No. G1

Date /

Problem	Ans	Reason
1	(a) 3×3	
	(b) 3×2	
	(c) 3×1	col vector
2	(a) 4×1	col vector
	(b) 4×2	
	(c) 2×2	
3	$A+B=\begin{pmatrix}6&6\\3&3\end{pmatrix}$	$A-B=\begin{pmatrix}4&-2\\-1&-3\end{pmatrix}$
4	$2A+B=\begin{pmatrix}6&5\\6&-7\end{pmatrix}$	$A-2B=\begin{pmatrix}-2&5\\-7&4\end{pmatrix}$
5	$3A+2B=\begin{pmatrix}7&1&7\\4&4&3\\12&-14&0\end{pmatrix}$	$2A-B=\begin{pmatrix}7&-4&7\\-2&5&2\\8&0&-7\end{pmatrix}$
6	(a) $\begin{pmatrix}5\cdot1+2\cdot2 & 5\cdot4+2\cdot3\\ 1\cdot1+0\cdot2 & 1\cdot4+0\cdot3\end{pmatrix}=\begin{pmatrix}9&26\\1&4\end{pmatrix}$	

Problem	Ans	Reason

6 cont

(b)

$$\begin{pmatrix} 2\cdot 2+3\cdot 4 & 2(-1)+3(-3) \\ 1\cdot 2-2\cdot 4 & 1(-1)+(2)(-3) \end{pmatrix} = \begin{pmatrix} 16 & -11 \\ -6 & 5 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 3(-1)-1(2)+3(0) & 3(2)-1(-1)+3(-4) & 3(-1)-1(0)+3(3) \\ 0(-1)+2(2)+1(0) & 0(2)+2(-1)+1(-4) & 0(-1)+2(0)+1(3) \\ 4(-1)-2(2)-2(0) & 4(2)-2(-1)-2(-4) & 4(-1)-2(0)-2(3) \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -5 & 6 \\ 4 & -6 & 3 \\ -8 & 18 & -10 \end{pmatrix}$$

Problem	Ans	Reason
7	(a) not defined	

(b)
$$\left(\begin{array}{c|c|c} 2(1)-1(2)+0(3) & 2(-2)-1(1)+0(2) & 2(-1)-1(0)+0(3) \\ \hline 4(1)-2(2)+3(3) & 4(-2)-2(1)+3(2) & 4(-1)-2(0)+3(3) \end{array}\right)$$

$$= \begin{pmatrix} 0 & -5 & -2 \\ 9 & -4 & -13 \end{pmatrix}$$

(c) not defined

(d)
$$\begin{pmatrix} -4 & -5 & 2 \\ 7 & -10 & -15 \end{pmatrix}$$

Problem	Ans	Reason

8 (a)

$$\begin{bmatrix} 4 & -2 & 0 \\ 8 & -4 & 6 \end{bmatrix}$$

(b) not defined

(c)

$$\begin{bmatrix} -1 & -5 & -3 \\ 11 & -6 & -13 \end{bmatrix}$$

(d) $CB = (A+C)B - AB$

$$= \begin{pmatrix} -4 & 0 & 4 \\ -2 & -6 & -2 \end{pmatrix}$$

9 $C = 2B - 3A$

$$= \begin{bmatrix} -2-12 & 6+6 \\ 8-6 & 2-6 \\ 4-15 & -4+9 \end{bmatrix} = \begin{bmatrix} -14 & 12 \\ 2 & -4 \\ -11 & 5 \end{bmatrix}$$

Problem	Ans	Reason
10	$c=1$	$(1,2)$ entry of AB is $4(c)+1(2)+2(c)$ ie $6c+2$ Require $8=6c+2$ $6=6c$ $\quad c=1$
11	$A+2B$	For A, B as in Ex 6.3
12	(a) 6 (b) 4	$2a+3b=18$ $4a+b=16$ $2a+3(16-4a)=18$ $-10a=18-48=-30$ $a=3$ $\quad b=4$
13	(a) $\begin{bmatrix} 6 & 2 \\ -2 & 0 \\ 8 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 9 & 7 \\ -2 & -2 \\ 14 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 50 & 38 \\ 39 & 29 \\ 39 & 29 \end{bmatrix}$	

Problem	Ans	Reason

14

$$C = \frac{4A - 3B}{2}$$

$$= \frac{1}{2}\begin{pmatrix} 16-15 & -8-0 & 4-(-9) \\ 8-9 & 0-(-3) & -8-6 \\ 8-9 & -20-0 & 24-12 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -4 & \frac{13}{2} \\ -\frac{1}{2} & \frac{3}{2} & -7 \\ -\frac{1}{2} & -10 & 6 \end{pmatrix}$$

