HW Section 9.3

No. 9.3

Date 1

Problem	Ans	Reason

1

X = monthly payment

Monthly interest rate $.09/12$

payments $= 12$

#Mo elapsed	payment	present value
1	X	$\frac{X}{a}$
2	X	$\frac{X}{a^2}$
⋮	⋮	⋮
12	X	$\frac{X}{a^{12}}$

$a = 1 + \frac{.09}{12}$

5000 = sum of present value of payments

$$= \frac{X}{a^{12}}\left(1 + a + a^2 + \cdots + a^{11}\right)$$

$$= \frac{X}{a^{12}} \quad \frac{a^{12}-1}{a-1}$$

$$X = 5000\ a^{12}\ \frac{a-1}{a^{12}-1}$$

$$= 5000\left(1 + \frac{.09}{12}\right)^{12} \frac{.09/12}{\left(1+\frac{.09}{12}\right)^{12} - 1}$$

$$\left[\ X \simeq 437.26\ \right]$$

Problem	Ans	Reason

2

X = monthly payment

Monthly interest rate $\frac{.075}{12}$

payments $= 3 \cdot 12 = 36$

$$10\,000 = \frac{X}{a^{36}}\left(1 + a + \cdots + a^{35}\right) \qquad a = 1 + \frac{.075}{12}$$

$$X = 10\,000\left(1 + \frac{.075}{12}\right)^{36} \frac{.075/12}{\left(1 + \frac{.075}{12}\right)^{36} - 1}$$

$$\left[\; X \simeq 311.06 \;\right]$$

3

X = quarterly payment

Quarterly interest rate $\frac{.085}{4}$

payments $= 5 \cdot 4 = 20$

$$X = 50\,000\left(1 + \frac{.085}{4}\right)^{20} \cdot \frac{.085/4}{\left(1 + \frac{.085}{4}\right)^{20} - 1}$$

$$\left[\; X \simeq 3094.85 \;\right]$$

Problem	Ans	Reason

4

X = quarterly payment

Quarterly interest rate $.07/4$

payments $= 6 \cdot 4 = 24$

$$X = 150\,000\left(1+\frac{.07}{4}\right)^{24}\frac{.07/4}{\left(1+\frac{.07}{4}\right)^{24}-1}$$

$$\left[\; X \simeq 7707.85 \;\right]$$

5

X = yearly payment

int rate $.065$

payments $= 10$

$$X = 20\,000\left(1+.065\right)^{10}\frac{.065}{\left(1+.065\right)^{10}-1}$$

$$\left[\; X \simeq 2782.09 \;\right]$$

Problem	Ans	Reason

6

X = yearly payment

int rate .085

payments = 15

$$X = 60\,000 \left(1.085 \right)^{15} \frac{.085}{\left(1.085 \right)^{15} - 1}$$

$$\left[X \simeq 7225.23 \right]$$

7

X = quarterly payment

quarterly interest rate $.08/4$

payments $= 2 \cdot 4 = 8$

$$X = 25000 \left(1 + \frac{.08}{4} \right)^{8} \frac{.08/4}{\left(1 + \frac{.08}{4} \right)^{8} - 1}$$

$$\left[X \simeq 3412.74 \right]$$

Problem | Ans | Reason

8

type of payment	total amount paid		total amt paid (approx)
quarterly	$25000\left(1+\frac{.08}{4}\right)^8 \frac{\frac{.08}{4}}{\left(1+\frac{.08}{4}\right)^8-1}$	8	273019.60
S. annual	$25000\left(1+\frac{.08}{2}\right)^4 \frac{.08/2}{\left(1+\frac{.08}{2}\right)^4-1}$	4	275420.05
yearly	$25000\left(1.08\right)^2 \frac{.08}{(1.08)^2-1}$	2	280384.62

9

P = # dollars in Loan

5 year:

