Section 8.3 Solutions

No. 8.3
Date 1

Problem	Ans	Reason
1	(a) Y (b) N (c) N (d) Y	Prob vector: entries nonnegative and sum to 1
2	(a) .3 (b) .4 (c) $1/4 = .25$ (d) $1/11$	$x = 1 - .2 - .5 = .3$ $1 = 2x + .2$ $4x = 1$ $x(2+3+3+2+0) = 1$ $x \cdot 11 = 1$
3	$X = \left[\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{2}\right]$	$X = [a \; b \; c]$ $a+b+c=1$ $a=b$ $c=2a$ So $a+a+2a=1$ $\quad 4a=1$ $a=\frac{1}{4}$ $\quad b=\frac{1}{4}$ $\quad c=\frac{1}{2}$
4	$X = [.4 \; .4 \; .2 \; 0]$	$X = [a \; b \; c \; d]$ $a+b+c+d=1$ $a=2c$ $d=0$ $b=.4$ So $2c+.4+c+0=1$ $3c=.6$ $c=.2$ $\quad a=.4$

Problem	Ans	Reason
5	(a) $X = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}$	$P = \begin{matrix} H \\ L \end{matrix}\begin{bmatrix} .7 & .3 \\ .5 & .5 \end{bmatrix}$ $\quad\quad H \quad\quad L$ $X = [\, a \;\; b \,]$ $a+b=1$ $a=2b$ $a=2/3 \quad b=1/3$
	(b) $\frac{188}{300} = \frac{47}{75}$	Find XP^2 (entry H) $P^2 = \begin{bmatrix} .7 & .3 \\ .5 & .5 \end{bmatrix}\begin{bmatrix} .7 & .3 \\ .5 & .5 \end{bmatrix}\frac{1}{100}$ $= \begin{bmatrix} 49+15 & 21+15 \\ 35+25 & 15+25 \end{bmatrix}\frac{1}{100}$ $= \begin{bmatrix} 64 & 36 \\ 60 & 40 \end{bmatrix}\frac{1}{100}$ H-entry of $XP^2 = \frac{2}{3}\frac{64}{100} + \frac{1}{3}\frac{60}{100}$ $= \frac{188}{300}$
	(c) $\frac{1876}{3000} = \frac{469}{750}$	H-entry of XP^3 $P^3 = \begin{bmatrix} .7 & .3 \\ .5 & .5 \end{bmatrix}\begin{bmatrix} 64 & 36 \\ 60 & 40 \end{bmatrix}\frac{1}{1000}$

Problem	Ans	Reason

5, cont

$$P^3 = \left[\begin{array}{c|c} 448 + 180 & 252 + 120 \\ \hline 320 + 300 & 180 + 200 \end{array}\right] \frac{1}{1000}$$

$$= \begin{bmatrix} 628 & 372 \\ 620 & 380 \end{bmatrix} \frac{1}{1000}$$

H-entry of IP^3

$$= \frac{2}{3}\,\frac{628}{1000} + \frac{1}{3}\,\frac{}{1000}$$

$$= \frac{1876}{3000}$$

6 — 2 weeks from now

Find k such that (L,H)-entry of P^k is at least .6

$$P = \begin{bmatrix} * & * \\ .5 & * \end{bmatrix} \qquad P^2 = \begin{bmatrix} * & * \\ .6 & * \end{bmatrix}$$

Problem	Ans	Reason
7	(a) $X = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ $P = \begin{matrix} \\ M \\ W \end{matrix} \begin{matrix} \begin{matrix} M & W \end{matrix} \\ \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{matrix}$	
	(b) $\frac{55}{108}$	Find M-entry of XP^3

$$P^2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\frac{1}{9}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}\cdot\frac{1}{9}$$