15 (a)

$$\begin{bmatrix} 6+1 & 0-2 & -9-0 \\ 3-1 & 12-4 & 18-3 \end{bmatrix} = \begin{bmatrix} 7 & -2 & -9 \\ 2 & 8 & 15 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3.2+1.1 & 3.0+1.4 & 3.-3+4.6 \\ 2.2+0.1 & 2.0+0.4 & 2.-3+0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & -3 \\ 4 & 0 & -6 \end{bmatrix}$$

(c) not def

(d) not def

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Problem	Ans	Reasons
16	(a) Not def	
	(b) $AX = \begin{pmatrix} -10 \\ 25 \end{pmatrix}$	
	(c) $AX + CY = \begin{pmatrix} -5 \\ 29 \end{pmatrix}$	$\begin{pmatrix} -10 \\ 25 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ 29 \end{pmatrix}$
	(d) $CAX = \begin{pmatrix} -5 \\ \end{pmatrix}$	$\begin{pmatrix} 3(-10)+1(25) \\ 2(-10)+0(25) \end{pmatrix} = \begin{pmatrix} -5 \\ \end{pmatrix}$
17	$c = 3$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -2 & c \end{pmatrix}$
		$= \left(\begin{array}{c\|c} 3-2=1 & -3+c \\ \hline 2-2=0 & -2+c \end{array}\right)$
		need $c = 3$

Problem	Ans	Reason
18	$c=2,\ d=-1$	$3c-d=7$ $c+2d=0$ $3(-2d)-d=7$ $-7d=7$ $d=-1$ $c=2$ check $3\cdot 2-(-1)=7$ ✓ $2+2(-1)=0$ ✓
19	(a) $C=\begin{pmatrix}-6 & 5\\ -9 & 2\end{pmatrix}$	$C=B-2A=\left(\begin{array}{c\|c}-2-4 & 3+2\\ \hline -1-8 & 2-0\end{array}\right)$ $=\begin{pmatrix}-6 & 5\\ -9 & 2\end{pmatrix}$
	(b) $D=\begin{pmatrix}1 & -2\\ -3 & -1\end{pmatrix}$	$D=I-A-B=\left(\begin{array}{c\|c}1-2+2 & +1-3\\ \hline -4+1 & 1-0-2\end{array}\right)$ $=\begin{pmatrix}1 & -2\\ -3 & -1\end{pmatrix}$

Problem	Ans	Reason
20	$a = \frac{1}{2}$ $b = 3$	$a2 + 1 \cdot 3 = 4$ $-1 \cdot 2 + b \cdot 3 = 7$ $a = \frac{4-3}{2} = 1/2$ $b = \frac{7+2}{3} = \frac{9}{3} = 3$
21	For ex $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$A - B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ Ex $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$
22	For ex $A = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$ For ex $A = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$	

Problem	Ans	Reason
23	$A^2 = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$	$A^2 = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$
	$A^3 = \begin{pmatrix} 12 & 5 \\ 5 & 2 \end{pmatrix}$	$= \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$
		$A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$
		$= \begin{pmatrix} 12 & 5 \\ 5 & 2 \end{pmatrix}$
24		$P = \begin{pmatrix} 1/2 & 1/2 \\ 1 & 0 \end{pmatrix}$
		$P^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
		$P^4 = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
		$= \begin{pmatrix} \frac{11}{16} & \frac{5}{16} \\ \frac{5}{8} & \frac{3}{8} \end{pmatrix} = \frac{1}{16}\begin{pmatrix} 11 & 5 \\ 10 & 6 \end{pmatrix}$
		$P^8 = \frac{1}{256}\begin{pmatrix} 11 & 5 \\ 10 & 6 \end{pmatrix}\begin{pmatrix} 11 & 5 \\ 10 & 6 \end{pmatrix}$
		$= \frac{1}{256}\begin{pmatrix} 171 & 85 \\ 170 & 86 \end{pmatrix}$

Problem	Ans	Reasm
25		$\begin{pmatrix} 2 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1+1 & 1+0 \end{pmatrix} = (2,1)$
		$XP^2 = XPP = XP = X$
26		$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{4}+\frac{1}{4} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$
		$P^2X = PPX = PX = X$
27		$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{9} & \frac{2}{3} & \frac{1}{9} \end{pmatrix}$
		$\begin{pmatrix} 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{9} & \frac{2}{3} & \frac{1}{9} \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{9} & \frac{2}{3} & \frac{1}{9} \end{pmatrix} = \begin{pmatrix} \frac{2}{9} & \frac{2}{3} & \frac{1}{9} \\ \frac{2}{27} & \frac{2}{9} & \frac{19}{27} \\ \frac{38}{81} & \frac{2}{27} & \frac{37}{81} \end{pmatrix}$
28		$\begin{pmatrix} 1 & 1 & \frac{3}{2} \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3/2 \end{pmatrix}$
		$XP^n = (XP)P\cdots P$ $= XP\cdots P$ $= \cdots = X$