# months elapsed	payment	present value
1	2000	$\frac{2000}{a}$
2	2000	$\frac{2000}{a^2}$
⋮	⋮	
60	2000	$\frac{2000}{a^{60}}$

$12 \cdot 5 = 60$

$a = 1 + \frac{.09}{12}$

$$P = \frac{2000}{a^{60}}\left(1+a+a^2+\cdots+a^{59}\right)$$

$$= \frac{2000}{a^{60}} \frac{a^{60}-1}{a-1}$$

$$= \frac{2000}{\left(1+\frac{.09}{12}\right)^{60}} \frac{\left(1+\frac{.09}{12}\right)^{60}-1}{\frac{.09}{12}} \quad \left[\tilde{=} 96346.75\right]$$

Problem

9. cnt

10 year amortization

$$P = \frac{2000}{\left(1 + \frac{.09}{12}\right)^{120}} \frac{\left(1 + \frac{.09}{12}\right)^{120} - 1}{.09/12}$$

$$\left[\simeq 157\,883.39 \right]$$

10 skip

too vague

Problem	Ans	Reason

11

X = # dollars in each monthly payment

monthly int rate $\frac{.072}{12}$

payments $= 4 \cdot 12 = 48$

# months elapsed	payment	present value
0	2000	2000
1	X	$\frac{X}{a}$
2	X	$\frac{X}{a^2}$
⋮	⋮	⋮
48	X	$\frac{X}{a^{48}}$

$a = 1 + \frac{.072}{12}$

$$9500 = 2000 + \frac{X}{a^{48}}\left(1 + a + \cdots + a^{47}\right)$$

$$7500 = \frac{X}{a^{48}} \quad \frac{a^{48}-1}{a-1}$$

$$X = 7500\, a^{48} \frac{a-1}{a^{48}-1}$$

$$= 7500\left(1+\frac{.072}{12}\right)^{48} \frac{\frac{.072}{12}}{\left(1+\frac{.072}{12}\right)^{48}-1}$$

$\left[\quad \underline{?} \quad 180.29 \quad \right]$ total payment on Loan is

$180.29 \times 48 = 8653.92$

7500 principal $8653.92 - 7500 = 1153.92$ interest

Problem	Ans	Reason
12		

Monthly payment is

$$9000\left(1+\frac{.072}{12}\right)^{48}\frac{\frac{.072}{12}}{\left(1+\frac{.072}{12}\right)^{48}-1}$$

$$\left[\simeq 216.35\right]$$

total payment on Loan is $216.35 \times 48 = 10384.91$

9000 principal

$10384.91 - 9000 = 1384.91$ interest

Problem	Ans	Reason

13

Monthly int rate $.078/12$ X = monthly payment

payments $= 5 \cdot 12 = 60$

#months elapsed	payment	present value
0	2000	2000
1	X	$\frac{X}{a}$
2	X	$\frac{X}{a^2}$
:	:	:
60	X	$\frac{X}{a^{60}}$

$a = 1 + \frac{.078}{12}$

$$9500 = 2000 + \frac{X}{a^{60}}\left(1 + a + a^2 + \cdots + a^{59}\right)$$

$$7500 = \frac{X}{a^{60}} \frac{a^{60}-1}{a-1}$$

$$X = 7500\, a^{60} \frac{a-1}{a^{60}-1}$$

$$= 7500\left(1 + \frac{.078}{12}\right)^{60} \frac{\frac{.078}{12}}{\left(1 + \frac{.078}{12}\right)^{60} - 1}$$

$$\left[\; X \doteq 151.36 \;\right]$$

Problem	Ans	Reason
14		

For $10000 over 10 years

Monthly payment is

$$10000\left(1+\frac{.06}{12}\right)^{120}\frac{\frac{.06}{12}}{\left(1+\frac{.06}{12}\right)^{120}-1}$$

$$\left[\simeq 111.02\right]$$

For $20000 over 25 years

Monthly payment is

$$20000\left(1+\frac{.06}{12}\right)^{300}\frac{.06/12}{\left(1+\frac{.06}{12}\right)^{300}-1}$$

