Math 320 Spring 2013/14

Lec 1 W, Jan 22

Math 320
Linear Algebra + Differential Equations
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In this course we will begin with diff eq's
then spend time on linear algebra
Then use this lin algebra to solve some deeper
problems in diff eq's

1.1 Differential equations and mathematical models

Here is a typical problem in diff eq's

Problem Find the general solution
to the equation

$$y' = 2y$$

Meaning

- It is assumed that y is a function

$$y = f(x)$$

that is defined on the real numbers $\mathbb{R}$

[or perhaps some interval in $\mathbb{R}$]

- y' means the derivative of y with respect to x:

$$y' = \frac{dy}{dx} = f'(x)$$

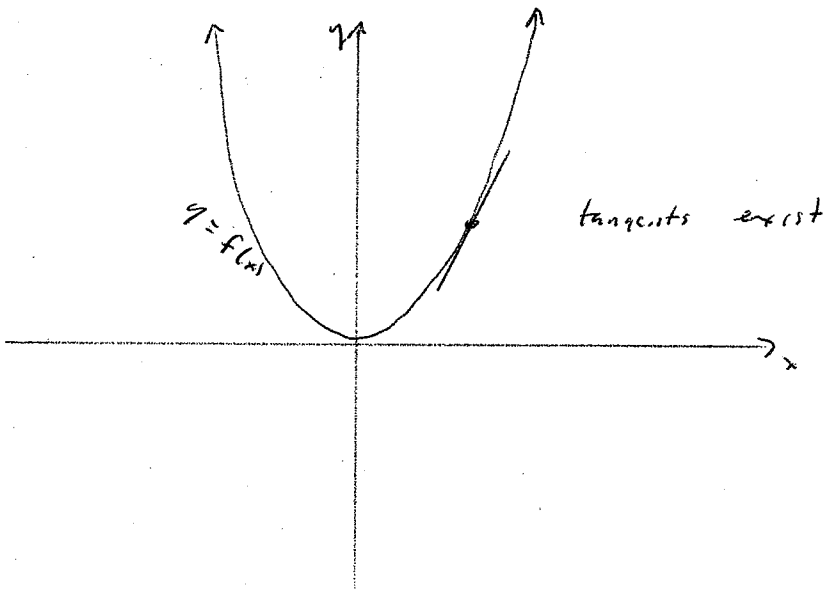
Recall the meaning: For all x the value $f'(x)$ is the slope of the line tangent to the graph of f at x

- It is assumed that $f'(x)$ exists for all x where $f(x)$ is defined "y is differentiable"

So the graph of $y = f(x)$ is "smooth."

