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HW Solutions 6.2

Problem	Ans	Reason
1	$B = A^{-1}$	check $AB = I$, $BA = I$

$$\underset{A}{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}} \underset{B}{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}} = \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}\right]$$

$$\underset{B}{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}} = \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}\right]$$

Problem	Ans	Reason
2	$B = A^{-1}$	check $AB = I$, $BA = I$

$$\underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \underset{B}{\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}} = \underset{I}{\left[\begin{array}{c|c} -2+3=1 & 1-1=0 \\ \hline -6+6=0 & 3-2=1 \end{array}\right]}$$

$$\underset{B}{\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \underset{I}{\left[\begin{array}{c|c} -2+3=1 & -4+4=0 \\ \hline \frac{3}{2}-\frac{3}{2}=0 & 3-2=1 \end{array}\right]}$$

Problem	Ans	Reason
3	$y = 5$	Write $B = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$ $AX = B$ so $X = A^{-1}B$ so $y = $ Coord 2 of X $=$ row 2 of A^{-1} times B $= 1(-1) + 2(4) - 2(1)$ $= 5$
4	$B = A^{-1}$	$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
5	✓	$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 9-8=1 & -6+6=0 \\ 12-12=0 & -8+9=1 \end{bmatrix} = I$ $\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 9-8=1 & 6-6=0 \\ -12+12=0 & -8+9=1 \end{bmatrix} = I$

Problem	Ans	Reason

6

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{bmatrix} = \left[\begin{array}{c|c|c} 0+0+1=1 & -2+0+2=0 & 2+0-2=0 \\ \hline 0+1-1=0 & -2+5-2=1 & 2-4+2=0 \\ \hline 0+1-1=0 & -3+5-2=0 & 3-4+2=1 \end{array}\right] = I$$

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix} = \left[\begin{array}{c|c|c} 0-2+3=1 & 0-1+1=0 & 0+1-1=0 \\ \hline 2+10- =0 & 0+5-4=1 & 1-5+4=0 \\ \hline 2+4-6=0 & 0+2-2=0 & 1-2+2=1 \end{array}\right] = I$$

7

$C^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array}\right] \quad r_2' = r_2 - 2r_1$$

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{1}{5} \end{array}\right] \quad r_2' = \frac{1}{5} r_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & \frac{1}{5} \\ 0 & 1 & \frac{-2}{5} & \frac{1}{5} \end{array}\right] \quad r_1' = r_1 + r_2$$

check

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} = \left[\begin{array}{c|c} \frac{3}{5}+\frac{2}{5}=1 & \frac{1}{5}-\frac{1}{5}=0 \\ \hline \frac{6}{5}-\frac{6}{5}=0 & \frac{2}{5}+\frac{3}{5}=1 \end{array}\right] = I \checkmark$$

Problem	Ans	Reason

7, cont

$D^{-1} = \begin{bmatrix} 20 & -10 \\ -\frac{50}{4} & \frac{30}{4} \end{bmatrix}$

$$\left[\begin{array}{cc|cc} .3 & .4 & 1 & 0 \\ .5 & .8 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 3 & 4 & 10 & 0 \\ 5 & 8 & 0 & 10 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 4/3 & \frac{10}{3} & 0 \\ 5 & 8 & 0 & 10 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 4/3 & \frac{10}{3} & 0 \\ 0 & 4/3 & -\frac{50}{3} & 10 \end{array}\right] \quad r_2' = r_2 - 5r_1$$

$$\left[\begin{array}{cc|cc} 1 & \frac{4}{3} & \frac{10}{3} & 0 \\ 0 & 1 & -\frac{50}{4} & \frac{30}{4} \end{array}\right] \quad r_2' = r_2 \frac{3}{4}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 20 & -10 \\ 0 & 1 & -\frac{50}{4} & \frac{30}{4} \end{array}\right] \quad r_1' = r_1 - 4/3\, r_2$$

check

$$\frac{1}{10}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} 20 & -10 \\ -\frac{50}{4} & \frac{30}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Problem	Ans	Reason
8	$c=2$	First note A^{-1} exists

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix} \frac{1}{4}$$

So AB has an inverse if and only if B has an inverse

Find B^{-1}

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & c & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & c-2 & -2 & 1 \end{array}\right] \quad r_2' = r_2 - 2r_1$$

