5,R Solutions

Problem	Ans	Reason
1	2.5 years later	t = # years elapsed

growth: $y = 3 + .6t$

inflation: $y = 2 + 1t$

$2 + t = 3 + .6t$

$.4t = 1$

$t = \frac{1}{.4} = \frac{10}{4} = 5/2$

2	\$.90	c = cost of call \$ t = # minutes called

$c = mt + b$

$.7 = m20 + b$

$1.00 = m35 + b$

$.3 = 15m$

$m = \frac{.3}{15} = \frac{.1}{5} = \frac{1}{50} = \frac{2}{100} = .02$

$b = .7 - 20(.02)$

$= .7 - .4 = .3$

For $t = 30$ $\quad c = (.02)30 + .3$

$= .6 + .3 = .9$

Problem	Ans	Reason
3	33	y = # pages written x = # minutes spent $y = mx + b$ $9 = m\,120 + b$ $17 = m\,200 + b$ $8 = m\,80$ $m = \frac{1}{10}$ $b = 9 - \frac{120}{10} = 9 - 12 = -3$ For $x = 360$ $y = \frac{360}{10} - 3$ $= 36 - 3 = 33$
4	$3x + 5y = 16$	line thru $(-2, -1)$ and $(3, -4)$ has slope $m = \frac{-4 - (-1)}{3 - (-2)} = \frac{-3}{5}$ Line thru $(-3, 5)$ with slope $\frac{-3}{5}$ has eq $\frac{y - 5}{x + 3} = \frac{-3}{5}$ $5(y - 5) = -3(x + 3)$ $\quad 3x + 5y = 16$

Problem	ans	Reason
5	$x=2 \quad y=3$	$y = \frac{16-5x}{2}$
		$6x + 3\frac{16-5x}{2} = 21$
		$-\frac{3}{2}x = -3$
		$x = 2 \quad y = 3$
		check $5.2 + 2.3 = 16$ ✓ $6.2 + 3.3 = 21$ ✓
6	$x = -2$ $y = 5/2$	$y = \frac{-1-3x}{2}$
		$5x + 4\frac{-1-3x}{2} = 0$
		$5x + 2(-1-3x) = 0$
		$-x = 2 \quad x = -2$
		$y = \frac{-1+6}{2} = 5/2$
		check $3(-2) + 2\frac{5}{2} = -1$ ✓ $5(-2) + 4\frac{5}{2} = 0$ ✓

Problem	Ans	Reason
7	$x = 2$ $y = -5/2$	$x + 2y = -3$ $x + y = -1/2$ $y = -3 + 1/2 = -5/2$ $x = -1/2 + 5/2 = 2$ check $-2 \cdot 2 - 4\left(\frac{-5}{2}\right) = 6$ ✓ $6 \cdot 2 + 6 \cdot \left(\frac{-5}{2}\right) = -3$ ✓
8	20 brochures 50 Newspapers	time requirements

	hours marketing time	hours design time
brochure	5	10
newspaper	4	11
total	300	750

x = # brochures made

y = # newspaper supplements made

$300 = 5x + 4y$

$750 = 10x + 11y$

$x = \frac{300 - 4y}{5}$

$750 = 10\frac{300 - 4y}{5} + 11y$

$150 = 3y$

$50 = y$

$20 = x$

Problem	Ans	Reason
9	11.25 Laps	t = # minutes elapsed
		d = #Laps travelled by tma
		$d+1$ = # ... stephanie
		$d(2.5) = t$ (tma)
		$(d+1)(2.25) = t$ (steph)
		$d\,2.5 = d\,2.25 + 2.25$
		$d(.25) = 2.25$
		$d = \frac{225}{25} = 9$
		$t = (2.5)\,9 = 22.5$
		#Laps for Rachael = $\left(\frac{1}{2}\text{ Lap/min}\right)(22.5\text{ min})$
		$= 11.25$ Laps

No. 5.R

Date 6

Problem	Ans	Reason
10	(a) 11480 $	t = #years since car purchased
	(b) $v = 18900 - t1855$	v = value

$$v = 18900 + tm$$

$$350 = 18900 + 10m$$

$$m = \frac{350 - 18900}{10} = 35 - 1890 = -1855$$

at $t = 4$

$$v = 18900 - 4 \cdot 1855$$
$$= 18900 - 7420$$
$$= 11480$$

Problem	Ans	Reason

11 9 years later

t = # years elapsed

Computer 1

yearly dep $\dfrac{1{,}000{,}000 - 50{,}000}{10}$

$= 100{,}000 - 5{,}000$

$= 95{,}000$

Computer 1 value at time t is

$1\,000\,000 - t\,95\,000$

Computer 2:

yearly dep $= \dfrac{2\,225\,000 - 75\,000}{10}$

$= 222500 - 7500$

$= 215{,}000$

Computer 2 value at time t is

$2\,225\,000 - t\,215\,000$

$2225000 - t\,215000 = 2(1000000 - t\,95000)$

$225000 = 25000t$

$t = \frac{225}{25} = 9$

No. 5.R

Date 8

Problem	Ans	Reason
12	(a) $\begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -2 \end{pmatrix}$	
	(b) $\begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$	
B	(a) $1(4) + 1(3) + 0(-1) + 3(-1) = 4$ ✓ $0(4) + 1(3) + -1(-1) + 0(-1) = 4$ ✓ $1(4) + 0(3) + 1(-1) + -2(-1) = 5$ ✓	
	(b) $1(4) + 0(2) + 1(-1) = 3$ ✓ $1(4) + -1(2) + 0(-1) = 2$ ✓ $0(4) + 1(2) + -1(-1) = 3$ ✓	

Problem	Ans	Reason
14	$x=3$ $y=-1$ $z=4$	$\begin{bmatrix} 2 & 4 & 1 & 6 \\ 1 & 3 & 1 & 4 \\ 2 & -2 & -1 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 & 1 & 4 \\ 2 & 4 & 1 & 6 \\ 2 & -2 & -1 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & -2 & -1 & -2 \\ 0 & -8 & -3 & -4 \end{bmatrix}$ $r_2' = r_2 - 2r_1$, $r_3' = r_3 - 2r_1$ $\begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & -8 & -3 & -4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ $r_1' = r_1 - 3r_2$, $r_3' = r_3 + 8r_2$ $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ $r_1' = r_1 + \frac{1}{2}r_3$, $r_2' = r_2 - \frac{1}{2}r_3$

$x=3, \quad y=-1, \quad z=4$

Check

$2(3) + 4(-1) + 1(4) = 6$ ✓

$1(3) + 3(-1) + 1(4) = 4$ ✓

$2(3) + -2(-1) + -1(4) = 4$ ✓

Problem	Ans	Reason
15	$x_1 = 8/5$ $x_2 = 6/5$	$\begin{bmatrix} 2 & 4 & 8 \\ 4 & -2 & 4 \\ 6 & 2 & 12 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 4 \\ 4 & -2 & 4 \\ 6 & 2 & 12 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 4 \\ 0 & -10 & -12 \\ 0 & -10 & -12 \end{bmatrix}$ $r_2' = r_2 - 4r_1$, $r_3' = r_3 - 6r_1$ $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6/5 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 8/5 \\ 0 & 1 & 6/5 \\ 0 & 0 & 0 \end{bmatrix}$ $r_1' = r_1 - 2r_2$ $x_1 = 8/5$ $x_2 = 6/5$

check

$2(8/5) + 4(6/5) = 8$ ✓

$4(8/5) + -2(6/5) = 4$ ✓

$6(8/5) + 2(6/5) = 12$ ✓

No. S.R

Date 11

Problem	Ans	Reason
16	No sol	

$$\begin{bmatrix} 2 & 2 & -4 & 12 \\ 1 & 1 & 1 & 6 \\ 3 & 3 & -3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 1 & 1 & 1 & 6 \\ 1 & 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & -5 \end{bmatrix} \quad r_2' = r_2 - r_1 \quad r_3' = r_3 - r_1$$

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix} \quad r_1' = r_1 + 2r_2 \quad r_3' = r_3 - r_1$$

No sol

No. 5, R

Date 12

Problem	Ans	Reason
17	$x = 5 - 5z$ $y = 2 - 2z$ $z =$ free	$\begin{bmatrix} 2 & -5 & 0 & 0 \\ 1 & -3 & -1 & -1 \\ -1 & 2 & -1 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & -3 & -1 & -1 \\ 2 & -5 & 0 & 0 \\ -1 & 2 & -1 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & -3 & -1 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & -2 \end{bmatrix}$ $r_2' = r_2 - 2r_1$, $r_3' = r_3 + r_1$ $\begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $r_1' = r_1 + 3r_2$, $r_3' = r_3 + r_2$