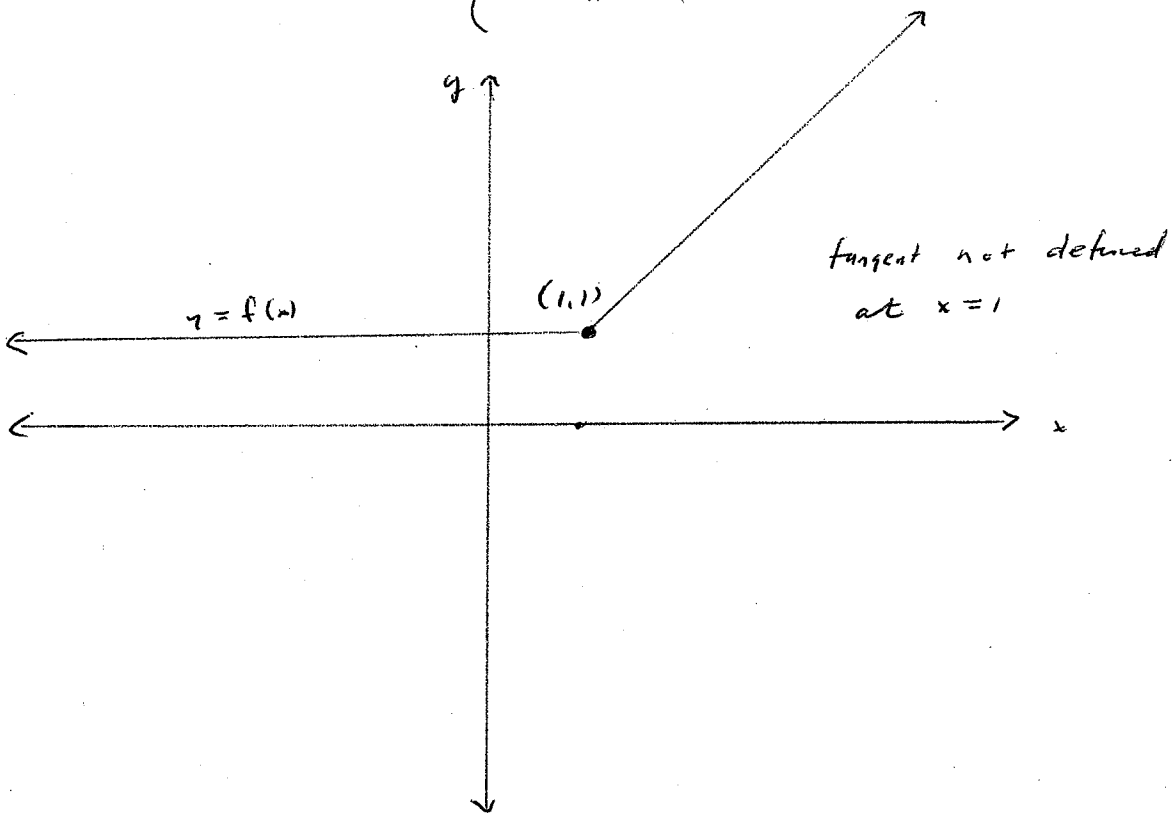
Ex this function is differentiable:

$f(x) = x^2$ $\quad x \in \mathbb{R}$



This function is not differentiable:

$$f(x) = \begin{cases} 1 & \text{if } x \leq 1 \\ x & \text{if } x \geq 1 \end{cases} \qquad x \in \mathbb{R}$$



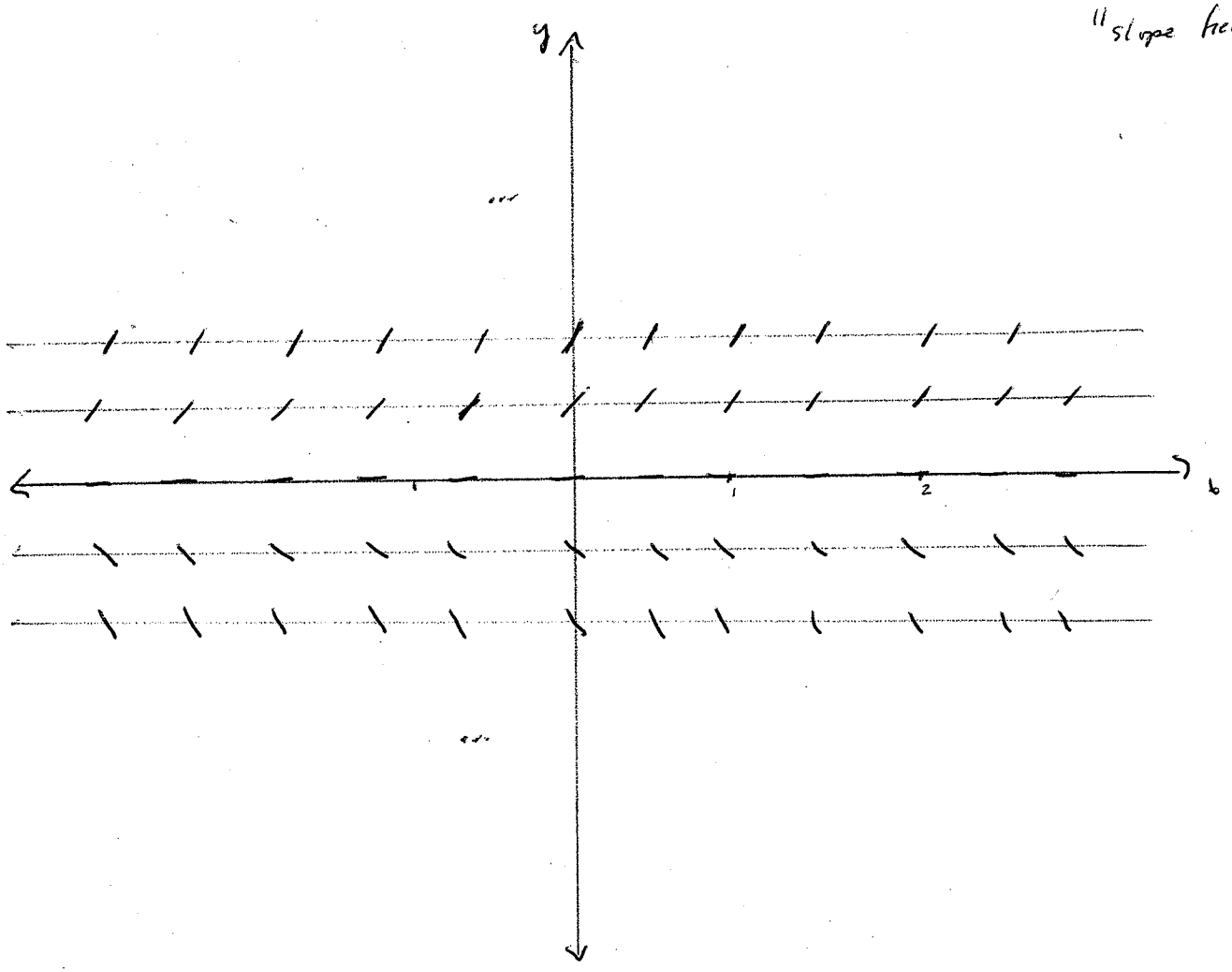
- We seek all the functions $y = f(x)$ that satisfy

