8.2 HW Solutions

No. 8.2
Date 1

Problem	Ans	Reason
1.	(a) $P(2) = P^2 = \begin{bmatrix} .5 & .5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} .5 & .5 \\ 1 & 0 \end{bmatrix}$	
	$= \begin{bmatrix} .75 & .25 \\ .5 & .5 \end{bmatrix}$	
	(b) $p_{12}(2) = .25$ $\quad p_{22}(2) = .5$	
2	(a) $P(2) = P^2 = \begin{bmatrix} 0 & 1 \\ .2 & .8 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ .2 & .8 \end{bmatrix}$	
	$= \begin{bmatrix} .2 & .8 \\ .16 & .84 \end{bmatrix}$	
	(b) $p_{11}(2) = .2$ $\quad p_{22}(2) = .84$	
3	$P(3) = P\,P(2) = \begin{bmatrix} .5 & .5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} .75 & .25 \\ .5 & .5 \end{bmatrix}$	
	$= \begin{bmatrix} .625 & .375 \\ .75 & .25 \end{bmatrix}$	
	$P(4) = P\,P(3) = \begin{bmatrix} .5 & .5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} .625 & .375 \\ .75 & .25 \end{bmatrix}$	
	$= \begin{bmatrix} .6875 & .3125 \\ .625 & .375 \end{bmatrix}$	

..oblem	Ans	Reason
3. cont		check $P[2]\,P[2] \stackrel{?}{=} P[4]$

$$\begin{bmatrix} .75 & .25 \\ .5 & .5 \end{bmatrix}\begin{bmatrix} .75 & .25 \\ .5 & .5 \end{bmatrix} = \begin{bmatrix} .6875 & .3125 \\ .625 & .375 \end{bmatrix} \checkmark$$

4 (a) $P(2) = P^2 = \begin{bmatrix} .6 & .4 \\ .4 & .6 \end{bmatrix}\begin{bmatrix} .6 & .4 \\ .4 & .6 \end{bmatrix}$

$$= \left[\begin{array}{c|c} .36+.16 & .24+.24 \\ \hline .24+.24 & .16+.36 \end{array}\right] = \begin{bmatrix} .52 & .48 \\ .48 & .52 \end{bmatrix}$$

(b) $P(3) = P\,P(2) = \begin{bmatrix} .6 & .4 \\ .4 & .6 \end{bmatrix}\begin{bmatrix} .52 & .48 \\ .48 & .52 \end{bmatrix}$

$$= \left[\begin{array}{c|c} .312+.192 & .288+.208 \\ \hline .208+.288 & .192+.312 \end{array}\right] = \begin{bmatrix} .504 & .496 \\ .496 & .504 \end{bmatrix}$$

5 .444 this (1,1)-entry of P^3

$$P^2 = \begin{bmatrix} .6 & .4 \\ .3 & .7 \end{bmatrix}\begin{bmatrix} .6 & .4 \\ .3 & .7 \end{bmatrix} = \begin{bmatrix} .48 & .52 \\ .39 & .61 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} .6 & .4 \\ .3 & .7 \end{bmatrix}\begin{bmatrix} .48 & .52 \\ .39 & .61 \end{bmatrix} = \begin{bmatrix} .444 & * \\ * & * \end{bmatrix}$$

