Section 5.3 HW Solutions

Problem | Ans | Reason

1

	grams plastic	inches wood strips	gms steel
airplane	100	10	200
boat	50	100	50
car	50	0	150

x = # planes made
y = # boats ..
z = # cars ..

plastic: $10500 = x\,100 + y\,50 + z\,50$
wood: $1500 = x\,10 + y\,100 + z\,0$
steel: $25,500 = x\,200 + y\,50 + z\,150$

Problem	Ans	Reason

2

	stocks	bonds	MoMktFunds
Rachael	$1/2$	$\frac{1}{4}$	$\frac{1}{4}$
Stephanie	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Tina	$\frac{1}{2}$	$\frac{1}{2}$	0
Return	9%	6%	3%

x = number of dollars invested by Rachael

y = " " Stephanie

z = " " Tina

Rachael $10000 = x\frac{1}{2}\ \frac{9}{100} + \frac{x}{4}\ \frac{6}{100} + \frac{x}{4}\ \frac{3}{100}$

Steph $10000 = \frac{y}{3}\ \frac{9}{100} + \frac{y}{3}\ \frac{6}{100} + \frac{y}{3}\ \frac{3}{100}$

Tina $10000 = \frac{z}{2}\ \frac{9}{100} + \frac{z}{2}\ \frac{6}{100}$

blem	Ans	Reasm

3

Requirements per package

	grams Raisins	grams peanuts	grams Choc chips
Hikers Mix	60	20	10
Bikers Mix	40	40	20
Special Mix	30	30	30
Total	18000	14000	7000

x = # packages of Hikers Mix
y = ... Bikers ..
z = ... Special ..

Raisins: $18000 = x60 + y40 + z30$
Peanuts: $14000 = x20 + y40 + z30$
Choc Chp: $7000 = x10 + y20 + z30$

Problem	Ans	Reason

4

Requirement per gallon ice cream

	quarts milk	quarts cream	oz vanilla	cacao
Deluxe V	3	1	2	0
Regular V	3.5	.5	1	0
Deluxe C	3.25	.75	0	2
total	100	25	80	160

16 oz / lb

x = # gallons Deluxe V
y = " " Regular V
z = " " Deluxe C

milk: $100 = x\,3 + y\,3.5 + z\,3.25$
cream: $25 = x\,1 + y\,.5 + z\,.75$
van: $80 = x\,2 + y\,1 + z\,0$
Cacao: $160 = x\,0 + y\,0 + z\,2$

Problem	Ans	Reason
5	(a) No	(1,1) - entry not 1
	(b) yes	
	(c) NO	Leading 1's in same column
6	(a) yes	
	(b) NO	Leading 1's in same col
	(c) NO	In Last col need to elim 3, 7, 9
7	(a) $\left[\begin{array}{ccc\|c} 2 & -1 & -1 & 2 \\ 1 & 1 & 2 & 4 \end{array}\right]$	
	(b) $\left[\begin{array}{cc\|c} 3 & -2 & 4 \\ 1 & 3 & -1 \end{array}\right]$	

Problem	Ans	Reason
8	(a) $\left[\begin{array}{ccc\|c} 3 & 2 & -1 & 8 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 5 \end{array}\right]$	
	(b) $\left[\begin{array}{ccc\|c} -1 & 2 & 0 & 5 \\ -1 & 0 & 3 & 2 \\ 2 & 0 & -1 & 6 \end{array}\right]$	

Problem	Ans	Reason

9 (a) $\left[\begin{array}{ccc|c} 3 & -1 & 6 & 8 \\ 2 & 1 & 4 & 7 \end{array}\right]$

(b) $3(-1) - (1) + 6(2) \stackrel{?}{=} 8$

$-3 \quad -1 \quad +12 \quad = 8$ ✓

$2(-1) + 1(1) + 4(2) \stackrel{?}{=} 7$

$-2 + 1 + 8 \quad = 7$ ✓

(c) For $x_2 = 0$ require

$3x_1 + 6x_3 = 8$

$2x_1 + 4x_3 = 7$

$3\left(\frac{7 - 4x_3}{2}\right) + 6x_3 = 8$

$\frac{21}{2} = 8$

$21 = 16$ No sol.