B^{-1} exists as long as $c \neq 2$

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Problem	Ans	Reason
9	$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & 1 \end{bmatrix}$	Find A^{-1} $\quad A = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$
	$X = \begin{bmatrix} \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$	$\left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$
	$X = \begin{bmatrix} \frac{1}{2} \\ -\frac{5}{2} \end{bmatrix}$	$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{3}{2} & 1 \end{array}\right] \quad r_2' = r_2 - 3r_1$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & 1 \end{bmatrix}$$

check $\begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$AX = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ so $X = A^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$X = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -3/2 \end{bmatrix}$$

$AX = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ so $X = A^{-1}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{5}{2} \end{bmatrix}$$

Problem	Ans	Reason
10	$A^{-1}=\begin{bmatrix}3 & -1\\ -\frac{5}{3} & \frac{2}{3}\end{bmatrix}$	
	$X=\begin{bmatrix}7\\ -\frac{11}{3}\end{bmatrix}$	
	$X=\begin{bmatrix}13\\ -\frac{22}{3}\end{bmatrix}$	

$$\left[\begin{array}{cc|cc}2 & 3 & 1 & 0\\ 5 & 9 & 0 & 1\end{array}\right]$$

$$\left[\begin{array}{cc|cc}1 & \frac{3}{2} & \frac{1}{2} & 0\\ 5 & 9 & 0 & 1\end{array}\right]$$

$$\left[\begin{array}{cc|cc}1 & \frac{3}{2} & \frac{1}{2} & 0\\ 0 & \frac{3}{2} & -\frac{5}{2} & 1\end{array}\right] \quad r_2' = r_2 - 5r_1$$

$$\left[\begin{array}{cc|cc}1 & \frac{3}{2} & \frac{1}{2} & 0\\ 0 & 1 & -\frac{5}{3} & \frac{2}{3}\end{array}\right] \quad r_2' = r_2 \tfrac{2}{3}$$

$$\left[\begin{array}{cc|cc}1 & 0 & 3 & -1\\ 0 & 1 & -\frac{5}{3} & \frac{2}{3}\end{array}\right] \quad r_1' = r_1 - \tfrac{3}{2} r_2$$

check $\begin{bmatrix}2 & 3\\ 5 & 9\end{bmatrix}\begin{bmatrix}3 & -1\\ -\frac{5}{3} & \frac{2}{3}\end{bmatrix} = \left[\begin{array}{c|c}1 & 0\\ \hline 0 & 1\end{array}\right] = I$ ✓

$$X = \begin{bmatrix}3 & -1\\ -\frac{5}{3} & \frac{2}{3}\end{bmatrix}\begin{bmatrix}3\\ 2\end{bmatrix} = \begin{bmatrix}7\\ -\frac{11}{3}\end{bmatrix}$$

$$X = \begin{bmatrix}3 & -1\\ -\frac{5}{3} & \frac{2}{3}\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = \begin{bmatrix}13\\ -\frac{22}{3}\end{bmatrix}$$

Problem	Ans	Reason

11 A^{-1} does not exist

$$\left[\begin{array}{cc|cc} 3 & -2 & 1 & 0 \\ -6 & 4 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -2/3 & \frac{1}{3} & 0 \\ -6 & 4 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 2 & 1 \end{array}\right] \quad r_2' = r_2 + 6r_1$$

12 (c) is correct

Find H^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & k & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & k-2 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad r_2' = r_2 - r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & k-2 & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & k-2 & -1 & 1 & 1 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - r_2 \\ r_3' = r_3 + r_2 \end{array}$$

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Problem	Ans	Reason
13.	$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$	

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1 \leftrightarrow r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1' = \frac{1}{2} r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - \frac{1}{2} r_3 \\ r_2' = r_2 - r_3 \end{array}$$

check

$$\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

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Problem	Ans	Reason
14	$A^{-1}=\begin{bmatrix} 0 & 2 & 4 \\ 1 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}$	

$$\left[\begin{array}{ccc|ccc} 0 & 2 & 4 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1 \leftrightarrow r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & -3 & 1 \end{array}\right] \quad r_3' = r_3 - 3r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{3}{2} & \frac{-1}{2} \end{array}\right] \quad \begin{array}{l} r_2' = \frac{1}{2} r_2 \\ r_3' = -\frac{1}{2} r_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & -3 & 1 \\ 0 & 0 & 1 & 0 & \frac{3}{2} & \frac{-1}{2} \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - r_3 \\ r_2' = r_2 - 2r_3 \end{array}$$

check

$$\begin{bmatrix} 0 & 2 & 4 \\ 1 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -3 & 1 \\ 0 & \frac{3}{2} & \frac{-1}{2} \end{bmatrix} \overset{\checkmark}{=} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