Problem	Ans	Reason
6	$\frac{36}{64}$	$p_{13}(3) = (1,3)\text{-entry of } P^3 = \frac{36}{64}$
7	(a)	$P(2) = P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
	(b)	$P(3) = P\,P(2) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
8	.24	$p_{32}(3) = (3,2)\text{-entry of } P^3 = [\text{row 3 of } P][\text{col 2 of } P^2] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} .24 \\ .4 \\ .3 \end{bmatrix} = .24$
9	(a)	$P[2] = P^2 = \begin{bmatrix} 0 & .8 & .2 \\ 0 & 1 & 0 \\ .5 & 0 & .5 \end{bmatrix}\begin{bmatrix} 0 & .8 & .2 \\ 0 & 1 & 0 \\ .5 & 0 & .5 \end{bmatrix} = \begin{bmatrix} .1 & .8 & .1 \\ 0 & 1 & 0 \\ .25 & .4 & .35 \end{bmatrix}$
		$P(3) = P\,P(2) = \begin{bmatrix} 0 & .8 & .2 \\ 0 & 1 & 0 \\ .5 & 0 & .5 \end{bmatrix}\begin{bmatrix} .1 & .8 & .1 \\ 0 & 1 & 0 \\ .25 & .4 & .35 \end{bmatrix} = \begin{bmatrix} .05 & .88 & .07 \\ 0 & 1 & 0 \\ .175 & .6 & .225 \end{bmatrix}$

Problem	Ans	Reason

9, cont

$$P(4) = P\,P(3) = \begin{bmatrix} 0 & .8 & .2 \\ 0 & 1 & 0 \\ .5 & 0 & .5 \end{bmatrix}\begin{bmatrix} .05 & .88 & .07 \\ 0 & 1 & 0 \\ .175 & .6 & .225 \end{bmatrix}$$

$$= \begin{bmatrix} .035 & .92 & .095 \\ 0 & 1 & 0 \\ .1125 & .74 & .1475 \end{bmatrix}$$

(b) $p_{22}(2) = 1$

$p_{22}(3) = 1$

$p_{22}(4) = 1$

(c) $p_{22}(k) = 1$ for all k

10 (a) For each row of $P(2)$, $P(3)$, $P(4)$ the entries are nonneg and add up to 1

(b) $p_{12}(2) < p_{12}(3) < p_{12}(4)$

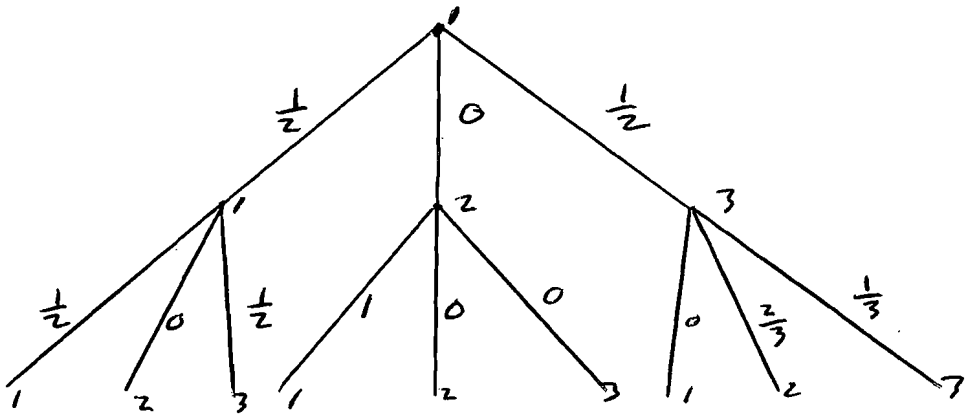
$.8 \qquad .88 \qquad .92$

(c) $p_{32}(2) < p_{32}(3) < p_{32}(4)$

$.4 \qquad .6 \qquad .74$

Problem	Ans	Reason

11 (a)