Problem	Ans	Reason
10	(a)	$2x + y + 3z = 11$ $6x - 3y + 4z = 12$ $4x - 4y + 1z = 1$
	(b)	$2\ \frac{1}{2} + 1 \cdot 1 + 3 \cdot 3 = 11$ ✓ $6\ \frac{1}{2} + -3 \cdot 1 + 4 \cdot 3 = 12$ ✓ $4\ \frac{1}{2} - 4 \cdot 1 + 1 \cdot 3 = 1$ ✓
	(c)	For $x = 1/2$ and $y = 0$ Require $1 + 3z = 11 \rightarrow z = 10/3$ $3 + 4z = 12 \rightarrow z = 9/4$ $2 + z = 1 \rightarrow z = -1$ No sol.

No. 5.3

Date 9

Problem	Ans	Reason
11	$x=2$ $y=0$ $z=1/2$	$\frac{1+6z}{2} + (4-8z) + 2z = 3$ $-3z = -3/2$ $z = 1/2$ $x = \frac{1+3}{2} = 2$ $y = \frac{4-8/2}{3} = 0$

check $2 \cdot 2 - 6 \cdot \frac{1}{2} = 1$ ✓

$2 + 3 \cdot 0 + 2 \cdot \frac{1}{2} = 3$ ✓

$3 \cdot 0 + 8 \cdot \frac{1}{2} = 4$ ✓

Problem	Ans	Reason
12	$x = \frac{1}{2}$ $y = 3/2$ $z = -1$	$\begin{bmatrix} -1 & 3 & 2 & 2 \\ 4 & 2 & 1 & 4 \\ -2 & 4 & -1 & 6 \end{bmatrix}$
		$\begin{bmatrix} 1 & -3 & -2 & -2 \\ 4 & 2 & 1 & 4 \\ -2 & 4 & -1 & 6 \end{bmatrix}$ $r_1' = -r_1$
		$\begin{bmatrix} 1 & -3 & -2 & -2 \\ 0 & 14 & 9 & 12 \\ 0 & -2 & -5 & 2 \end{bmatrix}$ $r_2' = r_2 - 4r_1$ $r_3' = r_3 + 2r_1$
		$\begin{bmatrix} 1 & -3 & -2 & -2 \\ 0 & 1 & \frac{9}{14} & \frac{12}{14} \\ 0 & -2 & -5 & 2 \end{bmatrix}$ $r_2' = \frac{1}{14} r_2$
		$\begin{bmatrix} 1 & 0 & -1/14 & 4/7 \\ 0 & 1 & \frac{9}{14} & \frac{12}{14} \\ 0 & 0 & -\frac{26}{7} & \frac{26}{7} \end{bmatrix}$ $r_1' = r_1 + 3r_2$ $r_3' = r_3 + 2r_2$
		$\begin{bmatrix} 1 & 0 & \frac{-1}{14} & \frac{4}{7} \\ 0 & 1 & 9/14 & \frac{6}{7} \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $r_3' = \frac{-7}{26} r_3$
		$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $r_1' = r_1 + \frac{1}{14} r_3$ $r_2' = r_2 - \frac{9}{14} r_3$

Problem	Ans	Reason
13	z free $y = -\frac{z}{2} - \frac{3}{4}$ $x = -\frac{3}{2}z + 2$	$\begin{bmatrix} 2 & -4 & 1 & 7 \\ 0 & 4 & 2 & -3 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & \frac{1}{2} & \frac{7}{2} \\ 0 & 1 & \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 3/2 & 2 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$ $r_1' = r_1 + 2r_2$

z free

$y = -\frac{z}{2} - 3/4$

$x = -\frac{3z}{2} + 2$

check

$2\left(-\frac{3}{2}z + 2\right) - 4\left(-\frac{z}{2} - \frac{3}{4}\right) + z \overset{?}{=} 7$ ✓

