Section 6.R Solutions

No. 6.R
Date 1

Problem	Ans	Reason
1	(a) 4×2	
	(b) 2×3	
	(c) 1×4	
	(d) 3×1	
2	(a) Not def	$D\ A\ B$: $5\times5\ \ 3\times5\ \ 5\times3$ — NO
	(b) def	$C\ A\ B$: $3\times3\ \ 3\times5\ \ 5\times3$
	(c) def	$A\ B\ C$: $3\times5\ \ 5\times3\ \ 3\times3$
	(d) def	$B\ A\ D$: $5\times3\ \ 3\times5\ \ 5\times5$
3	$5A+B=\begin{bmatrix}0 & 34\\ 12 & 83\\ 65 & -38\end{bmatrix}$	$5A+B=\begin{bmatrix}-1(5)+5=0 & 7(5)-1=34\\ 0(5)+12=12 & 15(5)+8=83\\ 12(5)+5=65 & -8(5)+2=-38\end{bmatrix}$
	$A-2B=\begin{bmatrix}-11 & 9\\ 24 & -1\\ 2 & -12\end{bmatrix}$	$A-2B=\begin{bmatrix}-1-2(5)=-11 & 7-2(-1)=9\\ 0-2(12)=24 & 15-2(8)=-1\\ 12-2(5)=2 & -8-2(2)=-12\end{bmatrix}$
4	$AX=B$ since	
	$10(-1)+3\cdot5=5$ ✓	
	$0(-1)+5\cdot5=25$ ✓	
	$-2(-1)+4\cdot5=22$ ✓	

Problem	Ans	Reason
5	$Y = \begin{bmatrix} 3 \\ 10 \\ 16 \end{bmatrix}$	$Y = \frac{1}{2}(B - X)$ $= \frac{1}{2}\begin{bmatrix} 5+1 \\ 25-5 \\ 22+10 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 16 \end{bmatrix}$
	$Z = \begin{bmatrix} 7 \\ 15 \\ 42 \end{bmatrix}$	$Z = B - 2X$ $= \begin{bmatrix} 5 & -2(-1) \\ 25 & -2(5) \\ 22 & -2(-10) \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \\ 42 \end{bmatrix}$
6	$2A+3B = \begin{bmatrix} 8 & -3 & 9 \\ 2 & 11 & -1 \\ -12 & 1 & -7 \end{bmatrix}$	$\left[\begin{array}{c\|c\|c} 2(1)+3(2) = 8 & 2(0)+3(-1) = -3 & 2(3)+3(1) = 9 \\ \hline 2(1)+3(0) = 2 & 2(1)+3(3) = 11 & 2(1)+3(-1) = -1 \\ \hline 2(-3)+3(-2) = -12 & 2(2)+3(-1) = 1 & 2(1)+3(-3) = -7 \end{array}\right]$
7	$p = -1$ $q = -1$	Require $1 \cdot p + 2q = -3$ ((1,1)-entry) $0 \cdot p - q = 1$ ((1,2)-entry so $q = -1$ $p = -3 - 2q$ $= -3 + 2 = -1$

Problem	Ans	Reason
8	(a) 3×3	A B C 3×5 5×3 3×3
	(b) 5×5	B A D 5×3 3×5 5×5
	(c) Not def	A B D 3×5 5×3 5×5 ≠
	(d) Not def	D B A C 5×5 5×3 3×5 ≠ 3×3
19	(a) $\begin{bmatrix} 6 & 12 \\ 2 & 10 \end{bmatrix}$	
	(b) not def	
	(c) $\begin{bmatrix} 8 & 7 \\ 3 & 5 \\ 0 & 6 \end{bmatrix}$	
	(d) $\begin{bmatrix} 33 & 75 \\ 5 & 25 \\ -2 & 8 \end{bmatrix}$ $=$ $\begin{bmatrix} 30+3 = 33 & 60+15 = 75 \\ 0+5 = 5 & 0+25 = 25 \\ -6+4 = -2 & -12+20 = 8 \end{bmatrix}$	
	(e) not def	
	(f) $\begin{bmatrix} 36 \\ 10 \\ 2 \end{bmatrix}$	