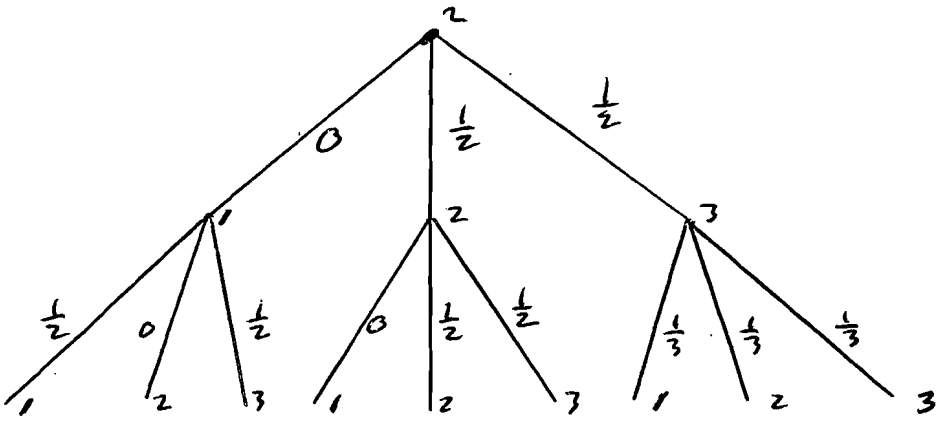
(b) $\left[\frac{1}{2}\cdot\frac{1}{2} + 0\cdot 1 + \frac{1}{2}\cdot 0 \;\middle|\; \frac{1}{2}\cdot 0 + 0\cdot 0 + \frac{1}{2}\cdot\frac{2}{3} \;\middle|\; \frac{1}{2}\cdot\frac{1}{2} + 0\cdot 0 + \frac{1}{2}\cdot\frac{1}{3}\right]$

$= \left[\frac{1}{4}, \frac{1}{3}, \frac{5}{12}\right]$

(c) $P(2) = P^2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

$= \left[\begin{array}{c|c|c} \frac{1}{4}+0+0 & 0+0+\frac{1}{3} & \frac{1}{4}+0+\frac{1}{6} \\ \hline \frac{1}{2}+0+0 & 0+0+0 & \frac{1}{2}+0+0 \\ \hline 0+\frac{2}{3}+0 & 0+0+\frac{2}{9} & 0+0+\frac{1}{9} \end{array}\right]$

$= \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{5}{12} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{2}{3} & \frac{2}{9} & \frac{1}{9} \end{bmatrix}$

Problem	Ans	Reason
12	(a) 	

(b)

$$\left[\begin{array}{ccc|ccc|ccc} 0 & + & 0 & + & \frac{1}{6} & 0 & + & \frac{1}{4} & + & \frac{1}{6} & 0 & + & \frac{1}{4} & + & \frac{1}{6} \end{array}\right]$$

$$= \left[\frac{1}{6}, \frac{5}{12}, \frac{5}{12}\right]$$

(c)

$$P(2) = P^2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$= \left[\begin{array}{c|c|c} \frac{1}{4} + \frac{1}{6} & \frac{1}{6} & \frac{1}{4} + \frac{1}{2} \\ \hline \frac{1}{6} & \frac{1}{4} + \frac{1}{6} & \frac{1}{4} + \frac{1}{6} \\ \hline \frac{1}{6} + \frac{1}{4} & \frac{1}{6} + \frac{1}{4} & \frac{1}{6} + \frac{1}{6} + \frac{1}{9} \end{array}\right]$$

$$= \begin{bmatrix} \frac{5}{12} & \frac{1}{6} & \frac{5}{12} \\ \frac{1}{6} & \frac{5}{12} & \frac{5}{12} \\ \frac{5}{18} & \frac{5}{18} & \frac{4}{9} \end{bmatrix}$$