$4\left(-\frac{z}{2} - \frac{3}{4}\right) + 2z \overset{?}{=} -3$ ✓

Problem	Ans	Reason
14	$x = 4 - \frac{7}{2}z$ $y = -\frac{7}{2}$ z free	$\left[\begin{array}{ccc\|c} 2 & -8 & 3 & 8 \\ 3 & -9 & 6 & 12 \\ 1 & -3 & 2 & 4 \end{array}\right]$ $\left[\begin{array}{ccc\|c} 1 & -3 & 2 & 4 \\ 3 & -9 & 6 & 12 \\ 2 & -8 & 3 & 8 \end{array}\right]$ $\left[\begin{array}{ccc\|c} 1 & -3 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right]$ $r_2' = r_2 - 3r_1$, $r_3' = r_3 - 2r_1$ $\left[\begin{array}{ccc\|c} 1 & -3 & 2 & 4 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\left[\begin{array}{ccc\|c} 1 & -3 & 2 & 4 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\left[\begin{array}{ccc\|c} 1 & 0 & \frac{7}{2} & 4 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $r_1' = r_1 + 3r_2$

z free $\quad y = -\frac{7}{2} \quad x = 4 - \frac{7}{2}z$

check $2\left(4 - \frac{7}{2}z\right) - 8\left(-\frac{7}{2}\right) + 3z \overset{?}{=} 8$ ✓

$4 - \frac{7}{2}z - 3\left(-\frac{7}{2}\right) + 2z \overset{?}{=} 4$ ✓

Problem	Ans	Reason
15	$x = 3 - z$ $y = -\frac{1}{2}z$ $z =$ free	$\begin{bmatrix} 1 & -2 & 0 & 3 \\ 1 & 2 & 2 & 3 \\ 1 & 6 & 4 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 4 & 2 & 0 \\ 0 & 8 & 4 & 0 \end{bmatrix}$ $r_2' = r_2 - r_1$, $r_3' = r_3 - r_1$ $\begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $r_1' = r_1 + 2r_2$

z free

$y = -\frac{1}{2}z$

$x = 3 - z$

check

$3 - z - 2\left(-\frac{1}{2}z\right) = 3$ ✓

$3 - z + 2\left(-\frac{1}{2}z\right) + 2z = 3$ ✓

$3 - z + 6\left(-\frac{1}{2}z\right) + 4z = 3$ ✓

Problem	Ans	Reason

16

$x_1 = 1/2$
$x_2 = -1/2$
$x_3 = 1$
$x_4 = 2$

$x_1 \quad x_2 \quad x_3 \quad x_4$

$$\begin{bmatrix} 3 & 5 & -1 & 2 & 2 \\ 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & -3 \\ 1 & -3 & -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -1 & 0 & 1 \\ 0 & 2 & 0 & -1 & -3 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 5 & -1 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1/2 & -3/2 \\ 0 & 6 & 1 & 0 & -2 \\ 0 & 14 & 2 & 2 & -1 \end{bmatrix} \quad r_3' = r_3 - 2r_1 \quad r_4' = r_4 - 3r_1$$

$$\begin{bmatrix} 1 & 0 & -1 & -3/2 & -7/2 \\ 0 & 1 & 0 & -1/2 & -3/2 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 2 & 9 & 20 \end{bmatrix} \quad r_1' = r_1 + 3r_2 \quad r_3' = r_3 - 6r_2 \quad r_4' = r_4 - 14r_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 3/2 & 7/2 \\ 0 & 1 & 0 & -1/2 & -3/2 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 3 & 6 \end{bmatrix} \quad r_1' = r_1 + r_3 \quad r_4' = r_4 - 2r_3$$

Problem | ans | reason

16, cont

$$\begin{bmatrix} 1 & 0 & 0 & 3/2 & 7/2 \\ 0 & 1 & 0 & -1/2 & -3/2 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$r_1' = r_1 - 3/2\, r_4$
$r_2' = r_2 + 1/2\, r_4$
$r_3' = r_3 - 3\, r_4$

$x_1 = 1/2 \quad x_2 = -1/2 \quad x_3 = 1 \quad x_4 = 2$

check

$3 \cdot \frac{1}{2} + 5 \cdot \frac{-1}{2} - 1 \cdot 1 + 2 \cdot 2 \stackrel{?}{=} 2$ ✓

