Section 4.R HW solutions

Problem

1

Ans

X	Pr
-1	1/6
2	2/6
5	3/6

Reason

outcome	1	2	3	4	5	6
X	-1	2	2	5	5	5

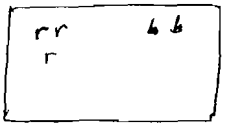
2

X	Pr
3	1/8
4	3/8
5	3/8
6	1/8

#H	#T	X	Pr
0	3	6	1/8
1	2	5	3/8
2	1	4	3/8
3	0	3	1/8

3

X	Pr
0	1/10
1	6/10
2	3/10



5 balls

Box

Chose 3

#red	#blue	X	Pr
3	0	0	$1/C(5,3)$
2	1	1	$2\,C(3,2)/C(5,3)$
1	2	2	$C(3,1)/C(5,3)$

Problem	Ans	Reason

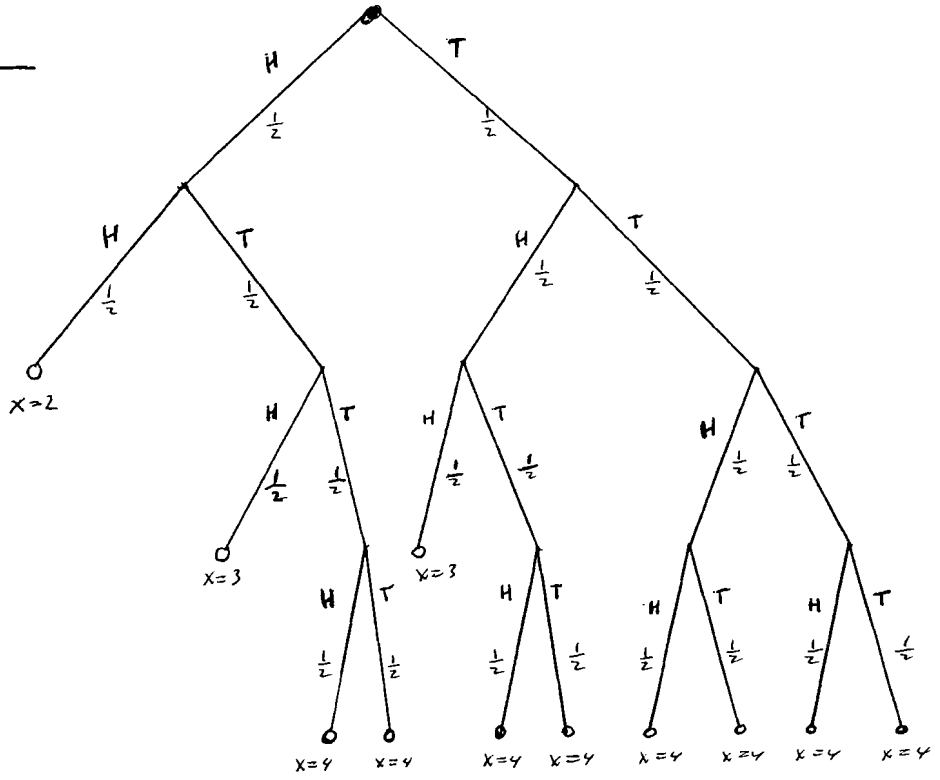
4

X	Pr
7	$\frac{1}{6}$
8	$\frac{1}{6}$
9	$\frac{2}{6}$
10	$\frac{1}{6}$
11	$\frac{1}{6}$

Outcome	34	35	36	45	46	56
X	7	8	9	9	10	11

5

X	Pr
2	$\frac{1}{4}$
3	$\frac{1}{4}$
4	$\frac{1}{2}$



$$Pr(x=2) = \frac{1}{2}\,\frac{1}{2} = \frac{1}{4}$$

$$Pr(x=3) = \frac{1}{2}\,\frac{1}{2}\,\frac{1}{2} + \frac{1}{2}\,\frac{1}{2}\,\frac{1}{2} = \frac{1}{4}$$

$$Pr(x=4) = \frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\ 8 = \frac{1}{2}$$