Problem	Ans	Reason
13	$P(2) = P^2 = \begin{bmatrix} 2 & 2 & 6 \\ 4 & 2 & 4 \\ 5 & 0 & 5 \end{bmatrix}\begin{bmatrix} 2 & 2 & 6 \\ 4 & 2 & 4 \\ 5 & 0 & 5 \end{bmatrix}$	$\frac{1}{100}$
	$= \left[\begin{array}{c\|c\|c} 4+8+30 & 4+4+0 & 12+8+30 \\ \hline 8+8+20 & 8+4+0 & 24+8+20 \\ \hline 10+0+25 & 10+0+0 & 30+0+25 \end{array}\right]$	$\frac{1}{100}$
	$= \begin{bmatrix} .42 & .08 & .50 \\ .36 & .12 & .52 \\ .35 & .10 & .55 \end{bmatrix}$	$.42+.08+.50=1$ ✓ $.36+.12+.52=1$ ✓ $.35+.10+.55=1$ ✓
	$P(3) = P^3 = P\,P(2) = \begin{bmatrix} 2 & 2 & 6 \\ 4 & 2 & 4 \\ 5 & 0 & 5 \end{bmatrix}\begin{bmatrix} 42 & 8 & 50 \\ 36 & 12 & 52 \\ 35 & 10 & 55 \end{bmatrix}$	$\frac{1}{1000}$
	$= \left[\begin{array}{c\|c\|c} 84+72+210 & 16+24+60 & 100+104+330 \\ \hline 168+72+140 & 32+24+40 & 200+196+220 \\ \hline 210+0+175 & 40+0+50 & 250+0+275 \end{array}\right]$	$\frac{1}{1000}$
	$= \begin{bmatrix} .366 & .100 & .534 \\ .380 & .096 & .524 \\ .385 & .090 & .525 \end{bmatrix}$	$.366+.100+.534=1$ ✓ $.380+.096+.524=1$ ✓ $.385+.090+.525=1$ ✓

Problem	Ans	Reason
14	(a) .6256 (b) 1−.6256 = .3744	$P = \begin{matrix} \\ H \\ L \end{matrix} \begin{matrix} \begin{matrix} H & L \end{matrix} \\ \begin{bmatrix} .7 & .3 \\ .5 & .5 \end{bmatrix} \end{matrix}$ Find (H,H)-entry of P^4 $P^2 = \begin{bmatrix} .7 & .3 \\ .5 & .5 \end{bmatrix}\begin{bmatrix} .7 & .3 \\ .5 & .5 \end{bmatrix} = \begin{bmatrix} .64 & .36 \\ .60 & .40 \end{bmatrix}$ $\begin{bmatrix} .64 & .36 \end{bmatrix}\begin{bmatrix} .64 \\ .60 \end{bmatrix} = .4096 + .2160$ $= .6256$
15	(a) .72 (b) .56	$P^2 = \begin{bmatrix} .8 & .2 \\ .4 & .6 \end{bmatrix}\begin{bmatrix} .8 & .2 \\ .4 & .6 \end{bmatrix}$ $= \begin{bmatrix} .72 & .28 \\ .56 & .44 \end{bmatrix}$

Problem	Ans	Reason
16	.553	Find $(1,2)$-entry of P^3

$$P^2 = \begin{bmatrix} .3 & .7 \\ .6 & .4 \end{bmatrix}\begin{bmatrix} .3 & .7 \\ .6 & .4 \end{bmatrix} = \begin{bmatrix} .51 & .49 \\ .42 & .58 \end{bmatrix}$$

$$\begin{bmatrix} .3 & .7 \end{bmatrix}\begin{bmatrix} .49 \\ .58 \end{bmatrix} = .553$$

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$$P(2\,1 = P^2 = \begin{bmatrix} 3 & 3 & 4 \\ 1 & 8 & 1 \\ 3 & 6 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 & 4 \\ 1 & 8 & 1 \\ 3 & 6 & 1 \end{bmatrix} \frac{1}{100}$$

$$= \begin{bmatrix} 9+3+12 & 9+24+24 & 12+3+4 \\ 3+8+3 & 3+64+6 & 4+8+1 \\ 9+6+3 & 9+48+6 & 12+6+1 \end{bmatrix} \frac{1}{100} = \begin{bmatrix} .24 & .57 & .19 \\ .14 & .73 & .13 \\ .18 & .63 & .19 \end{bmatrix}$$

