} A^{-1} A^*$$

For each row the two entries are inverses.

In each col the four entries are similar

Find the eigenspaces / eigenvalues.

Since $A, A^* \in T$ each $\neq e_0 V, e_0 V$

is left invariant by each of the above 8 matrices

Action on $e_0 V$:

Each of A, A^* acts on $e_0 V$ as $-I$

So each of above 8 elements act on $e_0 V$ as I

Action on eov:

Recall eov has basis $\hat{x}, \mathbb{1}$

Consider 4 vectors

$$\hat{x}, \mathbb{1} - \hat{x}, \mathbb{1}, n\hat{x} - \mathbb{1}$$

Recall meaning

vector	eigen vector for	eigenvalue
\hat{x}	A^*	$n-1$
$\mathbb{1} - \hat{x}$	A^*	-1
$\mathbb{1}$	A	$n-1$
$n\hat{x} - \mathbb{1}$	A	-1

Also

$$A \hat{x} = \mathbb{1} - \hat{x}$$

$$A^{-1}(\mathbb{1} - \hat{x}) = \hat{x}$$

$$A^* \mathbb{1} = n\hat{x} - \mathbb{1}$$

$$A^{*-1}(n\hat{x} - \mathbb{1}) = \mathbb{1}$$

Writing $W = eov$,

$$E_0^* W = \mathbb{C} \hat{x}$$

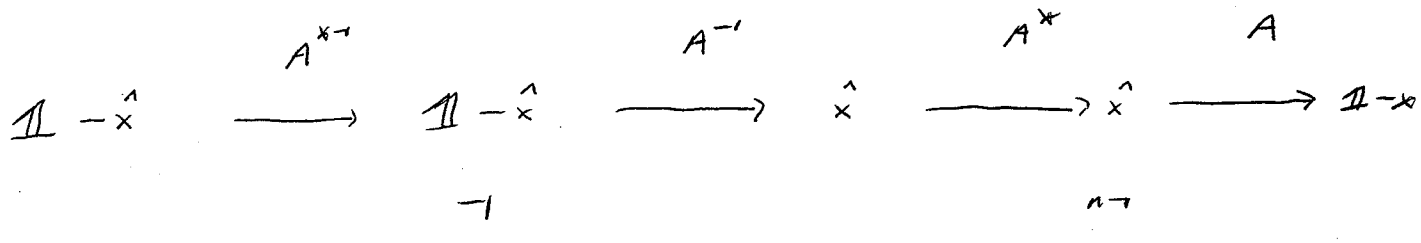
$$E_1^* W = \mathbb{C}(\mathbb{1} - \hat{x})$$

$$E_0 W = \mathbb{C} \mathbb{1}$$

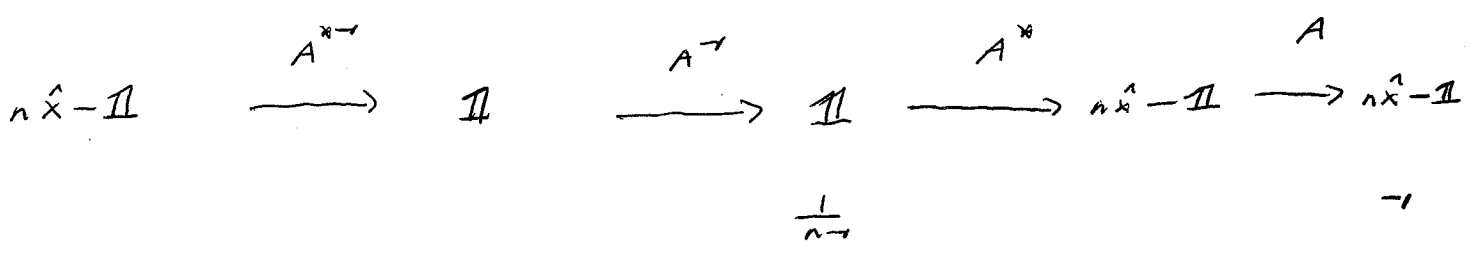
$$E_1 W = \mathbb{C}(n\hat{x} - \mathbb{1})$$

5/31/10

Find eigenvectors for $AA^*A^{-1}A^{*-1}$ in \mathbb{C}^n



$\mathbb{1} - \hat{x}$ is eigenvector with eigenval $1-n$



$n\hat{x} - \mathbb{1}$ is eigenvector with eigenval $\frac{1}{1-n}$

COR 57 For all 8 elements above the eigenvals are

eigval	$1-n$	$\frac{1}{1-n}$	1
mult	1	1	$n-2$

COR 58 For our 8 elements the eigenspaces are given below.

element	eigval $1-n$	eigval $\frac{1}{1-n}$	eigval 1
$AA^*A^*A^*$	$\mathbb{C}(\mathbb{1}-\hat{x})$	$\mathbb{C}(n\hat{x}-\mathbb{1})$	e, V
$A^*AA^*A^*$	$\mathbb{C}(n\hat{x}-\mathbb{1})$	$\mathbb{C}(\mathbb{1}-\hat{x})$	e, V
$A^*A^*A^*A$	$\mathbb{C}\hat{x}$	$\mathbb{C}(n\hat{x}-\mathbb{1})$	e, V
$A^*A^*AA^*$	$\mathbb{C}(n\hat{x}-\mathbb{1})$	$\mathbb{C}\hat{x}$	e, V
$A^*A^*AA^*$	$\mathbb{C}\hat{x}$	$\mathbb{C}\mathbb{1}$	e, V
$A^*AA^*A^*$	$\mathbb{C}\mathbb{1}$	$\mathbb{C}\hat{x}$	e, V
$A^*AA^*A^*$	$\mathbb{C}(\mathbb{1}-\hat{x})$	$\mathbb{C}\mathbb{1}$	e, V
$AA^*A^*A^*$	$\mathbb{C}\mathbb{1}$	$\mathbb{C}(\mathbb{1}-\hat{x})$	e, V