$$\left[\simeq 128.86\right]$$

2nd payment is Larger

Problem	Ans	Reason
15		

X = monthly payment

Loan for $\$75000$

Monthly interest $\frac{.068}{12}$

payments $25 \cdot 12 = 300$

$$X = 75000 \left(1 + \frac{.068}{12} \right)^{300} \Big/ \frac{\frac{.068}{12}}{\left(1 + \frac{.068}{12} \right)^{300} - 1}$$

$$\left[\simeq 520.55 \right]$$

No. 9.3

Date 12

Problem	Ans	Reason

16

(a) Monthly payment

$$80000\left(1+\frac{.06}{12}\right)^{180}\frac{\frac{.06}{12}}{\left(1+\frac{.06}{12}\right)^{180}-1}$$

$$\left[\simeq 675.09\right]$$

Total interest is $675.09 \quad 180 - 80\,000$

$$= 41515.38$$

(b) Monthly payment is

$$80000\left(1+\frac{.066}{12}\right)^{240}\frac{\frac{.066}{12}}{\left(1+\frac{.066}{12}\right)^{240}-1}$$

$$\left[\simeq 601.18\right]$$

Total interest is 64282.63

(c) Monthly payment is

$$80000\left(1+\frac{.072}{12}\right)^{300}\frac{\frac{.072}{12}}{\left(1+\frac{.072}{12}\right)^{300}-1}$$

$$\left[\simeq 575.67\right]$$

Total interest is 92701.29

Problem	Ans	Reason

17

Monthly payment $180

Monthly rate $\frac{.087}{12}$

N = # months

# months elapsed	payment	present value
1	180	$\frac{180}{a}$
2	180	$\frac{180}{a^2}$
⋮	⋮	⋮
N	180	$\frac{180}{a^N}$

$a = 1 + \frac{.087}{12}$

$$10000 = \frac{180}{a^N}\left(1 + a + a^2 + \cdots + a^{N-1}\right)$$

$$= \frac{180}{a^N}\,\frac{a^N - 1}{a - 1}$$

$$= 180\,\frac{1 - a^{-N}}{a - 1}$$

$$\frac{1000}{18}(a-1) = 1 - a^{-N}$$

$$a^{-N} = 1 - \frac{1000}{18}(a-1)$$

$$a^N = \frac{1}{1 - \frac{1000}{18}\left(\frac{.087}{12}\right)}$$

test values of N $\quad N = 72$

Problem	Ans	Reason

18

P = price of Condo

Option 1 Loan for 80% value of condo

$P - 12000$ = sum of present value of payments

# months elapsed	payment	present value
1	1000	$\frac{1000}{a}$
2	1000	$\frac{1000}{a^2}$
⋮	⋮	⋮
240	1000	$\frac{1000}{a^{240}}$

$a = 1 + \frac{.084}{12}$

$$P - 12000 = \frac{1000}{a^{240}} \frac{a^{240} - 1}{a - 1}$$

$$P = 12000 + \frac{1000}{\left(1 + \frac{.084}{12}\right)^{240}} \frac{\left(1 + \frac{.084}{12}\right)^{240} - 1}{\frac{.084}{12}}$$

$$\left[\approx 128076.00 \right]$$

Also 12000 is at least 20% of P

$$12000 \geq (.2)P$$

so $$60000 \geq P$$

Here $P = 60000$ is Max

Problem	Ans	Reason
18 cont	Option 2 Loan for 90% value of condo	

$$P = 12000 + \frac{1000}{\left(1+\frac{.09}{12}\right)^{240}} \frac{\left(1+\frac{.09}{12}\right)^{240}-1}{\frac{.09}{12}}$$

$$\left[\approx 123144.95 \right]$$

Also

$$(.2)P \geq 12000 \geq (.1)P$$

$$60000 \leq P \leq 120000$$

$P = 120000$ is Max.