$2 \cdot \frac{1}{2} \quad 0 \quad -\frac{1}{2} \quad -1 \cdot 1 + 0 = 0$ ✓

$2 \quad -\frac{1}{2} \quad -1 \cdot 2 = -3$ ✓

$1 \cdot \frac{1}{2} \quad -3 \quad -\frac{1}{2} \quad -1 \cdot 1 \quad 0 \cdot 2 = 1$ ✓

Problem	Ans	Reason
17	$x = 2$ $y = -2$ $z = -1$	$\begin{bmatrix} 1 & -1 & 1 & 3 \\ 2 & -2 & 1 & 7 \\ 1 & -2 & 3 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 2 & 0 \end{bmatrix}$ $r_2' = r_2 - 2r_1$, $r_3' = r_3 - r_1$ $\begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $r_1' = r_1 + r_2$ $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $r_1' = r_1 + r_3$, $r_2' = r_2 + 2r_3$ $x = 2 \quad y = -2 \quad z = 1$

check

$1 \cdot 2 - -2 + -1 \stackrel{?}{=} 3$ ✓

$2 \cdot 2 \quad -2 \cdot -2 + 1 \cdot -1 \stackrel{?}{=} 7$ ✓

$1 \cdot 2 + \quad -2 \cdot -2 + 3(-1) \stackrel{?}{=} 3$ ✓

Problem	Ans	Reason
18	z free $x = 7+5z$ $y = 2-z$	$\begin{bmatrix} 3 & 2 & -13 & 25 \\ 2 & -2 & -12 & 10 \end{bmatrix}$ $\begin{bmatrix} 1 & 4 & -1 & 15 \\ 1 & -1 & -6 & 5 \end{bmatrix}$ $r_1' = r_1 - r_2$, $r_2' = \frac{1}{2} r_2$ $\begin{bmatrix} 1 & 4 & -1 & 15 \\ 0 & -5 & -5 & -10 \end{bmatrix}$ $\begin{bmatrix} 1 & 4 & -1 & 15 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -5 & 7 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ $r_1' = r_1 - 4r_2$

z free

$y = 2 - z$

$x = 7 + 5z$

check:

$3(7+5z) + 2(2-z) - 13z \overset{?}{=} 25$ ✓

$2(7+5z) - 2(2-z) - 12z \overset{?}{=} 10$ ✓

Problem	Ans	Reason
19	$x = 3$ $y = 1$ $z = -2$	$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 3 & -2 & 4 & -1 \\ 2 & -4 & 2 & -2 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 & -1 \\ 2 & 1 & 3 & 1 \\ 3 & -2 & 4 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 5 & 1 & 3 \\ 0 & 4 & 1 & 2 \end{bmatrix}$ $r_2' = r_2 - 2r_1$, $r_3' = r_3 - 3r_1$ $\begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 4 & 1 & 2 \end{bmatrix}$ $r_2' = r_2 - r_3$ $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ $r_1' = r_1 + 2r_2$, $r_3' = r_3 - 4r_2$ $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ $r_1' = r_1 - r_3$

$x = 3 \quad y = 1 \quad z = -2$

$2 \cdot 3 + 1 \cdot 1 + 3 \cdot -2 = 1$ ✓

$3 \cdot 3 - 2 \cdot 1 + 4 \cdot -2 = -1$ ✓

$2 \cdot 3 - 4 \cdot 1 + 2 \cdot -2 = -2$ ✓

Problem	Ans	Reason
20	$x = 8$ $y = -2$ $z = -4$	$\begin{bmatrix} 1 & -1 & 2 & 2 \\ 1 & -5 & 5 & -2 \\ 1 & 3 & 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & -4 & 3 & -4 \\ 0 & 4 & -2 & 0 \end{bmatrix}$ $r_2' = r_2 - r_1$, $r_3' = r_3 - r_1$ $\begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & -1/2 & 0 \\ 0 & -4 & 3 & -4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 3/2 & 2 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 1 & -4 \end{bmatrix}$ $r_1' = r_1 + r_2$, $r_3' = r_3 + 4r_2$ $\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{bmatrix}$ $r_1' = r_1 - 3/2 r_3$, $r_2' = r_2 + 1/2 r_3$ $x = 8 \quad y = -2 \quad z = -4$ $1 \cdot 8 - 1(-2) + 2(-4) = 2$ ✓ $1 \cdot 8 - 5(-2) + 5(-4) = -2$ ✓ $1 \cdot 8 + 3(-2) + 0 = 2$ ✓