$$P^3 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}\frac{1}{27}$$

$$= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}\frac{1}{27}$$

M-entry of XP^3 is

$$\frac{3}{4}\ \frac{14}{27} + \frac{1}{4}\ \frac{13}{27}$$

$$= \frac{55}{108}$$

Problem	Ans	Reason

8

Next obs

$$X = [.1 \quad .9]$$

M-entry of XP = $\frac{1}{10}\frac{2}{3} + \frac{9}{10}\frac{1}{3}$

$= \frac{11}{30}$

9

$\left[\frac{5}{8} \quad \frac{3}{8}\right]$

Stable vector $W = [a \; b]$

$a + b = 1$

$WP = W$

$0 = W(P - I)$

$$[0 \; 0] = [a \; b]\begin{bmatrix} -.3 & .3 \\ .5 & -.5 \end{bmatrix}$$

$0 = -.3a + .5b$

$.3a = .5b$

$3a = 5b$

$3a = 5(1-a)$

$8a = 5$

$a = 5/8 \quad b = 3/8$

Problem	Ans	Reason

10

$$P = \begin{bmatrix} 0 & 1 \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 1 \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{5} & \frac{3}{5} \\ \frac{6}{25} & \frac{19}{25} \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 10 & 15 \\ 6 & 19 \end{bmatrix}\begin{bmatrix} 10 & 15 \\ 6 & 19 \end{bmatrix}\frac{1}{25^2}$$

$$= \begin{bmatrix} 100 + 90 & 150 + 285 \\ 60 + 114 & 90 + 361 \end{bmatrix}\frac{1}{625}$$

$$= \begin{bmatrix} 190 & 435 \\ 174 & 451 \end{bmatrix}\frac{1}{625}$$

$$P^8 = \begin{bmatrix} 190 & 435 \\ 174 & 451 \end{bmatrix}\begin{bmatrix} 190 & 435 \\ 174 & 451 \end{bmatrix}\frac{1}{625^2}$$

$$= \begin{bmatrix} 36100 + 75690 = 111790 & 82650 + 196185 = 278835 \\ * & * \end{bmatrix}\frac{1}{625^2}$$

$111790 / 278835 \simeq 2/5 \Rightarrow$ stable vector is $\begin{bmatrix} \frac{2}{7} & \frac{5}{7} \end{bmatrix}$

Problem	Ans	Reason
11	$\begin{bmatrix} \frac{4}{7} & \frac{3}{7} \end{bmatrix}$	$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

Stable vector $W = \begin{bmatrix} a & b \end{bmatrix}$

$$a + b = 1$$

$$W = WP$$

$$0 = W(P - I)$$

$$\begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

$$-\frac{1}{2}a + \frac{2}{3}b = 0$$

$$\frac{1}{2}a = \frac{2}{3}b$$

$$3a = 4b$$

$$3(1-b) = 4b \qquad 3 = 7b$$

$a = 4/7 \qquad b = 3/7$

check $\begin{bmatrix} \frac{4}{7} & \frac{3}{7} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix} \overset{\checkmark}{=} \begin{bmatrix} \frac{4}{7} & \frac{3}{7} \end{bmatrix}$

Problem	Ans	Reason
12	$\frac{1}{3}$	

Stable v. $W = [\,a \;\; b \;\; c\,]$

$$a + b + c = 1$$

$$0 = W(P - I)$$

$$[\,0 \;\; 0 \;\; 0\,] = [\,a \;\; b \;\; c\,] \begin{bmatrix} -1 & .5 & .5 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$0 = -a + .5b + .5c$$

$$0 = -b + c$$

$$0 = a - c \qquad \text{(red)}$$

$$a = b = c = \frac{1}{3}$$

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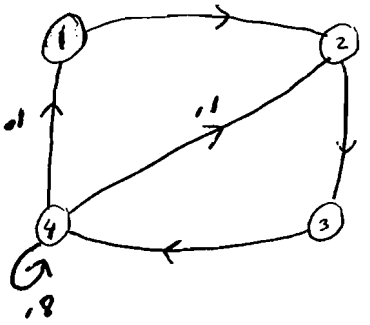
Problem	Ans	Reason
13	Regular $\begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$	$P = \begin{bmatrix} 0 & * \\ * & * \end{bmatrix}$ $P^2 = \begin{bmatrix} 0 & * \\ * & * \end{bmatrix}\begin{bmatrix} 0 & * \\ * & * \end{bmatrix}$ $= \begin{bmatrix} * & * \\ * & * \end{bmatrix}$ Stable vector $W = [a \; b]$ $[0 \; 0] = [a \; b]\begin{bmatrix} -1 & 1 \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$ $0 = -a + \frac{1}{3}b \qquad a = \frac{1}{3}b$ $b = 3a$ $a = \frac{1}{4}, \; b = \frac{3}{4}$