Problem	Ans	Reason
29	(a)	$(kX)P = k(XP)$ $= kX$
	(b)	Take $Y = k(1\ 1\ 3/2)$ with k chosen so that $k + k + k\,3/2 = 1$ $k = 2/7$ $Y = \left(\frac{2}{7}, \frac{2}{7}, \frac{3}{7}\right)$
30	(1,2)	$(a\ b)\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = (a\ b)$ $0a + \frac{b}{2} = a$ $a + b/2 = b$ so $a = b/2$ take $a = 1, b = 2$

Problem	Ans	Reason
31	[1, 2, 4]	Let $X = (x, y, z)$

$$(x \quad y \quad z)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix} = (x \quad y \quad z)$$

$$\frac{z}{4} = x$$

$$x + \frac{z}{4} = y$$

$$y + \frac{z}{2} = z$$

$$-1x + 0y + \frac{1}{4}z = 0$$
$$1x - 1y + \frac{1}{4}z = 0$$
$$0x + 1y - \frac{1}{2}z = 0$$

$$\begin{bmatrix} -1 & 0 & \frac{1}{4} & 0 \\ 1 & -1 & \frac{1}{4} & 0 \\ 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/4 & 0 \\ 1 & -1 & 1/4 & 0 \\ 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/4 & 0 \\ 0 & -1 & \frac{1}{2} & 0 \\ 0 & 1 & -1/2 & 0 \end{bmatrix} \quad r_2' = r_2 - r_1$$

Problem	Ans	Reason

31 cont

$$\begin{bmatrix} 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 1 & -1/2 & 0 \end{bmatrix} \qquad r_2' = -r_2$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x = 1/4\, z$
$y = \frac{1}{2} z$
z free

take $z = 4$

$$[x, y, z] = [1, 2, 4]$$

32 (a)

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & 4 & 9 \\ 4 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 19 \\ 29 \end{bmatrix}$$

Problem	Ans	Reason
33	$1(2+t)+2(1-t)+1(t)=4$ ✓ $2(2+t)+3(1-t)+1(t)=7$ ✓ $-2(2+t)+4(1-t)+6(t)=0$ ✓	
34	$\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find a, c

$$a+2c=1$$
$$3a+7c=0$$

$$3(1-2c)+7c=0$$
$$c=-3$$
$$a=1-2(-3)=7$$

Find b, d

$$b+2d=0$$
$$3b+7d=1$$

$$3(-2d)+7d=1$$
$$d=1$$
$$b=-2$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

Problem	Ans	Reason

35

$$\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 10 & -7 \\ -22 & 15 \end{bmatrix}$$

Find a, c

$$a - 2c = 10$$
$$-3a + 4c = -22$$

$$-3(10 + 2c) + 4c = -22$$
$$-2c = 30 - 22 = 8$$
$$c = -4$$
$$a = 2$$

Find b, d

$$b - 2d = -7$$
$$-3b + 4d = 15$$

$$-3(2d - 7) + 4d = 15$$
$$-2d = 15 - 21 = -6$$
$$d = 3$$
$$b = -7 + 2 \cdot 3$$
$$= -1$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

Problem	Ans	Reason
36	$X = \begin{bmatrix} \frac{24}{7} \\ \frac{10}{7} \\ \frac{10}{7} \end{bmatrix}$	$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & -1 \\ 0 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}$

$$\begin{bmatrix} 2 & -3 & 1 & 4 \\ 1 & 0 & -1 & 2 \\ 0 & 3 & 4 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & -3 & 1 & 4 \\ 0 & 3 & 4 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & 4 & 10 \end{bmatrix} \quad r_2' = r_2 - 2r_1$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 3 & 4 & 10 \end{bmatrix} \quad r_2' = \frac{-1}{3} r_2$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 7 & 10 \end{bmatrix} \quad r_3' = r_3 - 3r_2$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{10}{7} \end{bmatrix}$$

Problem	Ans	Reason

36, cont

$$\begin{bmatrix} 1 & 0 & 0 & \frac{24}{7} \\ 0 & 1 & 0 & \frac{10}{7} \\ 0 & 0 & 1 & \frac{10}{7} \end{bmatrix} \quad \begin{matrix} r_1' = r_1 + r_3 \\ r_2' = r_2 + r_3 \end{matrix}$$

$x = \frac{24}{7} \qquad y = \frac{10}{7} \qquad z = \frac{10}{7}$

check

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & -1 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} \frac{24}{7} \\ \frac{10}{7} \\ \frac{10}{7} \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix} \checkmark$$