$(3,3)$-entry of P^3 =

$$\begin{bmatrix} .3 & .6 & .1 \end{bmatrix}\begin{bmatrix} .19 \\ .13 \\ .19 \end{bmatrix} = .154$$

Problem	Ans	Reason
18	$\frac{14}{27}$	Find $p_{ww}(3)$ = (w,w)-entry of P^3

$$P = \begin{matrix} \\ M \\ W \end{matrix} \begin{matrix} \begin{matrix} M & \quad W \end{matrix} \\ \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{matrix}$$

$$P^2 = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{5}{9} \end{bmatrix}$$

(w,w)-entry of $P^3 = \frac{4}{9}\,\frac{1}{3} + \frac{5}{9}\,\frac{2}{3} = \frac{14}{27}$

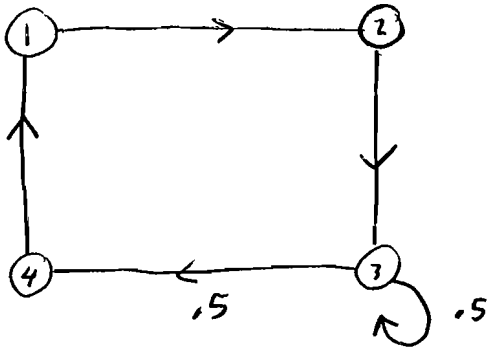
19	.18 Most Likely in Central district.	$P = \begin{matrix} \\ E \\ C \\ W \end{matrix} \begin{matrix} \begin{matrix} E & \quad C & \quad W \end{matrix} \\ \begin{bmatrix} .5 & .4 & .1 \\ .1 & .6 & .3 \\ .3 & .6 & .1 \end{bmatrix} \end{matrix}$

$$P^2 = \begin{bmatrix} .32 & .50 & .18 \\ & \times & \\ & & \end{bmatrix}$$

Problem	Ans	Reason

20

State	Prob
1	.5
2	0
3	.25
4	.25



Path	2341	2334	2333
Prob	.5	.25	.25

21

$$P = \begin{array}{c} \\ I \\ B \\ S \end{array} \begin{array}{c} \begin{array}{ccc} I & B & S \end{array} \\ \begin{bmatrix} .90 & .05 & .05 \\ .10 & .80 & .10 \\ .10 & .30 & .60 \end{bmatrix} \end{array}$$

(a) $P(2) = P^2 =$

$$\begin{bmatrix} 90 & 5 & 5 \\ 10 & 80 & 10 \\ 10 & 30 & 60 \end{bmatrix} \begin{bmatrix} 90 & 5 & 5 \\ 10 & 80 & 10 \\ 10 & 30 & 60 \end{bmatrix} \frac{1}{10000}$$

$$= \frac{1}{10000} \begin{bmatrix} 8100+50+50 & 450+400+150 & 450+50+300 \\ 900+800+100 & 50+6400+300 & 50+800+600 \\ 900+300+600 & 50+2400+1800 & 50+300+3600 \end{bmatrix}$$

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Problem	Ans	Reason
2 cmt	$= \begin{bmatrix} 8200 & 1000 & 800 \\ 1800 & 6750 & 1450 \\ 1800 & 4250 & 3950 \end{bmatrix} \frac{1}{10000}$	
	$= \begin{bmatrix} .820 & .100 & .080 \\ .180 & .675 & .145 \\ .180 & .425 & .395 \end{bmatrix}$	
	(b) .100	
22	Small bus .180 .425 .395 Max	
23	self employed .080 .145 .395 Max	

Problem	Ans	Reason
24		

$$P = \begin{array}{c} \\ C \\ L \\ M \end{array} \begin{array}{c} \begin{array}{ccc} C & L & M \end{array} \\ \begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix} \end{array}$$

$$2b = 3a$$

$$a + 2b = 1$$

$$\frac{2}{3}b + 2b = 1$$

$$b\,\frac{8}{3} = 1 \qquad b = 3/8 \qquad a = \frac{1}{4}$$

$$P = \begin{bmatrix} \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{4} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{4} \end{bmatrix}$$