Problem	Ans	Reason
21	free $x = 9 - 5z$ $y = -4 + 2z$	$\begin{bmatrix} 2 & 1 & 8 & 14 \\ 3 & -1 & 17 & 31 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 & 8 & 14 \\ 1 & -2 & 9 & 17 \end{bmatrix}$ $r_2' = r_2 - r_1$ $\begin{bmatrix} 1 & -2 & 9 & 17 \\ 2 & 1 & 8 & 14 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 9 & 17 \\ 0 & 5 & -10 & -20 \end{bmatrix}$ $r_2' = r_2 - 2r_1$ $\begin{bmatrix} 1 & -2 & 9 & 17 \\ 0 & 1 & -2 & -4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 5 & 9 \\ 0 & 1 & -2 & -4 \end{bmatrix}$ $r_1' = r_1 + 2r_2$

free
$y = -4 + 2z$
$x = 9 - 5z$

check

$2(9-5z) + 1(-4+2z) + 8z = 14$ ✓
$3(9-5z) - 1(-4+2z) + 17z = 31$ ✓

Problem	Ans	Reason
22	$x = 0$ $y = 4$ $z = 0$	$\begin{bmatrix} 2 & 2 & -1 & 8 \\ -2 & 1 & 1 & 4 \\ 4 & -1 & 2 & -4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & -\frac{1}{2} & 4 \\ -2 & 1 & 1 & 4 \\ 4 & -1 & 2 & -4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & -\frac{1}{2} & 4 \\ 0 & 3 & 0 & 12 \\ 0 & -5 & 4 & -20 \end{bmatrix}$ $r_2' = r_2 + 2r_1$, $r_3' = r_3 - 4r_1$ $\begin{bmatrix} 1 & 1 & -\frac{1}{2} & 4 \\ 0 & 1 & 0 & 4 \\ 0 & -5 & 4 & -20 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 4 & 0 \end{bmatrix}$ $r_1' = r_1 - r_2$, $r_3' = r_3 + 5r_2$ $\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $r_1' = r_1 + \frac{1}{2} r_3$ $x = 0 \quad y = 4 \quad z = 0$

Problem	Ans	Reason
23	$x = 50$ $y = 10$ $z = 100$	$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 105 \\ 1 & 10 & 0 & 150 \\ 2 & \frac{1}{2} & \frac{3}{2} & 255 \end{bmatrix}$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 105 \\ 0 & 9\frac{1}{2} & -\frac{1}{2} & 45 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 45 \end{bmatrix}$ $r_2' = r_2 - r_1$, $r_3' = r_3 - 2r_1$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 105 \\ 0 & 1 & -1 & -90 \\ 0 & \frac{19}{2} & -\frac{1}{2} & 45 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 1 & 150 \\ 0 & 1 & -1 & -90 \\ 0 & 0 & 9 & 900 \end{bmatrix}$ $r_1' = r_1 - \frac{1}{2}r_2$, $r_3' = r_3 - \frac{19}{2}r_2$ $\begin{bmatrix} 1 & 0 & 1 & 150 \\ 0 & 1 & -1 & -90 \\ 0 & 0 & 1 & 100 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 100 \end{bmatrix}$ $r_1' = r_1 - r_3$, $r_2' = r_2 + r_3$ $x = 50 \quad y = 10 \quad z = 100$

Problem	Ans	Reason
24	$x = \frac{4000000}{27}$ $y = \frac{3000000}{18}$ $z = \frac{2000000}{15}$	

Problem	Ans	Reason
25	$x = 100$ $y = 300$ $z = 0$	$\begin{bmatrix} 1 & 2 & 3 & 700 \\ 2 & 4 & 3 & 1400 \\ 6 & 4 & 3 & 1800 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 700 \\ 0 & 0 & -3 & 0 \\ 0 & -8 & -15 & -2800 \end{bmatrix}$ $r_2' = r_2 - 2r_1$, $r_3' = r_3 - 6r_1$ $\begin{bmatrix} 1 & 2 & 3 & 700 \\ 0 & 1 & \frac{15}{8} & 300 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -\frac{3}{4} & 100 \\ 0 & 1 & 0 & 300 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 300 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $x = 100 \quad y = 300 \quad z = 0$