Problem	Ans	Reason
14	Regular $\left[\frac{1}{6} \quad \frac{2}{6} \quad \frac{3}{6}\right]$	$P^2 = \begin{bmatrix} 0 & * & 0 \\ 0 & * & * \\ * & 0 & * \end{bmatrix}\begin{bmatrix} 0 & * & 0 \\ 0 & * & * \\ * & 0 & * \end{bmatrix}$ $= \begin{bmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$ $P^4 = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$ Reg Stable vector $w = [a \; b \; c]$ $a+b+c=1$ $[0\;0\;0] = [a \; b \; c]\begin{bmatrix} -1 & 1 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & 0 & -\frac{1}{3} \end{bmatrix}$ $0 = -a + \frac{c}{3}$ $\quad c = 3a$ $0 = a - \frac{1}{2}b$ $\quad b = 2a$ $a = \frac{1}{6}$ $\quad b = \frac{2}{6}$ $\quad c = \frac{3}{6}$

Problem	Ans	Reason
15	Not Reg	P^k Lower triangular for all k.
16	Reg	

$$\left[\frac{1}{15} \quad \frac{2}{15} \quad \frac{2}{15} \quad \frac{10}{15}\right]$$



Stable vector $W = [a\ b\ c\ d]$

$a+b+c+d=1$

$0 = W(P-I)$

$$[0\ 0\ 0\ 0] = [a\ b\ c\ d]\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ .1 & .1 & 0 & -.2 \end{bmatrix}$$

$0 = -a + .1d \qquad a = .1d$

$0 = a - b + .1d$

$0 = b - c \qquad b = .2d$

$0 = c - .2d \qquad c = .2d$

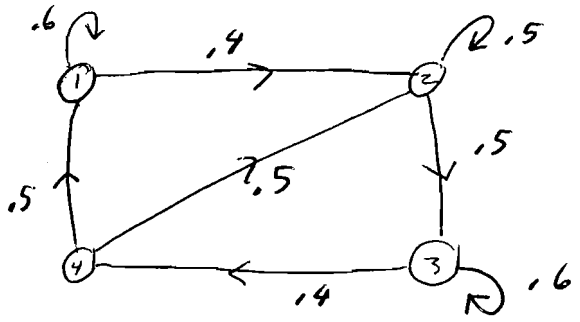
$(.1 + .2 + .2 + 1)d = 1$

$\frac{3}{2}d = 1 \qquad d = \frac{2}{3}$

$a = \frac{2}{30} \quad b = \frac{4}{30} \quad c = \frac{4}{30} \quad d = \frac{20}{30}$

Problem	Ans	Reason
17	Regular	

$$\begin{bmatrix} \frac{5}{23} & \frac{4}{23} & \frac{10}{23} & \frac{4}{23} \end{bmatrix}$$



Stable vector $W = [a\ b\ c\ d]$

$$[0\ 0\ 0\ 0] = [a\ b\ c\ d]\begin{bmatrix} -.4 & .4 & 0 & 0 \\ 0 & -.5 & .5 & 0 \\ 0 & 0 & -.4 & .4 \\ .5 & 0 & .5 & -1 \end{bmatrix}$$

$.4a = .5d$ $\quad 4a = 5d$

$.4a = .5b$ $\quad 4a = 5b$

$.4c = d$ $\quad 4c = 10d$

$a = \frac{5}{4}d$

$b = d$

$c = \frac{10}{4}d$

$\left(\frac{5}{4} + \frac{4}{4} + \frac{10}{4} + \frac{4}{4}\right) d = 1$

$a = \frac{5}{23}$ $\quad b = \frac{4}{23}$ $\quad c = \frac{10}{23}$ $\quad d = \frac{4}{23}$

Problem	Ans	Reason
18	all states equally likely	Stable vector $W = [a \ b \ c \ d]$

$a + b + c + d = 1$

$W = wP$

$$P - I = \begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -1 & \frac{1}{2} \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