Problem	Ans	Reason
6		

X	Pr
0	.352
10	.648

Bernoulli with $n=3$

success : Homer attends

$p = .6$ $q = .4$

#success	X	Pr
0	0	$1\,(.6)^0(.4)^3$
1	0	$3\,(.6)^1(.4)^2$
2	10	$3\,(.6)^2(.4)^1$
3	10	$1\,(.6)^3(.4)^0$

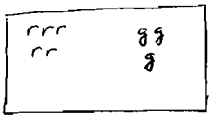
$$(.4)^3 + 3(.6)(.4)^2 = (.4)^2(.4 + 3(.6))$$
$$= (.16)(.4 + 1.8)$$
$$= (.16)(2.2) = .352$$

$$3(.6)^2(.4) + (.6)^3 = (.6)^2(3(.4) + .6)$$
$$= (.36)(1.2 + .6)$$
$$= (.36)(1.8) = .648$$

Problem | Ans | Reason

7

X	Pr
−.4	10/56
−.1	30/56
.2	15/56
.5	1/56



8 balls

Box

choose 3

#red	#green	X	Pr
3	0	$3(.2)+0(.5)-1=-.4$	$C(5,3)C(3,0)/C(8,3)$
2	1	$2(.2)+1(.5)-1=-.1$	$C(5,2)C(3,1)/C(8,3)$
1	2	$1(.2)+2(.5)-1=.2$	$C(5,1)C(3,2)/C(8,3)$
0	3	$0(.2)+3(.5)-1=.5$	$C(5,0)C(3,3)/C(8,3)$

8

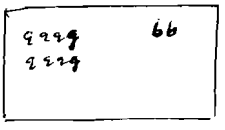
$E(x) = 9.5$

X	Pr	X Pr
5	.5	2.5
10	.3	3.0
20	.2	4.0
		$E(x) = 9.5$

9

Expected gain is

$ 2.40



10 balls

Box

Choose 1

Color	X	Pr	X Pr
g	5	8/10	40/10
b	−8	2/10	−16/10
			$E(x) = \frac{24}{10} = 2.4$

Problem	Ans	Reason
10	24.5	$10(.8)1 + 12(.5)2 + 5(.3)3$ $= 8.0 + 12.0 + 4.5$ $= 24.5$

11

(a)

Y	Pr
350	2/35
400	5/35
450	12/35
500	9/35
550	6/35
600	1/35

200g fish	150g fish	100g fish
3	2	2

7 fish

Aquarium

choose 3

#200g	#150g	#100g	Y	Pr
3	0	0	600	$1 \cdot 1 \cdot 1/35$
2	1	0	550	$3 \cdot 2 \cdot 1/35$
2	0	1	500	$3 \cdot 1 \cdot 2/35$
1	2	0	500	$3 \cdot 1 \cdot 1/35$
1	1	1	450	$3 \cdot 2 \cdot 2/35$
1	0	2	400	$3 \cdot 1 \cdot 1/35$
0	2	1	400	$1 \cdot 1 \cdot 2/35$
0	1	2	350	$1 \cdot 2 \cdot 1/35$

(b) $E(Y) = \frac{16500}{35} = \frac{3300}{7}$

(c) $Var(Y) =$

$$\left(350 - \tfrac{3300}{7}\right)^2 \tfrac{2}{35} + \left(400 - \tfrac{3300}{7}\right)^2 \tfrac{5}{35} + \left(450 - \tfrac{3300}{7}\right)^2 \tfrac{12}{35}$$

$$+ \left(500 - \tfrac{3300}{7}\right)^2 \tfrac{9}{35} + \left(550 - \tfrac{3300}{7}\right)^2 \tfrac{6}{35} + \left(600 - \tfrac{3300}{7}\right|^2 \tfrac{1}{35}$$

