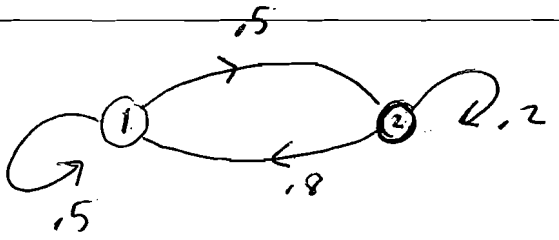
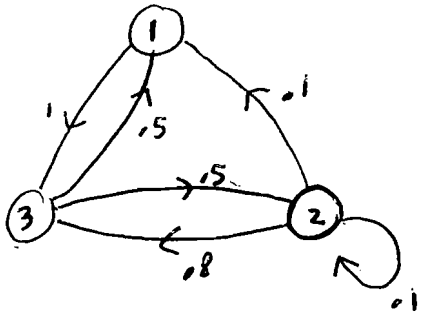
8.R Solutions for 1-12

Problem	Ans	Reason
1	(a) 	
	(b) .44	$(.2)^2 + (.8)(.5) = .44$
2	(a) 	
	(b) .5	$(1)(.5) = .5$
	.455	$(1)(.5)(1)(.5) +$
		$(1)(.5)(.8)(.5) +$
		$(1)(.5)(.1)(.1)$
		$= .25 + .20 + .005 = .455$

Problem	Ans	Reason
3	(a) $P = \begin{bmatrix} .2 & .4 & .4 \\ 0 & 1 & 0 \\ 0 & .8 & .2 \end{bmatrix}$	
	(b) .80	$(.4)(.8) + (.2)(.4) + (.4)(1) = .80$
4	(a) state 1. with prob .5409	$P = \begin{bmatrix} 9 & 1 \\ 2 & 8 \end{bmatrix} \frac{1}{10}$

$$P^2 = \begin{bmatrix} 9 & 1 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 9 & 1 \\ 2 & 8 \end{bmatrix} \frac{1}{100}$$

$$= \begin{bmatrix} 83 & 17 \\ 34 & 66 \end{bmatrix} \frac{1}{100}$$

$$P^3 = \begin{bmatrix} 83 & 17 \\ 34 & 66 \end{bmatrix}\begin{bmatrix} 9 & 1 \\ 2 & 8 \end{bmatrix} \frac{1}{1000}$$

$$= \left[\begin{array}{cc|c} 747 & 34 & 83 + 136 \\ \hline 306 + 132 & & 34 + 528 \end{array}\right] \frac{1}{1000}$$

$$= \begin{bmatrix} .781 & .219 \\ .438 & .562 \end{bmatrix}$$

Problem	Ans	Reason

4, cont

$$[.3 \quad .7] \begin{bmatrix} .781 & .219 \\ .438 & .562 \end{bmatrix} = [.5409, \quad .4591]$$

(b) Stable vector $w = [x \quad y]$

$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

$x + y = 1$

$wP = w \qquad w(P - I) = 0$

$$[x \quad y] \begin{bmatrix} -.1 & .1 \\ .2 & -.2 \end{bmatrix} = [0 \quad 0]$$

$-.1x + .2y = 0$

$x = 2y$

$x = \frac{2}{3} \qquad y = \frac{1}{3}$

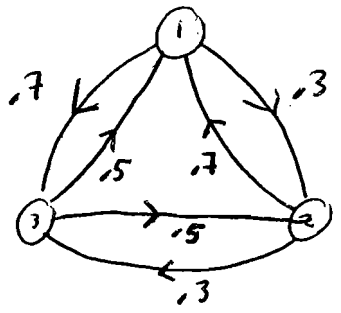
Problem	Ans	Reason
5	(a) $P(2) = P^2$	

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & .4 & .6 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & .4 & .6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & .4 & .6 \\ 0 & 0 & 1 \\ .4 & .24 & .36 \end{bmatrix}$$

(b) $$\begin{bmatrix} .3 & 0 & .7 \end{bmatrix} \begin{bmatrix} 0 & .4 & .6 \\ 0 & 0 & 1 \\ .4 & .24 & .36 \end{bmatrix}$$

$$= \begin{bmatrix} .280 & .288 & .432 \end{bmatrix}$$

Problem	Ans	Reason
6	(a) 	

