Section 9.1 HW SOLUTIONS

Problem		Ans	Reason
1	(a)	$100	$(.05)(2000) = 100.00$
	(b)	$120	$(.03)(4000) = 120.00$
2	(a)		$2000\left(\left(1+\frac{.05}{2}\right)^{2}-1\right)$
	(b)		$5000\left(\left(1+\frac{.06}{4}\right)^{4}-1\right)$

Problem	Ans	Reason
3	(a) $200\left(\left(1.01\right)^{8}-1\right)$	
	(b) $10000\left(\left(1.0125\right)^{20}-1\right)$	
4	(a) $500\left(\left(1.0225\right)^{4}-1\right)$	
	(b) $1000\left(\left(1.01625\right)^{16}-1\right)$	

$$4\overline{)6.5}\quad 1.625$$
$$\begin{array}{r} 4 \\ \hline 2.5 \\ 2.4 \\ \hline 10 \\ 8 \\ \hline 20 \end{array}$$

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Freq	init	Formula for P_n
Annual	1000	$1000(1.04)^{10}$
Semi An.	1000	$1000(1.02)^{20}$
Quart	1000	$1000(1.01)^{40}$
monthly	1000	$1000\left(1+\frac{.04}{12}\right)^{120}$

Problem | Ans | Reason

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Freq	Init	P_n
Annual	1000	$1000(1.075)^8$
S. Annual	1000	$1000(1.0375)^{16}$
Quart	1000	$1000(1.01875)^{32}$
Monthly	1000	$1000(1.00625)^{96}$

7 Profit of $13949

$$\text{Income} = 175\,000 - 100\,000$$
$$= 75\,000$$

$$\text{Expenses} = 100\,000\left((1.1)^5 - 1\right)$$

Observe

$$(1.1)^5 = (1 + .1)^5$$
$$= 1 + 5(.1) + 10(.1)^2 + 10(.1)^3 + 5(.1)^4 + (.1)^5$$
$$= 1 + .5 + .1 + .01 + .0005 + .00001$$
$$= 1.61051$$

$$100\,000\left((1.1)^5 - 1\right) = 100\,000\,(.61051)$$
$$= 61\,051 < 75000$$

She makes a profit of $75000 - 61051 = 13949$

Problem	Ans	Reason
8	$(1.85)^{1/5} - 1 \simeq .131$	

Let r denote the maximal annual interest rate

Require

$$100\,000\left((1+r)^5 - 1\right) - 75000 = 10\,000$$

$$100\,000\left((1+r)^5 - 1\right) = 85000$$

$$(1+r)^5 - 1 = .85$$

$$(1+r)^5 = 1.85$$

$$1+r = (1.85)^{1/5}$$

$$r = (1.85)^{1/5} - 1$$

Problem	Ans	Reason
9	Loss of about $ 33 000	

$$\text{Income} = 250\,000 - 100\,000$$
$$= 150\,000$$

$$\text{Expenses} = 100\,000\left((1.11)^{10} - 1\right)$$

$$\text{Profit} = \text{Income} - \text{Expenses}$$
$$= 100\,000\left(2.5 - (1.11)^{10}\right)$$

Observe

$$(1.11)^{10} = (1 + .11)^{10}$$
$$= 1 + 10(.11) + C(10,2)(.11)^2 + C(10,3)(.11)^3 + \cdots$$
$$= 1 + 1.1 + .5445 + .15972 + \cdots$$
$$> 2.5$$

Loss

$$1 + 1.1 + .5445 + .15972 - 2.5 \simeq .33$$

$$100\,000\,(.33) = 33\,000.$$

Will Loose approx $ 33000

Problem	Ans	Reason
10	Metro	

Principal after 5 years

Metro	4% NatL
$1000(1.0435)^5$	$1000\left(1+\frac{.042}{4}\right)^{20}$

Which is Larger?

$(1.0435)^5$ $\qquad$ $\left(1+\frac{.042}{4}\right)^{20}$

Obs

$$\left(1+\frac{.042}{4}\right)^{20} = \left(\left(1+\frac{.042}{4}\right)^4\right)^5$$

Which is Larger?