$$y' = 2y \qquad *$$

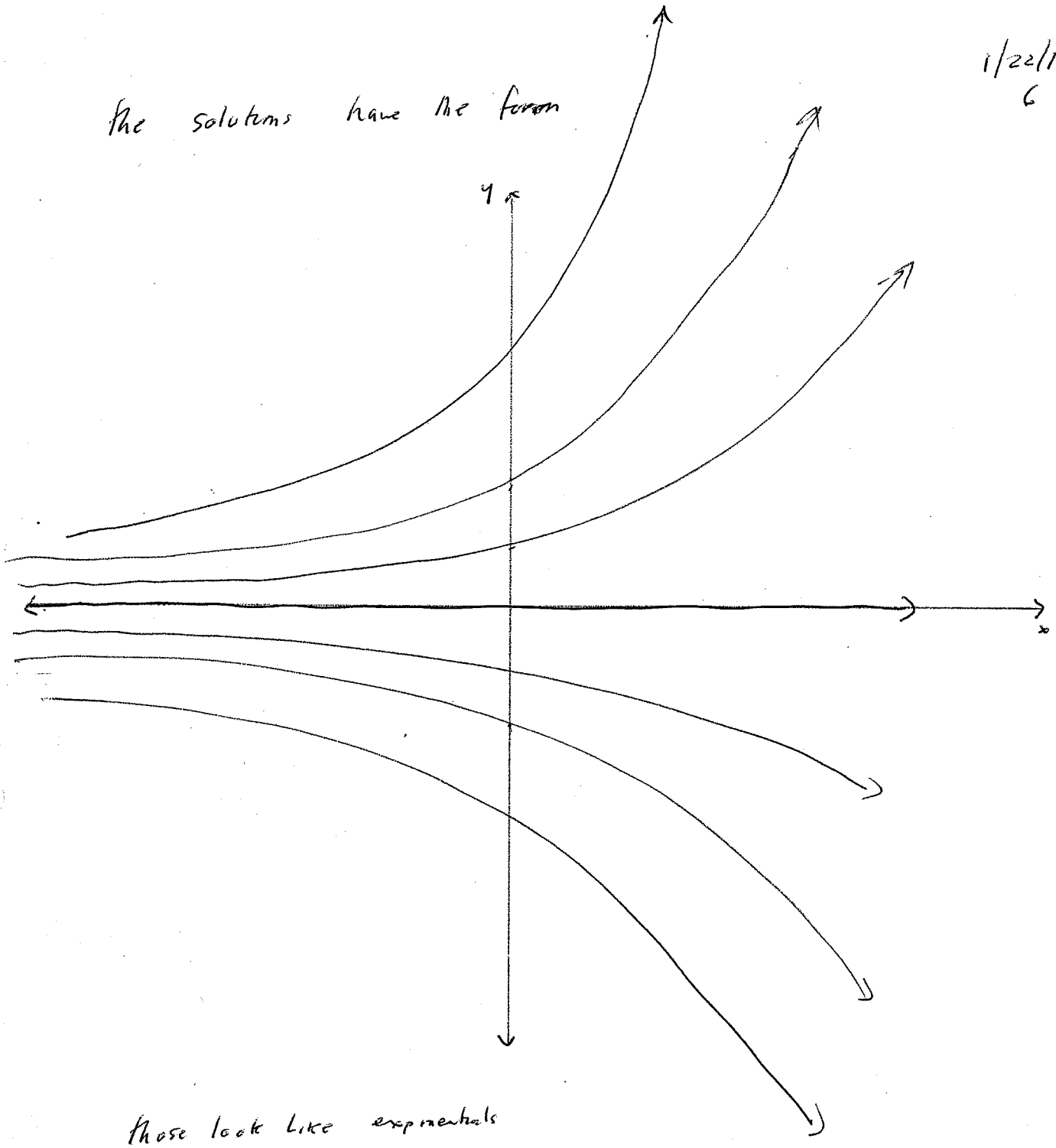
Eq $*$ tells us the slopes!