(b) $P(2) = P^2 = \begin{bmatrix} 0 & 3 & 7 \\ 7 & 0 & 3 \\ 5 & 5 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 & 7 \\ 7 & 0 & 3 \\ 5 & 5 & 0 \end{bmatrix} \frac{1}{100}$

$= \left[\begin{array}{c|c|c} 21+35 & 35 & 9 \\ \hline 15 & 21+15 & 49 \\ \hline 35 & 15 & 35+15 \end{array}\right] \frac{1}{100}$

$= \begin{bmatrix} .56 & .35 & .09 \\ .15 & .36 & .49 \\ .35 & .15 & .50 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 2 & 2 \end{bmatrix}\begin{bmatrix} 0 & 3 & 7 \\ 7 & 0 & 3 \\ 5 & 5 & 0 \end{bmatrix} \frac{1}{100}$

$= \begin{bmatrix} .24 & .28 & .48 \end{bmatrix}$

Problem	Ans	Reason
7	.1	$P_{21} P_{11} = (.2)(.5)$ $= .1$
8	(a)	$P(2) = P^2 = \begin{bmatrix} 5 & 5 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 5 & 5 \\ 2 & 8 \end{bmatrix}\frac{1}{100}$ $= \left[\begin{array}{c\|c} 25+10 & 25+40 \\ \hline 10+16 & 10+64 \end{array}\right]\frac{1}{100}$ $= \begin{bmatrix} .35 & .65 \\ .26 & .74 \end{bmatrix}$ $P(4) = P^4 =$ $\begin{bmatrix} 35 & 65 \\ 26 & 74 \end{bmatrix}\begin{bmatrix} 35 & 65 \\ 26 & 74 \end{bmatrix}\frac{1}{10000}$ $= \left[\begin{array}{c\|c} 1225+1690 & 2275+4810 \\ \hline 910+1924 & 1690+5476 \end{array}\right]\frac{1}{10000}$ $= \begin{bmatrix} .2915 & .7085 \\ .2834 & .7166 \end{bmatrix}$

Problem	Ans	Reason
8, cont	(b) $\begin{bmatrix} \frac{2}{7} & \frac{5}{7} \end{bmatrix}$	stable vector $W = \begin{bmatrix} x & y \end{bmatrix}$

$$x + y = 1$$

$$P - I = \begin{bmatrix} -.5 & .5 \\ .2 & -.2 \end{bmatrix}$$

$$0 = W(P - I)$$

$$0 = -.5x + .2y$$

$$5x = 2y = 2(1-x)$$

$$7x = 2 \qquad x = \frac{2}{7} \qquad y = \frac{5}{7}$$

(c)

$$\begin{array}{r|l} & .2857 \\ \hline 7 & 2.00000 \\ & 14 \\ \hline & 60 \\ & 56 \\ \hline & 40 \\ & 35 \\ \hline & 50 \\ & 49 \\ \hline & 10 \end{array}$$

$.2915 - \frac{2}{7} \cong .0058$

$.7085 - \frac{5}{7} \cong -.0058$

Problem	Ans	Reason
9	(a)	

$$P = \begin{bmatrix} 0 & 0 & * \\ * & 0 & * \\ 0 & * & 0 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 0 & * \\ * & 0 & * \\ 0 & * & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & * \\ * & 0 & * \\ 0 & * & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & * & 0 \\ 0 & * & * \\ * & 0 & * \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0 & * & 0 \\ 0 & * & * \\ * & 0 & * \end{bmatrix} \begin{bmatrix} 0 & * & 0 \\ 0 & * & * \\ * & 0 & * \end{bmatrix}$$

$$= \begin{bmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$P^8 = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