Problem	Ans	Reason
10	(a) $\begin{bmatrix} 6 & -3 \\ 9 & 3 \end{bmatrix}$	
	(b) $\begin{bmatrix} 17 & 8 \\ 2 & -9 \\ 9 & 2 \end{bmatrix} = \begin{bmatrix} 16+1 & 8-0 \\ 4-2 & -4-5 \\ 12-3 & 0+2 \end{bmatrix}$	
	(c) Not def	
	(d) $\begin{bmatrix} 14 & -2 \\ -1 & -2 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 4(2)+2(3) = 14 & 4(-1)+2(1) = -2 \\ 1(2)-1(3) = -1 & 1(-1)-1(1) = -2 \\ 3(2)+0(3) = 6 & 3(-1)+0(1) = -3 \end{bmatrix}$	
	(e) $\begin{bmatrix} -2 \\ -5 \\ -6 \end{bmatrix} = \begin{bmatrix} 4(-2)+2(3) \\ 1(-2)+-1(3) \\ 3(-2)+0(3) \end{bmatrix}$	
	(f) $\begin{bmatrix} 2 \\ 11 \\ -12 \end{bmatrix} = \begin{bmatrix} -1(-2)+0(3) \\ 2(-2)+5(3) \\ 3(-2)+-2(3) \end{bmatrix}$	

Problem	Ans	Reason
11	(a) $\begin{bmatrix} 4 & 0 \\ 6 & -2 \\ -4 & 0 \end{bmatrix}$	
	(b) $\begin{bmatrix} -2 & 4 & -3 \\ -2 & -3 & 1 \end{bmatrix} = \left[\begin{array}{c\|c\|c} 2\cdot 1-4 & 2\cdot 2-0 & 2\cdot 4-1 \\ \hline 2\cdot 0-2 & 2\cdot -1-1 & 22-3 \end{array}\right]$	
	(c) not def	
	(d) $\begin{bmatrix} 1 & 0 & 3 \\ -2 & -7 & 8 \end{bmatrix} = \left[\begin{array}{c\|c\|c} 1(1)+2(0) & 1(2)+2(-1) & 1(7)+2(2) \\ \hline -2(1)+3(0) & -2(2)+3(-1) & -2(7)+3(2) \end{array}\right]$	
	(e) not def	
	(f) $\begin{bmatrix} 10 & -2 \\ -7 & 1 \end{bmatrix} = \left[\begin{array}{c\|c} 2+6+2 & 0-2+0 \\ \hline 0-3-4 & 0+1+0 \end{array}\right]$	

Problem	Ans	Reason
12	(a) $\begin{bmatrix} 5 & 3 \\ -1 & -2 \\ -2 & 4 \end{bmatrix}$	
	(b) Not def	
	(c) Not def	
	(d) $\begin{bmatrix} 11 \\ 12 \end{bmatrix} = \begin{bmatrix} 1+20-10 \\ 5-8+15 \end{bmatrix}$	
	(e) Not def	
	(f) $\begin{bmatrix} -3 & -1 \\ 12 & -7 \end{bmatrix} = \begin{bmatrix} 2-5+0 & -3+10-8 \\ 10+2+0 & -15-4+12 \end{bmatrix}$	

Problem	Ans	Reason
13	$A^2 = \begin{bmatrix} 7 & -6 \\ -18 & 19 \end{bmatrix}$	$\underbrace{\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}}_{A} = \left[\begin{array}{c\|c} 4+3 & -2-4 \\ \hline -6-12 & 3+16 \end{array}\right]$
	$AB = \begin{bmatrix} 9 & -6 & -1 \\ -16 & 14 & 4 \end{bmatrix} = \left[\begin{array}{c\|c\|c} 8+1 & -4-2 & 0-1 \\ \hline -12-4 & 6+8 & 0+4 \end{array}\right]$	
	$A(AB) = A^2B =$	
	$\begin{bmatrix} 34 & -26 & -6 \\ -91 & 74 & 19 \end{bmatrix} = \left[\begin{array}{c\|c\|c} 18+16 & -12-14 & -2-4 \\ \hline -27-64 & 18+56 & 3+16 \end{array}\right]$	