- slope is 0 on x-axis $(y=0)$
- slope is 1 at $y = \frac{1}{2}$
- " " -1 $y = -1/2$
- " " 2 $y = 1$
- " " -2 $y = -1$
- " " " "

"slope field"



the solutions have the form



those look like exponentials

try

$$f(x) = Ce^{kx}$$ k, C = const

From calc

$$f'(x) = Ce^{kx}k$$

$$= kf(x)$$

Require $k = 2$

Gen sol is

$$y = Ce^{2x} \qquad C \in \mathbb{R}$$

It is a <u>1-parameter family</u> of solutions

The parameter is C.

A member of the family is called a <u>particular solution</u>

<u>Problem</u> Solve the diff equation

$$y' = 2y$$

"initial condition"

$$y(0) = 3$$

"initial value problem"

Sol: Gen sol is

$$y = Ce^{2x}$$

Find C such that $y(0) = 3$

$$3 = y(0) = Ce^{2\cdot 0} = C$$

the particular sol

$$y = 3e^{2x}$$

is the answer

□

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Examples of diff equations

	order
$x y' + x^2 + 1 = 0$	1
$y' = \frac{1}{x^2}$	1
$y' + 2y^3 = 0$	1
second derivative → $y'' + 5y' + 4y = 0$	2
$xy'' + y^3 + x = 0$	2
$y^5 y^{(4)} + x^3 + 1 = 0$ ↑ y''''	4

the order of a diff equation is the order of the highest derivative that appears in it

Math models

Given a real world or geometric situation

- formulate the problem mathematically
- Solve the math problem
- interp the solution

Problem For a colony of bacteria the time-rate of change of the population is proportional to the population. Describe the population growth over time