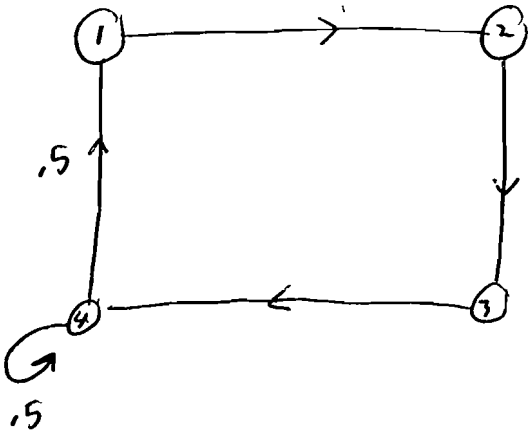
Find (C,C) - entry of P^4

$$P^2 = \begin{bmatrix} \frac{22}{64} & \frac{21}{64} & \frac{21}{64} \\ \frac{21}{64} & \frac{22}{64} & \frac{21}{64} \\ \frac{21}{64} & \frac{21}{64} & \frac{22}{64} \end{bmatrix}$$

$$(C,C)\text{-entry of } P^4 = \frac{22}{64}\frac{22}{64} + \frac{21}{64}\frac{21}{64} + \frac{21}{64}\frac{21}{64}$$

$$= \frac{484}{4096} + \frac{441}{4096} + \frac{441}{4096}$$

$$= \frac{1366}{4096}$$

Problem	Ans	Reason
25	Cadillac	$P^4 = \begin{pmatrix} \frac{1366}{4096} & c & c \\ & & \end{pmatrix}$ $2c + \frac{1366}{4096} = 1$ $\frac{1366}{4098}$ $4098 > 4096$ $c < \frac{1366}{4096}$
26	.5625	 $Pr = .5 + (.5)^4$ $= .5625$

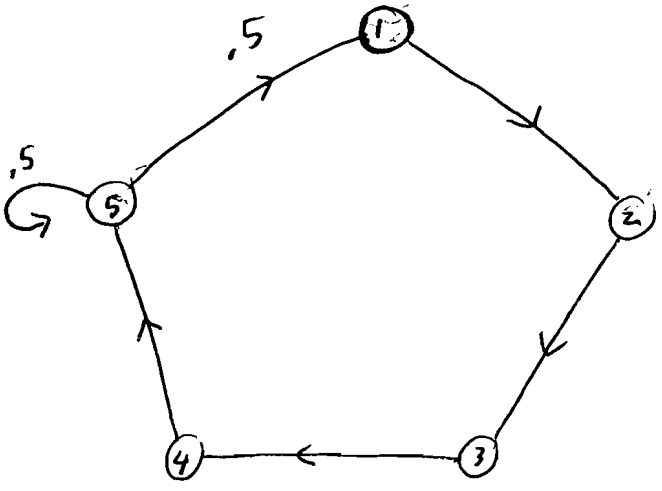
Problem	Ans	Reason

27 | 4

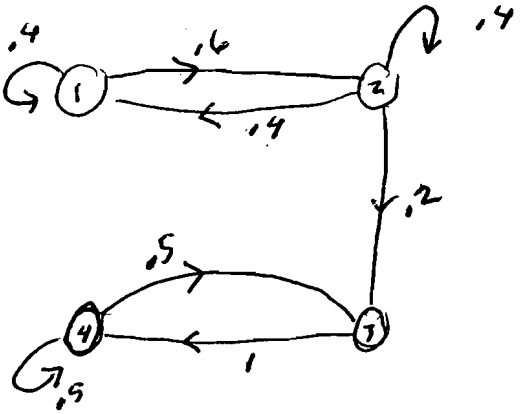
Steps	Prob
41234	.5
44444	$(.5)^4$
44123	$(.5)^2$
44412	$(.5)^3$
44441	$(.5)^4$