Problem 26

No sol

$$\begin{bmatrix} 1 & .5 & .75 & 25 \\ 2 & 1 & 0 & 80 \\ 3 & 3.5 & 3.25 & 100 \\ 0 & 0 & 1 & 80 \end{bmatrix}$$

$$\begin{bmatrix} 1 & .5 & .75 & 25 \\ 0 & 0 & -1.5 & 30 \\ 0 & 2 & 1 & 25 \\ 0 & 0 & 1 & 80 \end{bmatrix} \quad r_2' = r_2 - 2r_1 \quad r_3' = r_3 - 3r_1$$

No sol

No. 5.3

Date 26

Problem	Ans	Reason
27	(a)	x_1 = hours/week spent on sales (old c) x_2 = " " sales (new c) x_3 = " " Office Manager x_4 = " " Long-Range Planning

$$x_1 + x_2 + x_3 + x_4 = 40$$
$$x_1 + x_2 = 20$$
$$x_1 - 2x_2 = 0$$
$$x_2 - 2x_4 = 0$$

(b) $x_1 = \frac{40}{3}$ $x_2 = \frac{20}{3}$ $x_3 = \frac{50}{3}$ $x_4 = \frac{10}{3}$

Problem	Ans	Reason
28	$x_1 = 15$ $x_2 = 5$ $x_3 = 15$ $x_4 = 10$	x_1 = # hours per week devoted to Math x_2 = ... English x_3 = ... Biology x_4 = ... Econ

$$x_1 + x_2 + x_3 + x_4 = 45$$

$$x_1 + x_3 = 2(x_2 + x_4)$$

$$x_4 = 2x_2$$

$$x_1 = x_3$$

$$x_1 + x_2 + x_1 + 2x_2 = 45$$

$$x_1 + x_1 - 2x_2 - 4x_2 = 0$$

$$2x_1 + 3x_2 = 45$$
$$2x_1 - 6x_2 = 0$$

$$2x_1 + 3x_2 = 45$$
$$x_1 - 3x_2 = 0$$

$$3x_1 = 45 \qquad x_1 = 15$$

$$x_2 = 15/3 = 5$$

$x_1 = 15$ $x_2 = 5$ $x_3 = 15$ $x_4 = 10$

Problem	Ans	Reason
29	(a) $x_2 = -\frac{3}{8} + \frac{1}{2}x_3 - \frac{5}{16}x_4$ $x_1 = \frac{7}{4} - \frac{7}{8}x_4$ x_3 free x_4 free	$\begin{bmatrix} 3 & -2 & 1 & 2 & 6 \\ 2 & 4 & -2 & 3 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & -6 & 3 & -1 & 4 \\ 2 & 4 & -2 & 3 & 2 \end{bmatrix}$ $r_1' = r_1 - r_2$ $\begin{bmatrix} 1 & -6 & 3 & -1 & 4 \\ 0 & 16 & -8 & 5 & -6 \end{bmatrix}$ $\begin{bmatrix} 1 & -6 & 3 & -1 & 4 \\ 0 & 1 & -\frac{1}{2} & \frac{5}{16} & -\frac{6}{16} \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & \frac{7}{8} & \frac{7}{4} \\ 0 & 1 & -\frac{1}{2} & \frac{5}{16} & -\frac{6}{16} \end{bmatrix}$ $r_1' = r_1 + 6r_2$ x_3 free x_4 free $x_2 = -\frac{6}{16} + \frac{1}{2}x_3 - \frac{5}{16}x_4$ $x_1 = \frac{7}{4} - \frac{7}{8}x_4$ $3\left(\frac{7}{4} - \frac{7}{8}x_4\right) - 2\left(-\frac{6}{16} + \frac{1}{2}x_3 - \frac{5}{16}x_4\right) + x_3 + 2x_4 = 6$ ✓ $2\left(\frac{7}{4} - \frac{7}{8}x_4\right) + 4\left(-\frac{6}{16} + \frac{1}{2}x_3 - \frac{5}{16}x_4\right) - 2x_3 + 3x_4 = 2$ ✓
	(b) $x_1 = 0$ $x_2 = -1 + \frac{1}{2}x_3$ $x_3 =$ free $x_4 = 2$	