Problem	Ans	Reason
14	$AB = \begin{bmatrix} 9 & -6 & 3 \\ -10 & 8 & -4 \\ 16 & -14 & 7 \end{bmatrix} = \left[\begin{array}{c\|c\|c} 8+1 & -4-2 & 2+1 \\ \hline -8-2 & 4+4 & -2-2 \\ \hline 12+4 & -6-8 & 3+4 \end{array}\right]$	
	$BA = \begin{bmatrix} 15 & -12 \\ -9 & 9 \end{bmatrix} = \left[\begin{array}{c\|c} 8+4+3 & -4-4-4 \\ \hline -2-4-3 & 1+4+4 \end{array}\right]$	
	$BAB = B(AB) = (BA)B =$ $\begin{bmatrix} 72 & -54 & 27 \\ -45 & 36 & -18 \end{bmatrix} = \left[\begin{array}{c\|c\|c} 36+20+16 & -24-16-14 & 12+8+7 \\ \hline -9-20-16 & 6+16+14 & -3-8-7 \end{array}\right]$	

Problem	Ans	Reason
15	$AC = \begin{bmatrix} 5 & 10 \\ 4 & 8 \end{bmatrix} = \left[\begin{array}{c\|c} 6-1 & 12-2 \\ \hline -4+8 & -8+16 \end{array}\right]$	
	$BC = \begin{bmatrix} 5 & 10 \\ 4 & 8 \end{bmatrix} = \left[\begin{array}{c\|c} 4+1 & 8+2 \\ \hline 0+4 & 0+8 \end{array}\right]$	
	$(A-B)C = O$ and $A-B \neq O$ so C^{-1} does not exist.	
16	$D = \frac{A-B}{2} = \begin{bmatrix} \frac{1}{2} & -1 \\ -1 & 2 \end{bmatrix}$	$\frac{1}{2}\left[\begin{array}{c\|c} 3-2 & -1-1 \\ \hline -2-0 & 8-4 \end{array}\right]$
17	$A^{-1} = \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$	$7 \cdot 3 - 4 \cdot 5 = 21 - 20 = 1$
18	$A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$	$6 \cdot 4 - 5 \cdot 5 = -1$
	$(A^2)^{-1} = A^{-2} = \begin{bmatrix} 41 & -50 \\ -50 & 61 \end{bmatrix}$	

Problem	Ans	Reason

19

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 - 2r_1 \\ r_3' = r_3 - 3r_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 3 & -1 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right] \quad r_1' = r_1 - r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & -1 & -1 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right] \quad r_1' = r_1 - r_3$$

check ✓

$$\left[\begin{array}{ccc} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{array}\right]\left[\begin{array}{ccc} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \left[\begin{array}{ccc} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{array}\right]$$

Problem	Ans	Reason

20

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 1 & 0 \\ -\frac{1}{5} & -\frac{1}{5} & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & -\frac{3}{10} & 1 & \frac{1}{5} & 0 & 1 \end{array}\right] \quad r_3' = r_3 + \frac{1}{5} r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & \frac{17}{20} & \frac{1}{5} & \frac{3}{10} & 1 \end{array}\right] \quad r_1' = r_1 + \frac{1}{2} r_2 \qquad r_3' = r_3 + \frac{3}{10} r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{17} & \frac{6}{17} & \frac{20}{17} \end{array}\right] \quad r_3' = r_3 \frac{20}{17}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{18}{17} & \frac{10}{17} & \frac{5}{17} \\ 0 & 1 & 0 & \frac{2}{17} & \frac{20}{17} & \frac{10}{17} \\ 0 & 0 & 1 & \frac{4}{17} & \frac{6}{17} & \frac{20}{17} \end{array}\right] \quad r_1' = r_1 + \frac{1}{4} r_3 \qquad r_2' = r_2 + \frac{1}{2} r_3$$

check

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} \\ -\frac{1}{5} & -\frac{1}{5} & 1 \end{bmatrix} \begin{bmatrix} 18 & 10 & 5 \\ 2 & 20 & 10 \\ 4 & 6 & 20 \end{bmatrix} \frac{1}{17} \overset{\checkmark}{=} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{17} \begin{bmatrix} 18 & 10 & 5 \\ 2 & 20 & 10 \\ 4 & 6 & 20 \end{bmatrix}$$

Problem	Ans	Reason

21

First find A^{-1}

$$3 \cdot 4 - 2 \cdot 5 = 12 - 10 = 2$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}$$