$a = d$

$\frac{1}{2}a - b + \frac{1}{2}c = 0$

$\frac{1}{2}a + \frac{1}{2}b - c = 0$

$\frac{1}{2}b + \frac{1}{2}c = d$

$a = b = c = d = \frac{1}{4}$

Problem	Ans	Reason

19

state	Hit	miss
Prob	$\frac{4}{5}$	$\frac{1}{5}$

$$P = \begin{matrix} \\ H \\ M \end{matrix} \begin{matrix} \begin{matrix} H & M \end{matrix} \\ \begin{bmatrix} .85 & .15 \\ .6 & .4 \end{bmatrix} \end{matrix} \begin{matrix} \\ H \\ M \end{matrix}$$

$$P - I = \begin{bmatrix} -.15 & .15 \\ .6 & -.6 \end{bmatrix}$$

Stable v. $W = [\, a \;\; b \,]$ $\quad a+b=1$

$$.15a = .6b$$
$$15a = 60b$$
$$a = 4b$$

$$a = \frac{4}{5} \qquad b = \frac{1}{5}$$

20 $\frac{2}{3}$

$$P = \begin{matrix} \\ G \\ B \end{matrix} \begin{matrix} \begin{matrix} G & B \end{matrix} \\ \begin{bmatrix} .6 & .4 \\ .8 & .2 \end{bmatrix} \end{matrix}$$

$$P - I = \begin{bmatrix} -.4 & .4 \\ .8 & -.8 \end{bmatrix}$$

Stable vector $W = [\, a \;\; b \,]$ $\quad a+b=1$

$$.4a = .8b$$
$$4a = 8b$$
$$a = 2b$$

$$a = \frac{2}{3} \qquad b = \frac{1}{3}$$

Problem	Ans	Reason
21	Prob $\frac{7}{13}$ of attending	$P = \begin{bmatrix} .3 & .7 \\ .6 & .4 \end{bmatrix}$
		$P - I = \begin{bmatrix} -.7 & .7 \\ .6 & -.6 \end{bmatrix}$
		Stab vectr $W = [a\ b]$ $\quad a+b=1$
		$.7a = .6b$
		$7a = 6b$
		$7a = 6(1-a)$
		$13a = 6$ $\quad a = \frac{6}{13}$ $\quad b = \frac{7}{13}$
22	(a) Regular	$P^2 = \begin{pmatrix} * & 0 & * \\ * & * & * \\ * & * & 0 \end{pmatrix}\begin{pmatrix} * & 0 & * \\ * & * & * \\ * & * & 0 \end{pmatrix}$
		$= \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$
	(b) Not R	$P^2 = \begin{pmatrix} * & 0 & * \\ * & * & * \\ * & 0 & * \end{pmatrix}\begin{pmatrix} * & 0 & * \\ * & * & * \\ * & 0 & * \end{pmatrix}$
		$= \begin{pmatrix} * & 0 & * \\ * & * & * \\ * & 0 & * \end{pmatrix}$

Problem	Ans	Reason
22, cont	(c) Regular	

$$P^2 = \begin{bmatrix} 0 & 0 & * \\ * & * & * \\ * & * & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & * \\ * & * & * \\ * & * & 0 \end{bmatrix}$$

$$= \begin{bmatrix} * & * & 0 \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$P^4 = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

(d) Not reg

$$P^2 = \begin{bmatrix} * & * & * \\ 0 & * & 0 \\ * & * & 0 \end{bmatrix}\begin{bmatrix} * & * & * \\ 0 & * & 0 \\ * & * & 0 \end{bmatrix}$$

$$= \begin{bmatrix} * & * & * \\ 0 & * & 0 \\ * & * & * \end{bmatrix}$$

$$P^4 = \begin{bmatrix} * & * & * \\ 0 & * & 0 \\ * & * & * \end{bmatrix}\begin{bmatrix} * & * & * \\ 0 & * & 0 \\ * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} * & * & * \\ 0 & * & 0 \\ * & * & * \end{bmatrix}$$

Problem	Ans	Reason
23 (tedious)	see bk	
24	Compartments 1 and 4 each with prob $\frac{3}{10}$	

$$P = \left[\begin{array}{cc|cc} \frac{1}{3} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \hline \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{1}{3} \end{array}\right]$$

$$P - I = \left[\begin{array}{cc|cc} \frac{-2}{3} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{1}{3} & \frac{-2}{3} & 0 & \frac{1}{3} \\ \hline \frac{1}{3} & 0 & \frac{-2}{3} & \frac{1}{3} \\ \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{3} \end{array}\right]$$

St. vector $W = [a\ b\ c\ d]$

$$a + b + c + d = 1$$

$$\frac{-2}{3}a + \frac{1}{3}b + \frac{1}{3}c + \frac{2}{9}d = 0$$

$$\frac{2}{9}a - \frac{2}{3}b + \frac{2}{9}d = 0$$

$$\frac{2}{9}a - \frac{2}{3}c + \frac{2}{9}d = 0$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ -6 & 3 & 3 & 2 & 0 \\ 2 & -6 & 0 & 2 & 0 \\ 2 & 0 & -6 & 2 & 0 \end{array}\right]$$