State	Prob	
1	.0625	
2	.125	
3	.250	
4	.5625	Max

28 | 8



Problem	Ans	Reason
29	(a) $P(4)=P^2=$ $\left[\begin{array}{cc\|cc} 4 & 6 & 0 & 0 \\ 4 & 4 & 2 & 0 \\ \hline 0 & 0 & 0 & 10 \\ 0 & 0 & 5 & 5 \end{array}\right]\left[\begin{array}{cc\|cc} 4 & 6 & 0 & 0 \\ 4 & 4 & 2 & 0 \\ \hline 0 & 0 & 0 & 10 \\ 0 & 0 & 5 & 5 \end{array}\right]$	$\frac{1}{100}$
	$=\left[\begin{array}{cc\|cc} 40 & 48 & 12 & 0 \\ 32 & 40 & 8 & 20 \\ \hline 0 & 0 & 50 & 50 \\ 0 & 0 & 25 & 75 \end{array}\right]$	$\frac{1}{100}$
	$=\left[\begin{array}{cc\|cc} .40 & .48 & .12 & 0 \\ .32 & .40 & .08 & .20 \\ \hline 0 & 0 & .50 & .50 \\ 0 & 0 & .25 & .75 \end{array}\right]$	
	To get $P(4)$, $P(8)$ use $P(4) = P(2)\ P(2)$ $P(8) = P(4)\ P(4)$	
	(b) NO	
	(c) NO	



Problem

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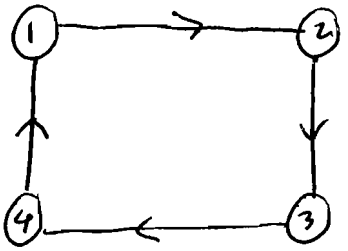

$P^4 = I$ so any power of P is one of

I, P, P^2, P^3

None of these has all entries pos.

31 $a = .1$

$$P = \begin{bmatrix} .6 & .4 \\ a & 1-a \end{bmatrix}$$

$p_{22}(3) = .825$

$$.4 = p_{11}(2) = (1,1)\text{-entry of } P^2$$

$$= \begin{bmatrix} .6 & .4 \end{bmatrix} \begin{bmatrix} .6 \\ a \end{bmatrix}$$

$$= .36 + .4a$$

$$.04 = .4a \quad \text{so} \quad a = .1$$

Problem

31, cont

$$P(2) = P^2 = \begin{bmatrix} .6 & .4 \\ .1 & .9 \end{bmatrix}\begin{bmatrix} .6 & .4 \\ .1 & .9 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} 36+4 & 24+36 \\ \hline 6+9 & 4+81 \end{array}\right] \frac{1}{100}$$

$$= \begin{bmatrix} .40 & .60 \\ .15 & .85 \end{bmatrix}$$

$p_{22}(3) =$ (2,2)-entry of $P(3) = P^3$

$$= \begin{bmatrix} .15 & .85 \end{bmatrix}\begin{bmatrix} .4 \\ .9 \end{bmatrix}$$

$$= .060 + .765 = .825$$

32

$.36 = p_{21}(2) =$ (2,1)-entry of P^2

$$= \begin{bmatrix} .2 & a & .8-a \end{bmatrix}\begin{bmatrix} .4 \\ .2 \\ .6 \end{bmatrix}$$

$$= .08 + .2a + .48 - .6a$$
$$= .56 - .4a$$

$$.4a = .20$$
$$a = .5$$

$p_{33}(2) =$ (3,3)-entry of $P^2 = \begin{bmatrix} .6 & .3 & .1 \end{bmatrix}\begin{bmatrix} .4 \\ .3 \\ .1 \end{bmatrix} = .34$

□