pf take for example

$$A^* A^{-1} A^{*-1} A = A^{-1} (A A^* A^{-1} A^{*-1}) A$$

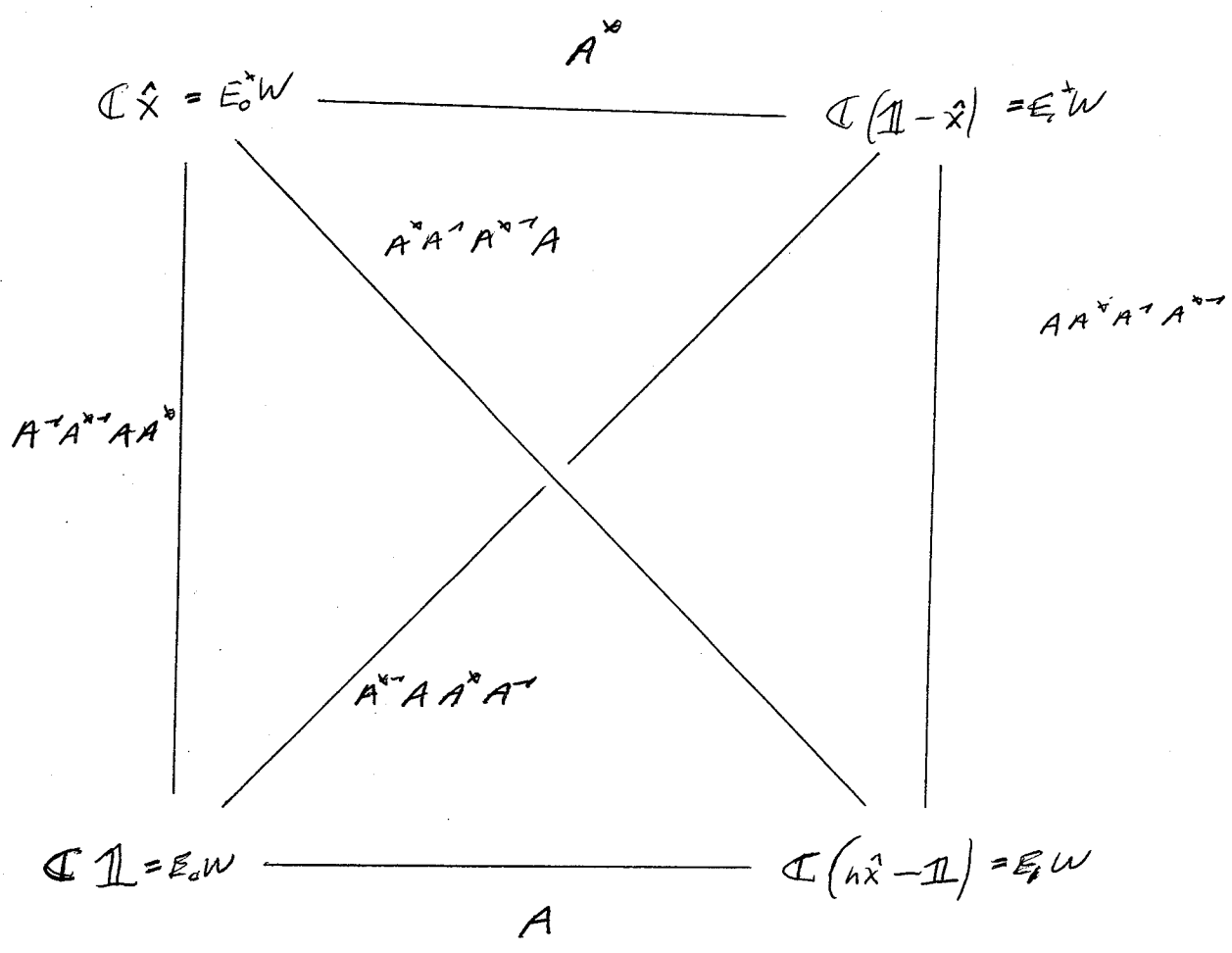
For the eigenval $1-n$ corresp eigenvector is

$$A^{-1} (\mathbb{I} - \hat{x}) = \hat{x}$$

For the eigenval $\frac{1}{1-n}$ corresp eigenvector is

$$A^{-1} (n\hat{x} - \mathbb{I}) = -(n\hat{x} - \mathbb{I})$$

Other entries similarly obtained (ex). □



Each edge in tetrahedron represents an eigenspace decomp of W for the corresp operator.

5/31/16

We have now solved problem 2 for Γ_n .

What about $\Gamma = H(A, \sigma)$?

View

$$\Gamma = \underbrace{\kappa_n \times \kappa_n \times \dots \times \kappa_n}_D$$

$$V(\Gamma) = V(\kappa_n)^{\otimes D}$$

View

$$A_D = \underbrace{a \otimes a \otimes \dots \otimes a}_P \\ = a^{\otimes P}$$

$$a = A(\kappa_n)$$

$$A_D^* = (a^*)^{\otimes P}$$

$$a^* = A^*(\kappa_n)$$

obs

$$A_D A_D^* A_D^{-1} A_D^{*-1} = (a a^* a^{-1} a^{*-1})^{\otimes P}$$

To get eigenspaces/eigenvals for

$$A_D A_D^* A_D^{-1} A_D^{*-1}$$

use our results for $a a^* a^{-1} a^{*-1}$.

The main problem is to relate this to the split decomposition for Γ .