1.0435 $\qquad$ $\left(1+\frac{.042}{4}\right)^4$

$\frac{.042}{4} = .0105$

$(1+.0105)^4 =$

$$1 + 4(.0105) + 6(.0105)^2 + 4(.0105)^3 + (.0105)^4$$

$$\approx 1.0426 \quad < \quad 1.0435$$

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Problem	Ans	Reason
11	Account 2 is more profitable	Principal after 5 years

Account 1	Account 2
$10000\left(1+\frac{.06}{365}\right)^{365\cdot 5}$	$10000\left(1+\frac{.0625}{4}\right)^{4\cdot 5}$

Which is Larger ?

$$\left(1+\frac{.06}{365}\right)^{365} \qquad \left(1+\frac{.0625}{4}\right)^{4}$$

$$\left(1+\frac{.06}{365}\right)^{365} \simeq 1.0618$$

$$\left(1+\frac{.0625}{4}\right)^{4} \simeq 1.06398$$

Problem	Ans	Reason

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After 10 years yield is

$$1000\left(1+\frac{.06}{365}\right)^{3650}$$

$$= 1000\left(1+3650\,\frac{.06}{365}+C(3650,2)\left(\frac{.06}{365}\right)^2+\cdots\right)$$

$$\approx 1000\left(1+.6+\frac{(3650)^2}{2}\frac{(.06)^2}{(365)^2}+\frac{(3650)^3}{3!}\frac{(.06)^3}{365^3}+\cdots\right)$$

$$\approx 1000\left(1+.6+\frac{(.6)^2}{2!}+\frac{(.6)^3}{3!}+\frac{(.6)^4}{4!}+\frac{(.6)^5}{5!}\right.$$

$$\approx 1000\left(1+.6+.18+.036+.0054+.000648\right.$$

$$\approx 1822$$

Problem	Ans	Reason
13	(a) $100\,000\left(1+\frac{.08}{360}\right)^{360}$	
	(b) $100\,000\left(1+\frac{.08}{365}\right)^{365}$	
14	WLOG initial deposit is 1 dollar	
	After 9 years yield is	
	$\left(1+\frac{.08}{4}\right)^{36} = 1 + 36\left(\frac{.08}{4}\right) + c(36,2)\left(\frac{.08}{4}\right)^2 + \cdots$	
	$\geq 1 + .72 + \frac{(.72)^2}{2} + \frac{(.72)^3}{3!} + \cdots$	
	$= 1 + .72 + .2592 + .062208 + \cdots$	
	≥ 2.0414	
	> 2	

Problem	Ans	Reason

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Old annual percentage yield is

$$\left(1 + \frac{.05}{4}\right)^4 - 1$$

New annual percentage yield is

$$\left(1 + \frac{.055}{4}\right)^4 - 1$$

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Call the desired rate R. Require

$$.045 = \left(1 + \frac{R}{4}\right)^4 - 1$$

$$R = 4\left((1.045)^{1/4} - 1\right)$$

R is a bit less than .045

$R \simeq .04426$

Problem	Ans	Reason
17	$(1.75)^{\frac{1}{15}} - 1$	$80\,000\left((1+r)^{15} - 1\right) = 60\,000$
		$(1+r)^{15} - 1 = \frac{6}{8}$
		$(1+r)^{15} = \frac{14}{8} = \frac{7}{4}$
		$r = \left(\frac{7}{4}\right)^{1/15} - 1$
18	$\left(\frac{17}{8}\right)^{\frac{1}{20}} - 1$	$80\,000\left((1+R)^{20} - 1\right) = 90{,}000$
		$R = \left(1 + \frac{9}{8}\right)^{\frac{1}{20}} - 1$
		$= \left(\frac{17}{8}\right)^{\frac{1}{20}} - 1$

Problem	Ans	Reasm

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$$\left(\left(2000\left(1+\frac{.044}{4}\right)^2-500\right)\left(1+\frac{.044}{4}\right)^2-500\right)\left(1+\frac{.044}{4}\right)^2$$

$$=\ 2000\left(1+\frac{.044}{4}\right)^6-500\left(1+\frac{.044}{4}\right)^4-500\left(1+\frac{.044}{4}\right)^2$$