Problem	Ans	Reason
30	(a) x free $y = \frac{3}{2}x - 4$ $z = \frac{7}{2}x - 10$ $w = \frac{7}{2}x - 10$	$\begin{bmatrix} 3 & -2 & 1 & -1 & 8 \\ 2 & 1 & -1 & 0 & 6 \\ 1 & -3 & 0 & 1 & 2 \end{bmatrix}$ $6x - 4y = 16$ $\frac{3}{2}x - y = 4$ x free $y = \frac{3}{2}x - 4$ $z = 2x + \left(\frac{3}{2}x - 4\right) - 6$ $= \frac{7}{2}x - 10$ $w = 3\left(\frac{3}{2}x - 4\right) + 2 - x$ $= \frac{7}{2}x - 10$
	(b) $x = \frac{20}{7}$ $y = \frac{2}{7}$ $z = 0$ $w = 0$	$w = 0 \rightarrow$ $\frac{7}{2}x = 10$ $x = \frac{20}{7}$
	(c) $x = 8/3$ $y = 0$ $z = -2/3$ $w = -2/3$	$\frac{3}{2}x = 4$ $x = \frac{8}{3}$

No. 5.3

Date 30

Problem	Ans	Reason

31 No sol

$$\begin{matrix} x & y & z & w \end{matrix}$$
$$\begin{bmatrix} 3 & -2 & 1 & -1 & 8 \\ 4 & -5 & 0 & 1 & 8 \\ 2 & 1 & -1 & 0 & 6 \\ 1 & -3 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{matrix} z & w & x & y \end{matrix}$$
$$\begin{bmatrix} 1 & -1 & 3 & -2 & 8 \\ 0 & 1 & 4 & -5 & 8 \\ -1 & 0 & 2 & 1 & 6 \\ 0 & 1 & 1 & -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 3 & -2 & 8 \\ 0 & 1 & 4 & -5 & 8 \\ 0 & -1 & 5 & -1 & 14 \\ 0 & 1 & 1 & -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 & -7 & 16 \\ 0 & 1 & 4 & -5 & 8 \\ 0 & 0 & 9 & -6 & 22 \\ 0 & 0 & -3 & 2 & -6 \end{bmatrix} \begin{matrix} r_1' = r_1 + r_2 \\ \\ r_3' = r_3 + r_2 \\ r_4' = r_4 - r_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 7 & -7 & 16 \\ 0 & 1 & 4 & -5 & 8 \\ 0 & 0 & 9 & -6 & 22 \\ 0 & 0 & 0 & 0 & \frac{4}{3} \end{bmatrix} r_4' = r_4 + \frac{1}{3} r_3$$

No Sol

Problem	Ans	Reason
32	(a) $x_1 = \frac{17}{3} + \frac{1}{3}x_4$ $x_2 = 0$ $x_3 = \frac{2}{3} + \frac{1}{3}x_4$ $x_4 =$ free	$\begin{bmatrix} 1 & -1 & 2 & -1 & 7 \\ 1 & 1 & -4 & 1 & 3 \\ 1 & 0 & -1 & 0 & 5 \\ 0 & 2 & 6 & -2 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 2 & -1 & 7 \\ 0 & 2 & -6 & 2 & -4 \\ 0 & 1 & -3 & 1 & -2 \\ 0 & 2 & 6 & -2 & 4 \end{bmatrix}$ $r_2' = r_2 - r_1$, $r_3' = r_3 - r_1$
	(b) $x_1 = 0$ $x_2 = 0$ $x_3 = -5$ $x_4 = -17$	$\begin{bmatrix} 1 & -1 & 2 & -1 & 7 \\ 0 & 1 & -3 & 1 & -2 \\ 0 & 1 & -3 & 1 & -2 \\ 0 & 1 & 3 & -1 & 2 \end{bmatrix}$
	(c) No	$\begin{bmatrix} 1 & 0 & -1 & 0 & 5 \\ 0 & 1 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -2 & 4 \end{bmatrix}$ $r_1' = r_1 + r_2$ $\begin{bmatrix} 1 & 0 & -1 & 0 & 5 \\ 0 & 1 & -3 & 1 & -2 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & \frac{17}{3} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $r_1' = r_1 + r_3$, $r_2' = r_2 + 3r_3$