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Problem	Ans	Reason
15		

(a)

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$AX = B$$

(b)

$x_1 = -11$
$x_2 = 5$

Find A^{-1}

$$\left[\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 2 & 4 & 1 & 0 \end{array}\right] \quad r_1 \leftrightarrow r_2$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & -2 & 1 & -2 \end{array}\right] \quad r_2' = r_2 - 2r_1$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & 1 & -1/2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 3/2 & -2 \\ 0 & 1 & -1/2 & 1 \end{array}\right] \quad r_1' = r_1 - 3r_2$$

check

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

$$X = A^{-1}B = \begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -11 \\ 5 \end{bmatrix}$$

Problem	Ans	Reason

16

(a)

$$A = \begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -2 \\ 0 & 2 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$$

$$AX = B$$

(b)

$$\left[\begin{array}{ccc|ccc} 2 & -2 & 0 & 1 & 0 & 0 \\ 3 & 0 & -2 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 3 & 0 & -2 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad r_1' = \frac{1}{2} r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 3 & -2 & -\frac{3}{2} & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad r_2' = r_2 - 3r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -1 & -3/2 & 1 & -1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad r_2' = r_2 - r_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 1 & -1 \\ 0 & 1 & -1 & -3/2 & 1 & -1 \\ 0 & 0 & 1 & 3 & -2 & 3 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 + r_2 \\ \\ r_3' = r_3 - 2r_2 \end{array}$$

Problem	Ans	Reason

16, cnt

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 2 \\ 0 & 1 & 0 & \frac{3}{2} & -1 & 2 \\ 0 & 0 & 1 & 3 & -2 & 3 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 + r_3 \\ r_2' = r_2 + r_3 \end{array}$$

check

$$\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -2 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 2 \\ \frac{3}{2} & -1 & 2 \\ 3 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$$A^{-1} = \begin{bmatrix} 2 & -1 & 2 \\ \frac{3}{2} & -1 & 2 \\ 3 & -2 & 3 \end{bmatrix}$$

(c)

$$X = A^{-1}B = \begin{bmatrix} 2 & -1 & 2 \\ \frac{3}{2} & -1 & 2 \\ 3 & -2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ -5 \\ -8 \end{bmatrix}$$

check

$$\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -2 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ -5 \\ -8 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} \checkmark$$

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Problem	Ans	Reason
17	$B = A^{-1}$ if and only if $x = 3$	$B = A^{-1}$ whenever $AB = I$

$$AB \overset{?}{=} I$$

$$\begin{pmatrix} 1 & -2 & 1 \\ -1 & x & -1 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \overset{?}{=} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ x-3 & x-2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

require $x = 3$

Problem	Ans	Reason
18	$A^{-1} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$	Find A^{-1}

$$\left[\begin{array}{cc|cc} 5 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 2 & 1 & 0 & 1 \end{array}\right] \quad r_1' = r_1 - 2r_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & 5 \end{array}\right] \quad r_2' = r_2 - 2r_1$$

check

$$\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

$B^{-1} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$ Find B^{-1}

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] \quad r_2' = r_2 - 2r_1$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right] \quad r_2' = -r_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & 1 & 2 & -1 \end{array}\right] \quad r_1' = r_1 - 3r_2$$

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Problem	Ans	Reason

18, cont

check

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

Find $(AB)^{-1}$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 25 \\ 4 & -9 \end{bmatrix}$$

Find $(BA)^{-1}$

$$(BA)^{-1} = A^{-1} B^{-1}$$

$$= \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 5 \\ 20 & -11 \end{bmatrix}$$

Problem	Ans	Reason
19	$A^{-1}B^{-1} = (BA)^{-1}$	
	$B^{-1}A^{-1} = (AB)^{-1}$	
20	(a) $B = \begin{bmatrix} \frac{1}{5} & \frac{9}{5} \\ \frac{8}{5} & \frac{12}{5} \end{bmatrix}$	If $AB = C$ then $B = A^{-1}C$

$$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-1}{5} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{5} & \frac{-1}{5} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} & \frac{9}{5} \\ \frac{8}{5} & \frac{12}{5} \end{bmatrix}$$

check

$$\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 9 \\ 8 & 12 \end{bmatrix}\frac{1}{5} \stackrel{?}{=} \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix} \checkmark$$