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$$\left(\left(\left(5000\left(1+\frac{.04}{4}\right)^4-500\right)\left(1+\frac{.04}{4}\right)^2-1000\right)\left(1+\frac{.04}{4}\right)^8$$

$$=\ 5000\left(1+\frac{.04}{4}\right)^{14}-500\left(1+\frac{.04}{4}\right)^{10}-1000\left(1+\frac{.04}{4}\right)^8$$

21 (a)

$$20000\left(1+\frac{.045}{4}\right)^{48}$$

(b)

$$\left(\left(\left(20000\left(1+\frac{.045}{4}\right)^{48}-5000\right)\left(1+\frac{.045}{4}\right)^4-5000\right)\left(1+\frac{.045}{4}\right)^4$$

$$-5000\left(1+\frac{.045}{4}\right)^4-5000$$

$$=\ 20000\left(1+\frac{.045}{4}\right)^{60}-5000\left(1+\frac{.045}{4}\right)^{12}-5000\left(1+\frac{.045}{4}\right)^8$$

$$-5000\left(1+\frac{.045}{4}\right)^4-5000$$

Problem	Ans	Reason

22 Let D = # dollars deposited now

require $D\left(1+\frac{.05}{4}\right)^{9\cdot 14} = 30\,000$

$$D = \frac{30\,000}{\left(1+\frac{.05}{4}\right)^{56}}$$

23 Let D = # dollars deposited now

$$D = \frac{30\,000}{\left(1+\frac{.05}{4}\right)^{56}} + \frac{1000}{\left(1+\frac{.05}{4}\right)^{72}}$$

24 $$\frac{7000}{\left(1+\frac{.05}{4}\right)^{56}} + \frac{7300}{\left(1+\frac{.05}{4}\right)^{60}} + \frac{7700}{\left(1+\frac{.05}{4}\right)^{64}} + \frac{8000}{\left(1+\frac{.05}{4}\right)^{68}}$$

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Problem	Ans	Reason
25	Yield after 5 years $1000(1+.05)(1+.04)(1+.05)(1+.05)(1+.06)$	
	$= 1000(1+r)^5$	r = annual percentage yield expressed as a decimal
	$r = \left((1.04)(1.05)^3(1.06)\right)^{1/5} - 1$	
26	$\left(\left(1+\frac{.05}{4}\right)^{16}\left(1+\frac{.06}{4}\right)^{16}\left(1+\frac{.07}{4}\right)^{8}\right)^{\frac{1}{40}} - 1$	
27	$\left((1.1)(.9)\right)^{\frac{1}{2}} - 1 \approx -.005$	

Problem	Ans	Reason

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r = effective yearly rate of increase

Require

$$275(1+r)^5 = 310$$

$$r = \left(\frac{310}{275}\right)^{\frac{1}{5}} - 1$$

Problem	Ans	Reason
29	Income is	

$$200 \qquad \text{(from stock sale)}$$

$$+ \quad 100\left(1+\frac{.04}{365}\right)^{212}$$

$$+ \quad 100\left(1+\frac{.04}{365}\right)^{122}$$

$$+ \quad 100\left(1+\frac{.04}{365}\right)^{30}$$

Mo	#days
Dec	31
Jan	31
Feb	28
March	31
Ap	30
May	31
June	30

r = annual percentage yield.

$$2000\left(\left(1+r\right)^{3/4}-1\right) = \text{above income}$$

$$r = \left(1+\frac{200+100\left(1+\frac{.04}{365}\right)^{212}+100\left(1+\frac{.04}{365}\right)^{122}+100\left(1+\frac{.04}{365}\right)^{30}}{2000}\right)^{4/3}$$

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Problem	Ans	Reason
30	$r = \left(1 + \frac{-200 + 100\left(1+\frac{.04}{365}\right)^{212} + 100\left(1+\frac{.04}{365}\right)^{122} + 100\left(1+\frac{.04}{365}\right)^{30}}{2000}\right)^{4/3}$	