$x = 5 - 5z$

$y = 2 - 2z$

z free

check

$2(5-5z) - 5(2-2z) + 0(z) = 0$ ✓

$1(5-5z) - 3(2-2z) - 1(z) = -1$ ✓

$-1(5-5z) + 2(2-2z) - 1(z) = -1$ ✓

Problem	Ans	Reason
18	$x_1 = -2$ $x_2 = 0$ $x_3 = 1$	$\begin{bmatrix} 1 & 5 & 2 & 0 \\ 2 & 7 & 1 & -3 \\ 0 & 2 & 3 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 5 & 2 & 0 \\ 0 & -3 & -3 & -3 \\ 0 & 2 & 3 & 3 \end{bmatrix}$ $r_2' = r_2 - 2r_1$ $\begin{bmatrix} 1 & 5 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 3 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -3 & -5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $r_1' = r_1 - 5r_2$, $r_3' = r_3 - 2r_2$ $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $r_1' = r_1 + 3r_3$, $r_2' = r_2 - r_3$ $x_1 = -2 \quad x_2 = 0 \quad x_3 = 1$

check

$1(-2) + 5(0) + 2(1) = 0$ ✓

$2(-2) + 7(0) + 1(1) = -3$ ✓

$0(-2) + 2(0) + 3(1) = 3$ ✓

Problem	Ans	Reason
19	$x_1 = 0$ $x_2 = -1$ $x_3 = -2$	$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & -1 & -1 & 3 \\ 0 & 2 & -3 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 2 & -3 & 4 \end{bmatrix}$ $r_2' = r_2 - 2r_1$ $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -3 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$ $r_3' = r_3 - 2r_2$ $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ $r_1' = r_1 + r_3$ $r_2' = r_2 + r_3$ $x_1 = 0 \quad x_2 = -1 \quad x_3 = -2$

check

$1(0) + 0(-1) - 1(-2) = 2$ ✓

$2(0) - 1(-1) - 1(-2) = 3$ ✓

$0(0) + 2(-1) - 3(-2) = 4$ ✓

Problem	Ans	Reason
20	$x_1 = \frac{3}{2} - \frac{5}{4}x_3 - x_4$ $x_2 = 1 + \frac{3}{2}x_3$ $x_3 =$ free $x_4 =$ free	$\begin{bmatrix} 2 & 1 & 1 & 2 & 4 \\ 2 & -1 & 4 & 2 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 1 & 2 \\ 2 & -1 & 4 & 2 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 1 & 2 \\ 0 & -2 & 3 & 0 & -2 \end{bmatrix}$ $r_2' = r_2 - 2r_1$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -3/2 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 5/4 & 1 & 3/2 \\ 0 & 1 & -3/2 & 0 & 1 \end{bmatrix}$ $r_1' = r_1 - \frac{1}{2}r_2$

$x_1 = \frac{3}{2} - \frac{5}{4}x_3 - x_4, x_2 = 1 + 3/2\,x_3$

x_3 free x_4 free

check

$2\left(\frac{3}{2} - \frac{5}{4}x_3 - x_4\right) + 1\left(1 + \frac{3}{2}x_3\right) + 1\left(x_3\right) + 2\left(x_4\right) = 4$

$2\left(\frac{3}{2} - \frac{5}{4}x_3 - x_4\right) - 1\left(1 + \frac{3}{2}x_3\right) + 4\left(x_3\right) + 2\left(x_4\right) = 2$

Problem	Ans	Reason
21	$x_1 = 3 + \frac{2}{5}x_2$ $x_2 =$ free $x_3 = 1 - \frac{1}{2}x_4$ $x_4 =$ free	$\begin{bmatrix} 5 & -2 & 2 & 1 & 17 \\ 5 & -2 & -10 & -5 & 5 \end{bmatrix}$ $\begin{bmatrix} 5 & -2 & -10 & -5 & 5 \\ 5 & -2 & 2 & 1 & 17 \end{bmatrix}$ $\begin{bmatrix} 1 & -2/5 & -2 & -1 & 1 \\ 0 & 0 & 12 & 6 & 12 \end{bmatrix}$ $\begin{bmatrix} 1 & -2/5 & -2 & -1 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & -2/5 & 0 & 0 & 3 \\ 0 & 0 & 1 & \frac{1}{2} & 1 \end{bmatrix}$ $r_1' = r_1 + 2r_2$