(a)

$$C = A^{-1}B = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} -2 & 8 \\ 4 & 5 \end{bmatrix}$$

$$= \frac{1}{2}\left[\begin{array}{c|c} -8-8 & 32-10 \\ \hline 10+12 & -40+15 \end{array}\right] = \frac{1}{2}\begin{bmatrix} -16 & 22 \\ 22 & -25 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 11 \\ 11 & -\frac{25}{2} \end{bmatrix}$$

(b) $D = BA^{-1} = \frac{1}{2}\begin{pmatrix} -2 & 8 \\ 4 & 5 \end{pmatrix}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}$

$$= \frac{1}{2}\left[\begin{array}{c|c} -8-40 & 4+24 \\ \hline 16-25 & -8+15 \end{array}\right]$$

$$= \frac{1}{2}\begin{bmatrix} -48 & 28 \\ -9 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -24 & 14 \\ -\frac{9}{2} & \frac{7}{2} \end{bmatrix}$$

No. G.R

Date 13

Problem	Ans	Reason
22	A^{-1} does not exist	We notice row 1 + row 2 = row 3 so $[1\ 1\ -1]A = O$
		Now A^{-1} does not exist otherwise
		$O = OA^{-1} = [1\ 1\ -1]AA^{-1} = [1\ 1\ -1]$
23	$k = -6 \Leftrightarrow$ A^{-1} does not exist	$3 \cdot 12 + 6k \neq 0 \Leftrightarrow A^{-1}$ exists
24		

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 7 & 0 & 1 & 0 \\ 3 & 8 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 2 & 0 & -3 & 0 & 1 \end{array}\right] \quad r_2' = r_2 - 2r_1 \quad r_3' = r_3 - 3r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right]$$

Problem | ans | Reason

24, cont

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 4 & 0 & -1 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right] \quad r_1' = r_1 - 2r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -3 & -1 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right] \quad r_1' = r_1 - 3r_3$$

check

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 8 & 9 \end{bmatrix} \begin{bmatrix} 10 & -3 & -1 \\ -3/2 & 0 & \frac{1}{2} \\ -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$$A^{-1} = \begin{bmatrix} 10 & -3 & -1 \\ -3/2 & 0 & \frac{1}{2} \\ -2 & 1 & 0 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 10 & -3 & -1 \\ -3/2 & 0 & \frac{1}{2} \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} 110 \\ -20 \\ -20 \end{bmatrix}$$

check

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 8 & 9 \end{bmatrix} \begin{bmatrix} 110 \\ -20 \\ -20 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -10 \end{bmatrix} \checkmark$$

Problem	Ans	Reason

25

$$X = \begin{bmatrix} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \\ 6 \end{bmatrix}$$

$$X = \begin{bmatrix} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -7 \\ 2 \\ 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Check

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} -11 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} -7 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \checkmark$$

No. 6.R

Date 16

Problem	Ans	Reason
26	(a) $B = A^{-1}C$	

$$= \begin{bmatrix} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 4 & 1 \\ -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -10 \\ -6 & 3 \\ -18 & 6 \end{bmatrix}$$

check $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} 29 & -10 \\ -6 & 3 \\ -18 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 4 & 1 \\ -3 & 3 \end{bmatrix}$

(b) $H = DA^{-1}$

No such H since D is 3×2 A^{-1} is 3×3

Problem	Ans	Reason
27	$0 < a < .8$	

$$A = \begin{bmatrix} .2 & a \\ a & .2 \end{bmatrix}$$

$$I - A = \begin{bmatrix} .8 & -a \\ -a & .8 \end{bmatrix}$$

Find $(I-A)^{-1}$:

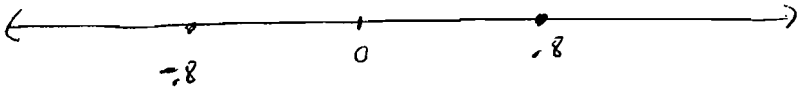
Require $(.8)(.8) - a^2 \neq 0$

$a \neq \mp .8$

$$(I-A)^{-1} = \frac{1}{(.8)^2 - a^2} \begin{bmatrix} .8 & a \\ a & .8 \end{bmatrix}$$

Require (1,1) entry is > 0

So $(.8)^2 - a^2 > 0$



So for $-.8 < a < .8$

Require (1,2)-entry is > 0

So $a > 0$