Problem	Ans	Reason

24, cont

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 9 & 9 & 8 & 6 \\ 0 & -8 & -2 & 0 & -2 \\ 0 & -2 & -8 & 0 & -2 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 + 6r_1 \\ r_3' = r_3 - 2r_1 \\ r_4' = r_4 - 2r_1 \end{array}$$

$r_4 \to -\frac{1}{2} r_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & -8 & -2 & 0 & -2 \\ 0 & 9 & 9 & 8 & 6 \end{array}\right] \quad r_2 \leftrightarrow r_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 30 & 0 & 6 \\ 0 & 0 & -27 & 8 & -3 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - r_2 \\ \\ r_3' = r_3 + 8r_2 \\ r_4' = r_4 - 9r_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 1 & 0 & \frac{1}{5} \\ 0 & 0 & -27 & 8 & -3 \end{array}\right] \quad r_3' = r_3 \frac{1}{30}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & \frac{3}{5} \\ 0 & 1 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & 0 & \frac{1}{5} \\ 0 & 0 & 0 & 8 & \frac{12}{5} \end{array}\right] \quad \begin{array}{l} r_1' = r_1 + 3r_3 \\ r_2' = r_2 - 4r_3 \\ \\ r_4' = r_4 + 27r_3 \end{array}$$

$d = \frac{3}{10} \qquad c = \frac{1}{5} \qquad b = \frac{1}{5} \qquad a = \frac{3}{10}$

Problem	Ans	Reason

24, cont

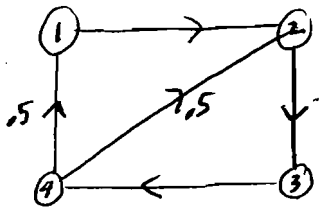
check

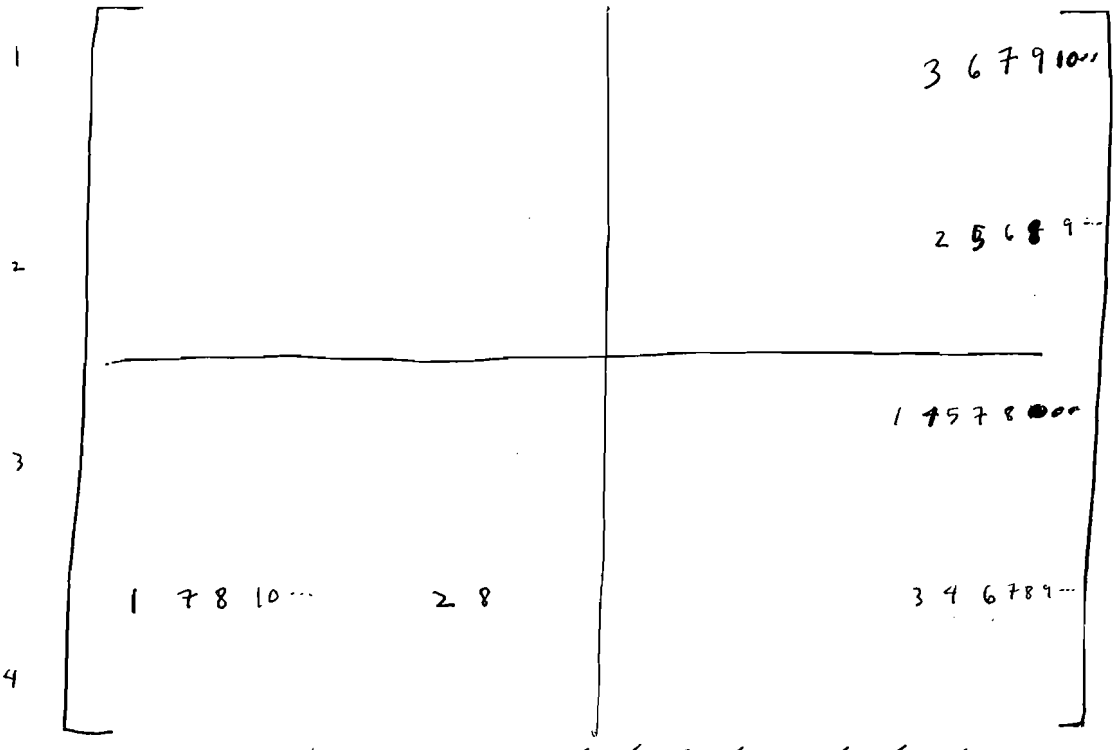
$$\begin{bmatrix} 3 & 2 & 2 & 3 \end{bmatrix} \left[\begin{array}{cc|cc} 3 & 2 & 2 & 2 \\ 3 & 3 & 0 & 3 \\ \hline 3 & 0 & 3 & 3 \\ 2 & 2 & 2 & 3 \end{array}\right] \frac{1}{10} \quad \frac{1}{9}$$