Regular

Problem	Ans	Reason
9, cont	(b)	Stable vector $W = [x \; y \; z]$

$$x + y + z = 1$$

$$W(P - I) = 0$$

$$P - I = \begin{bmatrix} -1 & 0 & 1 \\ .6 & -1 & .4 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\begin{aligned} 0 &= -x + .6y \\ 0 &= -y + z \\ 0 &= x + .4y - z \end{aligned}$$

$$y = z$$

$$x = .6z$$

$$.6z + z + z = 1$$

$$2.6z = 1$$

$$z = \frac{1}{2.6} \qquad y = \frac{1}{2.6} \qquad x = \frac{.6}{2.6}$$

$$W = \left[\frac{6}{26} \quad \frac{10}{26} \quad \frac{10}{26} \right]$$

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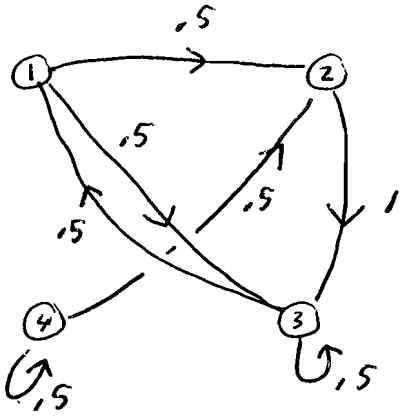
Problem	Ans	Reason

10 Not Regular

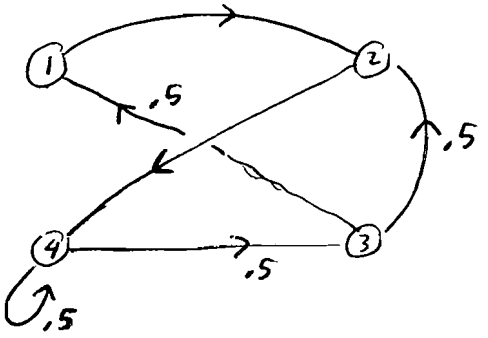


Can't get from 1 to 4

11 Regular

$\left[\frac{1}{9} \quad \frac{2}{9} \quad \frac{2}{9} \quad \frac{4}{9}\right]$

Most Likely State 4



St Vecta $W = [x \; y \; z \; w]$ $\quad x+y+z+w=1$

$O = W(P-I)$ $\quad P-I = \left[\begin{array}{cc|cc} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ \hline .5 & .5 & -1 & 0 \\ 0 & 0 & .5 & -.5 \end{array}\right]$

$x = .5z = .25w$ $\quad (.25+.5+.5+1)w=1$

$z = .5w$ $\quad w = \frac{1}{2.25}$

$y = .5w$

$x = \frac{.25}{2.25} = \frac{1}{9}$ $\quad y = \frac{2}{9}$ $\quad z = \frac{2}{9}$ $\quad w = \frac{4}{9}$ Max

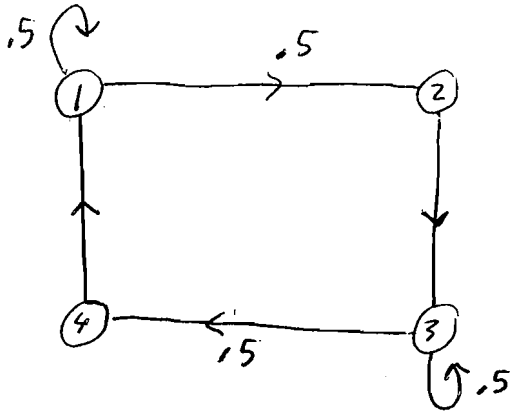
Problem	Ans	Reason

12

Regular

$\left[\frac{2}{6} \quad \frac{1}{6} \quad \frac{2}{6} \quad \frac{1}{6}\right]$

state 1, 3 Most Likely, each with prob $\frac{1}{3}$



Stable v: $W = [x \quad y \quad z \quad w]$

$x + y + z + w = 1$

$0 = W(P - I)$

$$P - I = \left[\begin{array}{cc|cc} -.5 & .5 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ \hline 0 & 0 & -.5 & .5 \\ 1 & 0 & 0 & -1 \end{array}\right]$$

$.5x = w$ $\quad$ $w = \frac{1}{2}x$

$.5x = y$ $\quad$ $y = \frac{1}{2}x$

$y = .5z$ $\quad$ $z = x$

$x(1 + \frac{1}{2} + 1 + \frac{1}{2}) = 1$

$x = \frac{1}{3}$ $\quad$ $y = \frac{1}{6}$

$z = \frac{1}{3}$ $\quad$ $w = \frac{1}{6}$

$W = \left[\frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{6}\right]$