Sol. Let $P(t)$ = population of bacteria at time t

Require

$$P'(t) = k P(t)$$

k = constant

Similar to first problem, the general solution is

$$P(t) = Ce^{kt} \qquad C = \text{cnst} \qquad k = \text{cnst}$$

Of course

$$P(t) \geq 0 \qquad \forall t$$

so

$$C \geq 0$$

— o —

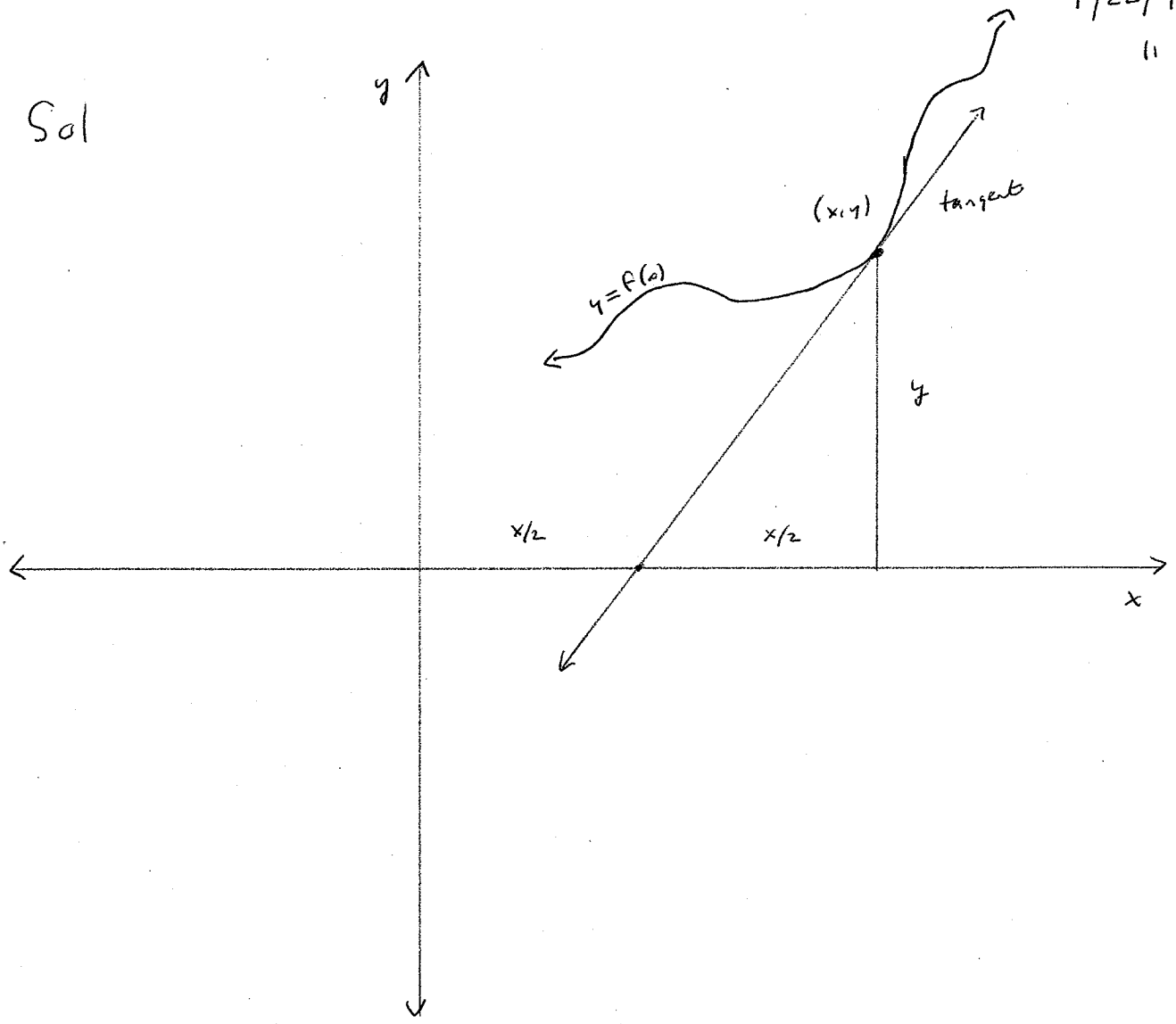
Problem Given a differentiable function

$$y = f(x) \qquad x \in \mathbb{R}$$

Assume that the line tangent to the graph $y = f(x)$ at each pt (x, y) intersects the x-axis at $\left(\frac{x}{2}, 0\right)$

Find $f(x)$

Sol



Require

$$y' = \text{slope of tangent}$$

$$= \frac{y}{x/2}$$

$$= \frac{2y}{x}$$

so

$$xy' = 2y \qquad x \in \mathbb{R}$$

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Using the "slope field" method one sees that the general solution is

$$y = Cx^2 \qquad C = \text{const}$$