$$= \begin{bmatrix} 3 & 2 & 2 & 3 \end{bmatrix} \frac{1}{10} \quad \checkmark$$

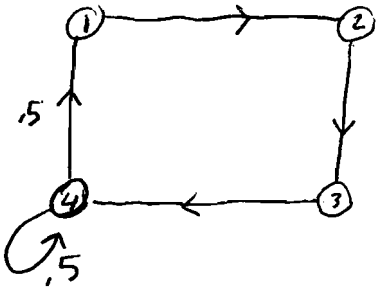
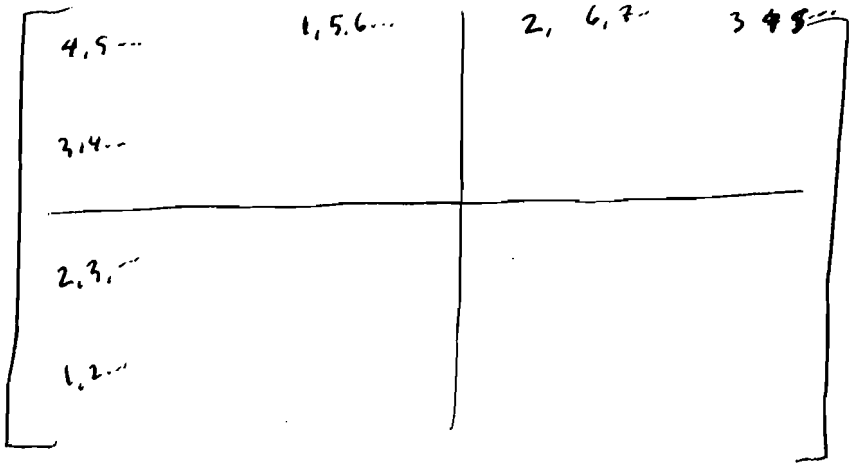
25

$k = 10$ P_1





poss k x x 3 4 5 6 x 8 9 10 11

Problem	Ans	Reason
25. cont	P_2   poss k : ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ 6 7 ...	$k = 6$

Problem	Ans	Reason

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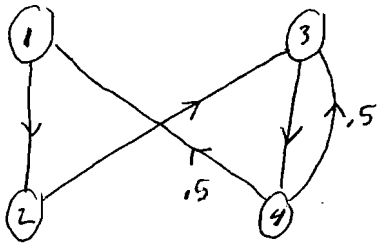
(a)



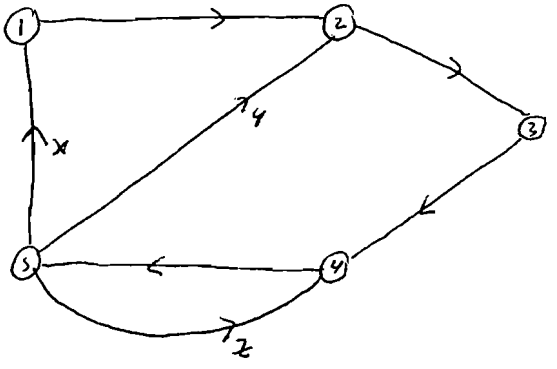
(b) clear from diag

(c)

Call states	1, 3	odd
call states	2, 4	even

Each transition switches parity of the state

27



Fa paths ① → ① poss path lengths are

5, 7, 9, 11, ...