$x_1 = 3 + \frac{2}{5}x_2$

$x_3 = 1 - \frac{1}{2}x_4$

x_2 free x_4 free

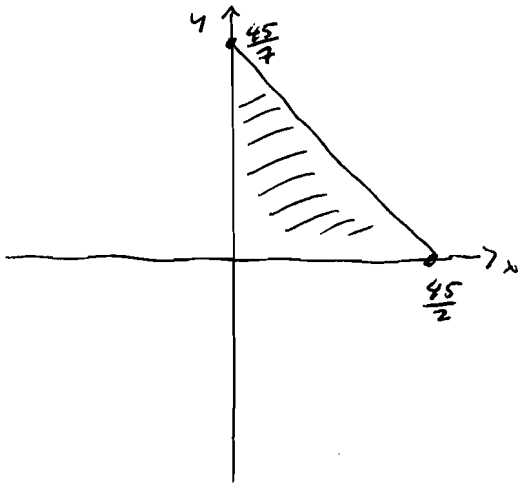
check ✓

$5(3 + \frac{2}{5}x_2) - 2(x_2) + 2(1 - \frac{1}{2}x_4) + 1(x_4) = 17$ ✓

$5(3 + \frac{2}{5}x_2) - 2(x_2) - 10(1 - \frac{1}{2}x_4) - 5(x_4) = 5$

Problem	Ans	Reason
22	(a) $x_1 = 1$ $x_2 = -4$ $x_3 = 0$	$\begin{bmatrix} 2 & -1 & -3 & 6 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 3 & -3 \end{bmatrix}$
	(b) The above solution has $x_1 = 1$	$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & -1 & -3 & 6 \\ 1 & 1 & 3 & -3 \end{bmatrix}$
		$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & -7 & 4 \\ 0 & 1 & 1 & -4 \end{bmatrix}$ $r_2' = r_2 - 2r_1$, $r_3' = r_3 - r_1$
		$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 7 & -4 \\ 0 & 1 & 1 & -4 \end{bmatrix}$
		$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 7 & -4 \\ 0 & 0 & -6 & 0 \end{bmatrix}$ $r_3' = r_3 - r_2$
		$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 7 & -4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
		$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $r_1' = r_1 - 2r_3$, $r_2' = r_2 - 7r_3$
	$x_1 = 1$ $x_2 = -4$ $x_3 = 0$	

Problem	Ans	Reason
23		c = dollar cost per year x = miles driven in 1000's y = age in years $c = 50 + 20x + 70y$ Lines intersect at $x = 10,\ y = 1,\quad c = 50 + 200 + 70 = 320$
24	$x \geq 0$ $y \geq 0$ $2x + 7y \leq 45$	Require $c \leq 500$ $20x + 70y \leq 450$ $2x + 7y \leq 45$



blem	Ans	Reason
25	(b) No Sol	
26	(c) 1 free var	
27	(a) Unique sol	
28	(a) Unique sol	
29	(c) 1 free var	
30	$k = \frac{1}{2}$	

$$\begin{bmatrix} 3 & 1 & -1 & 6 \\ 2 & 0 & -1 & 2 \\ 0 & 1 & k & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 2 & 0 & -1 & 2 \\ 0 & 1 & k & 3 \end{bmatrix} \quad r_1' = r_1 - r_2$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & -2 & -1 & -6 \\ 0 & 1 & k & 3 \end{bmatrix} \quad r_2' = r_2 - 2r_1$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & \frac{1}{2} & 3 \\ 0 & 1 & k & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & \frac{1}{2} & 3 \\ 0 & 0 & k-\frac{1}{2} & 0 \end{bmatrix}$$

Problem	Ans	Reason
31		In (a), (b), (c) the first two equations are the same. solve those first

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 3 & -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -1 & 1 & -1 \end{bmatrix} \quad r_2' = r_2 - 2r_1$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{bmatrix} \quad r_1' = r_1 - 2r_2$$

$$x_1 = 1 - x_3$$
$$x_2 = 1 + x_3$$

(a)

$x_1 = \frac{1}{2}$
$x_2 = \frac{3}{2}$
$x_3 = \frac{1}{2}$

$$2(1-x_3) + 4(1+x_3) - 4x_3 = 5$$
$$-2x_3 = -1$$
$$x_3 = 1/2$$

(b)

No sol

$$2(1-x_3) + 4(1+x_3) - 2x_3 = 5$$
$$0 = 1$$

(c)

$x_1 = 1 - x_3$
$x_2 = 1 + x_3$
$x_3 =$ free

$$3(1-x_3) + 5(1+x_3) - 2x_3 = 8$$
$$0 = 0$$

Problem	Ans	Reason
32	$k \neq 2$	$x_1 + x_3 - 2x_4 = 9$ $x_1 + x_3 - kx_4 = 8$ $(k-2)x_4 = 1$ For $k \neq 2$ we can solve for x_4. Then for any given value of x_3 we can solve for x_1 to get a solution to the system.
33 skip		
34	(a) $x = -5/3 + \frac{10}{3}z$ $y = 8/3 - 4/3\,z$ z = free $\frac{1}{2} \le z \le 2$	$2x + 5y = 10$ $3y + 4z = 8$ $x \ge 0,\ y \ge 0,\ z \ge 0$ $x = -\frac{10}{6} + \frac{20}{6}z$ $y = 8/3 - 4/3\,z$ z = free $0 \le -\frac{10}{6} + \frac{20}{6}z \rightarrow z \ge 1/2$ $0 \le 8/3 - 4/3\,z \quad z \le 2$
	(b) intersection with plane $x=0$ is $x=0$ $y=2$ $z=\frac{1}{2}$ intersection with plane $y=0$ is $x=5$ $y=0$ $z=2$	