Problem	Ans	Reason
33	(a) $x = 1 + \frac{1}{5}z$ $y = 3 - \frac{3}{5}z$ z free	$\begin{bmatrix} 5 & 5 & 2 & 20 \\ 3 & 1 & 0 & 6 \end{bmatrix}$ $\begin{bmatrix} -1 & 3 & 2 & 8 \\ 3 & 1 & 0 & 6 \end{bmatrix}$ $r_1' = r_1 - 2r_2$ $\begin{bmatrix} 1 & -3 & -2 & -8 \\ 3 & 1 & 0 & 6 \end{bmatrix}$ $\begin{bmatrix} 1 & -3 & -2 & -8 \\ 0 & 10 & 6 & 30 \end{bmatrix}$ $r_2' = r_2 - 3r_1$ $\begin{bmatrix} 1 & -3 & -2 & -8 \\ 0 & 1 & 3/5 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & -1/5 & 1 \\ 0 & 1 & 3/5 & 3 \end{bmatrix}$ $r_1' = r_1 + 3r_2$
	$x = 1 + \frac{1}{5}z$ $y = 3 - \frac{3}{5}z$ z free	
	(b) If $y=0$ then $z=5$, $x=2$ If $z=0$ then $x=1$, $y=3$ If $x=0$ then $y=6$ $z=-5$	
	(c) line thru x-y-z space	

Problem	Ans	Reason

34 | 80 Feathers

	ft Wood	ft string	Feathers
Bow	5	4	0
Arrow	3	0	4
total	100	32	a

Find a

x = # bows made
y = # arrows made

$100 = 5x + 3y$

$32 = 4x$

$a = 4y$

$x = 32/4 = 8$

$y = \frac{100 - 5x}{3}$

$= \frac{100 - 40}{3}$

$= \frac{60}{3}$

$= 20$

$a = 4 \cdot 20 = 80$

Problem	Ans	Reason

35 (a) $x = 100$

$y = 250$

$z = \frac{200}{3}$

$$\begin{bmatrix} 1 & 2 & 3 & 800 \\ 2 & 4 & 3 & 1400 \\ 6 & 4 & 3 & 1800 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 800 \\ 0 & 0 & -3 & -200 \\ 0 & -8 & -15 & -3000 \end{bmatrix} \quad r_2' = r_2 - 2r_1 \quad r_3' = r_3 - 6r_1$$

$$\begin{bmatrix} 1 & 2 & 3 & 800 \\ 0 & 1 & 15/8 & \frac{3000}{8} \\ 0 & 0 & 1 & \frac{200}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3/4 & 50 \\ 0 & 1 & 15/8 & \frac{3000}{8} \\ 0 & 0 & 1 & \frac{200}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 250 \\ 0 & 0 & 1 & \frac{200}{3} \end{bmatrix}$$

$x = 100$ $\quad y = 250$ $\quad z = \frac{200}{3}$

(b) $x = 100$

$y = 350$

$z = -\frac{200}{3}$

Require $z \geq 0$ so No sol

Problem	Ans	Reasn
35 cnt	(c) 2000 g peanuts Left.	take $z=0$

$$18000 = x60 + 40$$
$$6000 = x10 + 20$$

$$14000 - x20 - 40 = ?$$

$$1800 = 6x + 4y$$
$$600 = x + 2y$$

$$x = 600 - 2y$$

$$1800 = 6(600-2y) + 4y$$
$$8y = 1800$$
$$y = 1800/8 = 225$$
$$x = 600 - 2(225)$$
$$= 600 - 450$$
$$= 150$$

$$14000 - 20(150) - 40(225)$$
$$= 14000 - 3000 - 9000$$
$$= 2000$$