(b) $B = \begin{bmatrix} 4 & 2 \\ \frac{-8}{5} & \frac{-7}{5} \end{bmatrix}$

If $BA = C$ then $B = CA^{-1}$

$$B = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} \frac{1}{5} & \frac{-1}{5} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -\frac{8}{5} & -\frac{7}{5} \end{bmatrix}$$

check

$$\begin{bmatrix} 4 & 2 \\ -8/5 & -7/5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix} \checkmark$$

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Problem	Ans	Reason

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(a)

$$A^{-1} = \begin{bmatrix} -10 & 0 & -15 \\ -8 & 2 & -8 \\ -5 & 0 & -5 \end{bmatrix} \qquad A = \frac{1}{10}\begin{bmatrix} 2 & 0 & -6 \\ 0 & 5 & -8 \\ -2 & 0 & 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & -6 & 1 & 0 & 0 \\ 0 & 5 & -8 & 0 & 1 & 0 \\ -2 & 0 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ 0 & 5 & -8 & 0 & 1 & 0 \\ -2 & 0 & 4 & 0 & 0 & 1 \end{array}\right] \qquad r_1' = -r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ 0 & 5 & -8 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 & 0 & 1 \end{array}\right] \qquad r_3' = r_3 + 2r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -8/5 & 0 & 1/5 & 0 \\ 0 & 0 & 1 & -1/2 & 0 & -1/2 \end{array}\right] \qquad \begin{array}{l} r_2' = \frac{1}{5} r_2 \\ r_3' = -\frac{1}{2} r_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -3/2 \\ 0 & 1 & 0 & -4/5 & \frac{1}{5} & -\frac{4}{5} \\ 0 & 0 & 1 & -1/2 & 0 & -1/2 \end{array}\right] \qquad \begin{array}{l} r_1' = r_1 + 3r_3 \\ r_2' = r_2 + 8/5\, r_3 \end{array}$$

check

$$\begin{bmatrix} 2 & 0 & -6 \\ 0 & 5 & -8 \\ -2 & 0 & 4 \end{bmatrix}\begin{bmatrix} -1 & 0 & -3/2 \\ -4/5 & 1/5 & -4/5 \\ -1/2 & 0 & -1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

(b) $AA^{-1}A = A$

Problem	Ans	Reason
22		

Suppose $(A+B)^{-1} = A^{-1} + B^{-1}$

then
$$
\begin{aligned}
I &= (A+B)(A+B)^{-1} \\
&= (A+B)(A^{-1}+B^{-1}) \\
&= AA^{-1} + BA^{-1} + AB^{-1} + BB^{-1} \\
&= I + BA^{-1} + AB^{-1} + I
\end{aligned}
$$

so
$$BA^{-1} + AB^{-1} + I = 0$$

But $BA^{-1} + AB^{-1} + I =$

$$\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 13 \\ -8 & 21 \end{pmatrix} + \begin{pmatrix} -21 & 13 \\ -8 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -25 & 26 \\ -16 & 27 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

So $(A+B)^{-1} \neq A^{-1} + B^{-1}$

Problem	Ans	Reason

23 (a) $x=-2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ -1 & 3 & -2 & 0 & 1 & 0 \\ -1 & 2 & x & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & x+2 & 1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 + r_1 \\ r_3' = r_3 + r_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & x+2 & 1 & 0 & 1 \\ 0 & 2 & 0 & 1 & 1 & 0 \end{array}\right] \quad r_2 \leftrightarrow r_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & x+4 & 2 & 0 & 1 \\ 0 & 1 & x+2 & 1 & 0 & 1 \\ 0 & 0 & -2(x+2) & -1 & 1 & -2 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 + r_2 \\ \\ r_3' = r_3 - 2r_2 \end{array}$$