Problem

19 (a) Find interest over 5 years on 70 000

$$70\,000\left(\left(1+\frac{.10}{12}\right)^{60}-1\right)$$

X = monthly payment to handle this over 5 years

Require present value of interest = sum of present values of payments

$$\frac{70\,000\left(\left(1+\frac{.10}{12}\right)^{60}-1\right)}{\left(1+\frac{.10}{12}\right)^{60}}$$

$$= \frac{X}{\left(1+\frac{.10}{12}\right)^{60}} \cdot \frac{\left(1+\frac{.10}{12}\right)^{60}-1}{\frac{.10}{12}}$$

$$X = \frac{.10}{12}\ 70000$$

$$= \frac{7000}{12} \quad \underline{\sim}\ 583.33$$

(b)

$$\frac{70000\left(\left(1+\frac{.10}{12}\right)^{60}-1\right)+20000}{\left(1+\frac{.10}{12}\right)^{60}}$$

$$= X \cdot \frac{\left(1+\frac{.10}{12}\right)^{60}-1}{\frac{.10}{12}}$$

Problem	Ans	Reason
19 cnt	(6) $X = \dfrac{70000\left(\left(1+\frac{.10}{12}\right)^{60}-1\right)}{\left(1+\frac{.10}{12}\right)^{60}-1} + 20000 \quad \frac{.10}{12}$	
	$[X \approx 841.61]$	
20		

rate	Monthly payment
6%	$50000\left(1+\frac{.06}{12}\right)^{360}\frac{\frac{.06}{12}}{\left(1+\frac{.06}{12}\right)^{360}-1}$ ≈ 299.78
7%	$50000\left(1+\frac{.07}{12}\right)^{360}\frac{\frac{.07}{12}}{\left(1+\frac{.07}{12}\right)^{360}-1}$ ≈ 332.65
8%	$50000\left(1+\frac{.08}{12}\right)^{360}\frac{\frac{.08}{12}}{\left(1+\frac{.08}{12}\right)^{360}-1}$ ≈ 366.88

Problem	Ans	Reason
21	(a) Monthly payment is $5000\left(1+\frac{.18}{12}\right)^{24}\frac{\frac{.18}{12}}{\left(1+\frac{.18}{12}\right)^{24}-1}$ $\left[\pm \quad 249.62\right]$	
	(b) $24 \times$ Monthly payment $=$ 5990.89	
	(c) Monthly payment is $5000\left(1+\frac{.18}{12}\right)^{12}\frac{\frac{.18}{12}}{\left(1+\frac{.18}{12}\right)^{12}-1}$ $\left[\pm \quad 458.40\right]$	
	$12 \times$ Monthly payment $=$ 5500.80	

oblem | Ans | Reason

22

X = 5. annual payt

# ½years elapsed	payment	amount-value of payt in 5 years
1	X	$X\left(1+\frac{.065}{2}\right)^9$
2	X	$X\left(1+\frac{.065}{2}\right)^8$
⋮	⋮	⋮
10	X	X

$$80000 = X\left(1+a+a^2+\cdots+a^9\right) \qquad a = 1+\frac{.065}{2}$$

$$= X\,\frac{a^{10}-1}{a-1}$$

$$X = 80000\,\frac{a-1}{a^{10}-1}$$

$$= 80000\,\frac{.065/2}{\left(1+\frac{.065}{2}\right)^{10}-1}$$

$$\left[\ \approx 6898.49\ \right]$$

Problem	Ans	Reason
23		Amount to be paid in 3 years is $500\,000\,(1.09)^3$
		X = quarterly payment into sinking fund
		$X = 500\,000\,(1.09)^3 \dfrac{.06/4}{\left(1+\frac{.06}{4}\right)^{12}-1}$
	[≈ 49651.41]	
24		$X = 500\,000\,(1.09)^3 \dfrac{.06/2}{\left(1+\frac{.06}{2}\right)^{6}-1}$
	[≈ 100104.12]	

