Math 542 **Midterm Exam, Spring 2016**

Prof: Paul Terwilliger

Your Name (please print) ______________________________

NO CALCULATORS/ELECTRONIC DEVICES ALLOWED.

MAKE SURE YOUR CELL PHONE IS OFF.

Problem	Value	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Let R and S denote nonzero rings. Prove that the ring $R \times S$ is not a field.

2. Prove that the polynomial ring $\mathbb{Z}[x, y]$ is not a Euclidean domain.

3. Find all the ordered pairs r, s of positive integers such that $r^2 + s^2 = 999$.

4. Let $\mathbb{F}$ denote a field and consider the polynomial ring $R = \mathbb{F}[x, y]$. Consider the ideals $I = R(x - y^2)$ and $J = R(x^2 - y^2)$ in R. Prove that the quotient rings R/I and R/J are not isomorphic.

5. Let $\mathbb{F}$ denote a field. Let R denote the set of polynomials in $\mathbb{F}[x]$ that have x-coefficient zero. Note that R is a subring of $\mathbb{F}[x]$. Prove that R is not a UFD.

6. Prove that the polynomial $x^3 + nx + 2$ is irreducible in $\mathbb{Z}[x]$, provided that $n \neq 1, -3, -5$.

7. For the $\mathbb{Z}$-modules $M = \mathbb{Z}/7\mathbb{Z}$ and $N = \mathbb{Z}/6\mathbb{Z}$, find all the elements in $\mathrm{Hom}_{\mathbb{Z}}(M, N)$.

8. Let $n = 1000$. Find the order of the group of units for the ring $\mathbb{Z}/n\mathbb{Z}$.

9. Let R denote a ring with $1 \neq 0$. Let M denote an R-module, and let N denote an R-submodule of M. Define $J = \{r \in R | ra = 0 \text{ for all } a \in N\}$. Prove that J is a 2-sided ideal in R.

10. Let R denote a commutative ring with $1 \neq 0$. Let F denote a free R-module with finite rank. Prove that the R-modules $\mathrm{Hom}_R(F, R)$ and F are isomorphic.