(b) $\longrightarrow$ now assume $x \neq -2$

(2,2)-entry of A^{-1} is $\frac{1}{2}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & x+4 & 2 & 0 & 1 \\ 0 & 1 & x+2 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{2(x+2)} & \frac{-1}{2(x+2)} & \frac{1}{x+2} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3x+4}{2(x+2)} & \frac{x+4}{2(x+2)} & \frac{-2}{x+2} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2(x+2)} & \frac{-1}{2(x+2)} & \frac{1}{x+2} \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - (x+4) r_3 \\ r_2' = r_2 - (x+2) r_3 \end{array}$$

check

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 3 & -2 \\ -1 & 2 & x \end{bmatrix} \begin{bmatrix} 3x+4 & x+4 & -4 \\ x+2 & x+2 & 0 \\ 1 & -1 & 2 \end{bmatrix} \frac{1}{2(x+2)} \overset{?}{=} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

No. 6.2

Date 21

Problem	Ans	Reason
24	$\left[\begin{array}{cccc\|cccc} 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$	
	$\left[\begin{array}{cccc\|cccc} 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$	$r_1 \leftrightarrow r_2$
	$\left[\begin{array}{cccc\|cccc} 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$	$r_3' = r_3 - r_1$ $r_4' = r_4 - r_1$
	$\left[\begin{array}{cccc\|cccc} 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & -1 & -2 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$	$r_3' = r_3 - 2r_2$
	$\left[\begin{array}{cccc\|cccc} 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -5 & -2 & 3 & 1 & -4 \\ 0 & 0 & -1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$	$r_3' = r_3 - 4r_4$
	$\left[\begin{array}{cccc\|cccc} 1 & 0 & 0 & -6 & -2 & 4 & 1 & -4 \\ 0 & 1 & 0 & 11 & 5 & -6 & -2 & 8 \\ 0 & 0 & 1 & -5 & -2 & 3 & 1 & -4 \\ 0 & 0 & 0 & -4 & -2 & 2 & 1 & -3 \end{array}\right]$	$r_1' = r_1 + r_3$ $r_2' = r_2 - 2r_3$ $r_4' = r_4 + r_3$

Problem	Ans	Reason

24, cont

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -6 & -2 & 4 & 1 & -4 \\ 0 & 1 & 0 & 11 & 5 & -6 & -2 & 8 \\ 0 & 0 & 1 & -5 & -2 & 3 & 1 & -4 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{3}{4} \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{3}{4} \end{array}\right] \quad \begin{array}{l} r_1' = r_1 + 6r_4 \\ r_2' = r_2 - 11r_4 \\ r_3' = r_3 + 5r_4 \end{array}$$

check ✓

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & -1 & -1 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4} & -\frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & -\frac{1}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{3}{4} & \frac{-1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix}$$

Problem	Ans	Reason

25

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1' = r_1 + r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 1 & 0 \\ 0 & 3 & 2 & 1 & 2 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_2' = r_2 + r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 2 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_2' = r_2 + 2r_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -7 & -1 & -3 & -4 \\ 0 & 1 & 4 & 1 & 2 & 2 \\ 0 & 0 & 5 & 1 & 2 & 3 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - 2r_2 \\ \\ r_3' = r_3 + r_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -7 & -1 & -3 & -4 \\ 0 & 1 & 4 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{5} & \frac{-1}{5} & \frac{1}{5} \\ 0 & 1 & 0 & \frac{1}{5} & \frac{2}{5} & \frac{-2}{5} \\ 0 & 0 & 1 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} \end{array}\right] \quad \begin{array}{l} r_1' = r_1 + 7r_3 \\ r_2' = r_2 - 4r_3 \end{array}$$

Problem	Ans	Reason
25, cont	check	

$$\begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 1 & 2 & 3 \end{bmatrix}\frac{1}{5} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\underline{X} = \frac{1}{5}\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\underline{X} = \frac{1}{5}\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/5 \\ 4/5 \\ 4/5 \end{bmatrix}$$

check

$$\begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3/5 \\ 4/5 \\ 4/5 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \checkmark$$