$P(8) = P^8$ has (1,1) entry 0 for all x, y, z

28 too tedious

Problem	Ans	Reason

29

$$\begin{bmatrix} \frac{b}{1+b-a} & \frac{1-a}{1+b-a} \end{bmatrix} \begin{bmatrix} a & 1-a \\ b & 1-b \end{bmatrix}$$

$$= \begin{bmatrix} \frac{ab + (1-a)b}{1+b-a} & \frac{b(1-a) + (1-a)(1-b)}{1+b-a} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{b}{1+b-a} & \frac{1-a}{1+b-a} \end{bmatrix} \checkmark$$

30 $\begin{bmatrix} \frac{4}{7} & \frac{3}{7} \end{bmatrix}$ In #29 Take $a = 3/4$, $b = 1/3$

$$\frac{b}{1+b-a} = \frac{1}{3} \frac{1}{1 + \frac{1}{3} - 3/4}$$

$$= \frac{1}{3 + 1 - \frac{9}{4}}$$

$$= \frac{1}{4 - 9/4} \frac{4}{4} = \frac{4}{16-9} = \frac{4}{7}$$

$$\frac{1-a}{1+b-a} = \frac{1}{4} \frac{1}{1+\frac{1}{3} - 3/4} = \frac{1}{4 + \frac{4}{3} - 3}$$

$$= \frac{1}{1 + \frac{4}{3}} \frac{3}{3} = \frac{3}{3+4} = \frac{3}{7}$$

Problem	Ans	Reason
31	$\begin{bmatrix} \frac{b}{1+b} & \frac{1}{1+b} \end{bmatrix}$	$P-I = \begin{bmatrix} -1 & 1 \\ b & -b \end{bmatrix}$

St. vector $W = [x \; y]$ $\quad x+y=1$

$$0 = W(P-I)$$

$$-x + by = 0$$

$$x = by$$

$$x = b(1-x)$$

$$(1+b)x = b \qquad x = \frac{b}{1+b}$$

$$y = \frac{1}{1+b}$$

Problem	Ans	Reason

32

$$P - I = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ a & b & -a-b \end{bmatrix}$$

St. vector $W = [\, x \;\; y \;\; z \,]$

$$x + y + z = 1$$

$$W = PW \qquad 0 = W(P - I)$$

$$\begin{aligned} -x + az &= 0 \\ x - y + bz &= 0 \\ y - (a+b)z &= 0 \end{aligned}$$

$$x = az$$

$$y = x + bz = (a+b)z$$

$$1 = x + y + z = (a + a + b + 1)z$$

$$z = \frac{1}{1 + 2a + b}$$

$$x = \frac{a}{1 + 2a + b}$$

$$y = \frac{a+b}{1+2a+b}$$

$$\left[\frac{a}{1+2a+b} \quad \frac{a+b}{1+2a+b} \quad \frac{1}{1+2a+b} \right]$$

Problem	Ans	Reason
33	$a = .5$	$X_2 = X_0 P^2$

$$X_0 P = \begin{bmatrix} .6 - .2a & .32 + .2a & .08 \end{bmatrix}$$

$.1 =$ Last coord of X_2

$=$ $\cdots$ $(X_0 P) P$

$= .2(.6 - .2a)$

So

$10 = 2(6 - 2a)$

$= 12 - 4a$

$4a = 2$

$a = \frac{1}{2} = .5$

check $X_0 = \begin{bmatrix} .4 & .5 & .1 \end{bmatrix}$

$$X_0 P = \begin{bmatrix} .4 & .5 & .1 \end{bmatrix} \begin{bmatrix} 0 & .8 & .2 \\ .8 & .2 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} .5 & .42 & .08 \end{bmatrix}$$

$$X_0 P^2 = \begin{bmatrix} .5 & .42 & .08 \end{bmatrix} \begin{bmatrix} 0 & .8 & .2 \\ .8 & .2 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} .416 & .484 & \end{bmatrix}$$

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Problem	Ans	Reason

34 the prob is aprox $6/11$

$$P = \begin{matrix} \\ H \\ M \end{matrix} \begin{matrix} \begin{matrix} H & M \end{matrix} \\ \begin{bmatrix} .75 & .25 \\ .3 & .7 \end{bmatrix} \end{matrix}$$

stable vector $W = [\, a \; b \,]$ $\quad a+b=1$

$$P - I = \begin{bmatrix} -.25 & .25 \\ .3 & -.3 \end{bmatrix}$$

$$.25a = .3b$$

$$25a = 30b$$

$$5a = 6b$$

$$5a = 6(1-a)$$

$$11a = 6$$

$$a = \frac{6}{11} \qquad b = \frac{5}{11}$$