Problem	Ans	Reason
25		

quarterly int rate is $\frac{.08}{4} = .02$

payments $= 8 \cdot 4 = 32$

X = quarterly payment

$$X = 30\,000 \frac{.02}{(1.02)^{32} - 1}$$

$$[\ \pm \quad 678.32\ ]$$

oblem

26

Consider value A of the fund on the day Diane begins college 8 years from now

	# years elapsed	payment	value 8 years from now
Diane	8	3000	3000
	9	⋮	$\frac{3000}{(1.02)^4}$
	10	⋮	$\frac{3000}{(1.02)^8}$
	11	⋮	⋮
David	12	3000	⋮
	13	⋮	
	14	⋮	
	15	⋮	$\frac{3000}{(1.02)^{28}}$
		⋮	

Require

$$A = \frac{3000}{a^7}\left(1 + a + a^2 + \cdots + a^7\right) \qquad a = (1.02)^4$$

$$= \frac{3000}{a^7}\,\frac{a^8 - 1}{a - 1} = \frac{3000}{(1.02)^{28}}\,\frac{(1.02)^{32} - 1}{(1.02)^4 - 1}$$

Let X = # dollars the parents pay each quarter

As in #25 $\qquad X = A\,\frac{.02}{(1.02)^{32} - 1}$

$$X = \frac{3000}{(1.02)^{28}}\,\frac{.02}{(1.02)^4 - 1} \qquad \left[\approx 418.07\right]$$

oblem

27

N = # periods

# ½ year elapsed	paymt	amount after N periods
1	6000	$6000(1.05)^{N-1}$
2	6000	$6000(1.05)^{N-2}$
⋮	⋮	⋮
N	6000	6000

Require

$$100\,000 = 6000\left(1+a+a^2+\cdots+a^{N-1}\right) \quad a=1.05$$

$$= 6000\,\frac{a^N-1}{a-1}$$

$$\frac{100}{6} = \frac{a^N-1}{a-1}$$

$$\frac{(a-1)100}{6} = a^N-1$$

$$1+\frac{(.05)100}{6} = (1.05)^N$$

$$\frac{11}{6} = (1.05)^N$$

$$\boxed{N = 13}$$

Problem

28 skip

vague

29

Let X = # dollars deposited each year

# years elapsed	payment	amount value in 5 years
1	X	$X(1.08)^4$
2		$X(1.08)^3$
3		$X(1.08)^2$
4		$X(1.08)^1$
5		X

Require

$$2000\,000 = X(1+a+a^2+a^3+a^4) \qquad a = 1.08$$

$$= X\,\frac{a^5-1}{a-1}$$

$$X = 2000\,000\,\frac{a-1}{a^5-1}$$

$$= 2000\,000\,\frac{.08}{(1.08)^5-1}$$

[2 340 912,91]

Problem

30

Compare value after 10 years

cost / profit

Let R = rate of interest

Cost:

$$800\,000(1+R)^{10}$$

Profit:

# years elapsed	profit	value of profit after 10 years
6	200 000	$200\,000(1.06)^4$
7		$200\,000(1.06)^3$
8		⋮
9		
10		200 000

$$\text{Profit} = 200\,000\,(1 + a + a^2 + \cdots + a^4) \qquad a = 1.06$$

$$= 200\,000\,\frac{a^5 - 1}{a - 1}$$

$$= 200\,000\,\frac{(1.06)^5 - 1}{.06}$$

Require $\quad 800\,000(1+R)^{10} = 200\,000\,\dfrac{(1.06)^5 - 1}{.06}$

Get $\quad R = \left(\frac{1}{4}\,\dfrac{(1.06)^5 - 1}{.06}\right)^{\frac{1}{10}} - 1$

[$R \doteq .0349$ ie 3.49%]