Problem	Ans	Reason
26		

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 3 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 3 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 1 & -\frac{5}{2} & -\frac{3}{2} & 0 & 1 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 - 2r_1 \\ r_3' = r_3 - 3r_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & -1 & 1 \end{array}\right] \quad r_3' = r_3 - r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 & -2 \end{array}\right] \quad r_3' = -2r_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 & 5 & -4 \\ 0 & 0 & 1 & 1 & 2 & -2 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - \frac{1}{2}r_3 \\ r_2' = r_2 + 2r_3 \end{array}$$

check

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

Problem | Ans | Reason

26. cont

$$A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

check

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \checkmark$$

No. 6.2
Date 27

Problem	Ans	Reason
27		

def $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

Find A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right] \quad r_3' = r_3 - r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - r_2 \\ \\ r_3' = r_3 + r_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right] \quad r_3' = -r_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - 2r_3 \\ r_2' = r_2 + r_3 \end{array}$$

check

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

Problem	Ans	Reason
27. cont		

$$A^{-1} = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

(a)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 15 \\ 1 \\ 10 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

check

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 3 & 1 \\ 5 & 3 & 0 \\ 4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 15 & 9 & 0 \\ 1 & 0 & 1 \\ 10 & 6 & 0 \end{bmatrix} \checkmark$$

No. 6.2

Date 29

Problem	Ans	Reason
28		

Find col 1 of A^{-1}

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -1 & 2 & -1+a & 0 \\ -2 & 2 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & a & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 + r_1 \\ r_3' = r_3 + 2r_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1+a & 2 \\ 0 & 1 & a & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad r_1' = r_1 + r_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2a \\ 0 & 1 & 0 & 1-2a \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - (1+a) r_3 \\ r_2' = r_2 - a r_3 \end{array}$$

(3,1)-entry of A^{-1} is 2

6.2

No.

Date 30

Problem	Ans	Reason

29

$$A^2 = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 18 \\ 6 & 28 \end{bmatrix}$$

Find A^{-1}:

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 1 & 5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right] \quad r_2' = r_2 - r_1$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 5/2 & -3/2 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right] \quad r_1' = r_1 - 3r_2$$

$$A^{-1} = \begin{bmatrix} \frac{5}{2} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$$

$$(A^{-1})^2 = (A^{-1})(A^{-1}) = \begin{bmatrix} \frac{5}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{5}{2} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{28}{4} & \frac{-18}{4} \\ \frac{-6}{4} & \frac{4}{4} \end{bmatrix}$$

$$= (A^2)^{-1}$$

Problem	Ans	Reason

30

$$(A^n)^{-1} = (A^{-1})^n \quad \text{since}$$

$$A^n (A^{-1})^n = \underbrace{AA\cdots A}_{n}\ \underbrace{A^{-1}A^{-1}\cdots A^{-1}}_{n}$$

$$= I$$

31 $A^{-1} = \begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 1 & b & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -b \\ 0 & 1 & 0 & 1 \end{array}\right] \quad r_1' = r_1 - br_2$$

check $\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

32 $A^{-1} = \begin{bmatrix} \frac{1}{1-bc} & \frac{-b}{1-bc} \\ \frac{-c}{1-bc} & \frac{1}{1-bc} \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 1 & b & 1 & 0 \\ c & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & b & 1 & 0 \\ 0 & 1-bc & -c & 1 \end{array}\right] \quad r_2' = r_2 - cr_1$$

$$\left[\begin{array}{cc|cc} 1 & b & 1 & 0 \\ 0 & 1 & \frac{-c}{1-bc} & \frac{1}{1-bc} \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{1-bc} & \frac{-b}{1-bc} \\ 0 & 1 & \frac{-c}{1-bc} & \frac{1}{1-bc} \end{array}\right] \quad r_1' = r_1 - br_2$$

Problem	Ans	Reason

33

$$A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \frac{1}{ad-bc}$$

check :

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \frac{1}{ad-bc} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

34

$$A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$$

35

$$A^{-1} = \begin{bmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix} \quad \left[\begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & b-ac & 1 & -a & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1' = r_1 - a r_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} r_1' = r_1 - (b-ac) r_3 \\ r_2' = r_2 - c r_3 \end{array}$$

check

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

Problem	Ans	Reason

36

$ABC =$

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 5 \end{bmatrix} \begin{bmatrix} x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x & 1 \\ 5x & 5 \end{bmatrix}$$

Find $\begin{bmatrix} x & 1 \\ 5x & 5 \end{bmatrix}^{-1}$

$$\left[\begin{array}{cc|cc} x & 1 & 1 & 0 \\ 5x & 5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} x & 1 & 1 & 0 \\ \underbrace{0 \quad\quad 0} & & -5 & 1 \end{array}\right] \quad r_2' = r_2 - 5r_1$$

$(ABC)^{-1}$ does not exist no matter what is x.