$STD = \sqrt{Var(Y)}$

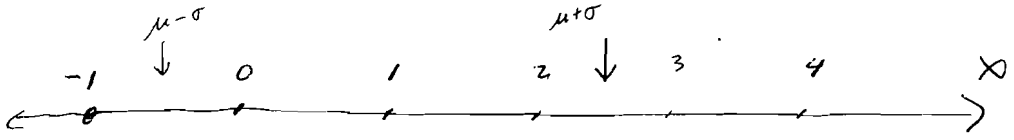
Problem	Ans	Reason
12	(a) 1.00	
	(b) 2.17	
	(c) $1.4 < \sqrt{2.17} < 1.5$	
	(d) .60	

X	Pr	X Pr
−1	.20	−.20
0	.35	0
1	.10	.10
2	.15	.30
4	.20	.80
		1.00

$$Var(X) = (-1-1)^2(.20) + (0-1)^2(.35) + (1-1)^2(.10) + (2-1)^2(.30) + (4-1)^2(.80)$$

$$= .8 + .35 + 0 + .30 + .72$$

$$= 2.17$$



$$Pr(x=0) + Pr(x=1) + Pr(x=2) = .35 + .10 + .15 = .60$$

$$\mu = 1 \qquad \sigma = \sqrt{2.17}$$

Problem	Ans	Reason
13	(a) 146	Bernoulli $n = 365$
	(b) $\sqrt{87.6}$	success = late del $p = .40$

X = # successes

$$\text{mean} = E(X) = np$$
$$= 365(.4)$$
$$= 146$$

$$\text{var}(x) = npq$$
$$= 360(.4)(.6)$$
$$= 146(.6)$$
$$= 87.6$$

$$STD = \sqrt{87.6}$$

14 $E(x) = 5.5728$

Bernoulli $n = 5$ $p = .6$

success : heads

var(x) as below

STD = $\sqrt{\text{var}(x)}$

# Heads	X	Pr	X Pr
0	0	$1(.6)^0(.4)^5$	0
1	0	$5(.6)^1(.4)^4$	0
2	0	$10(.6)^2(.4)^3$	0
3	0	$10(.6)^3(.4)^2$	0
4	20	$5(.6)^4(.4)$	$100(.6)^4(.4)$
5	5	$(.6)^5$	$5(.6)^5$
			5.5728

$$\text{var}(x) =$$

$$(.6)^5(5 - 5.5728)^2 + 5(.6)^4(.4)(20 - 5.5728)^2$$
$$+ (1 - (.6)^5 - 5(.6)^4(.4))(0 - 5.5728)^2$$

Problem	Ans	Reason

15 $\frac{67}{12}$

X: red die, green die

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	4	4	4	4	4	4
3	5	5	5	5	5	5
4	6	6	6	6	6	6
5	7	7	7	7	7	7
6	8	8	8	8	8	8

$E(X) = \frac{1}{36}$ times sum of entries in above table

$= \frac{1}{36}\left(1+2+3+4+5+6 + 4\cdot 6 + 5\cdot 6 + 6\cdot 6 + 7\cdot 6 + 8\cdot 6\right)$

$= \frac{1}{36}\left(21 + 6\cdot 30\right) = \frac{201}{36} = \frac{67}{12}$

16 29

Bernoulli process Length n Find n

$p = .65 \qquad q = .35$

$E(\text{hard wrists}) = n(.35)$

Find n such that

$n(.35) \geq 10$

$n \geq \frac{10}{.35} = \frac{1000}{35} = 28.88\ldots$

29 days must pass

Item	Ans	Reason
17	(a) $2/5$	$100(.6) = 150p$
	(b) Y has Larger variance	$\frac{60}{150} = p$
		$\frac{2}{5} = p$
		$Var(x) = 100(.6)(.4) = 24$
		$Var(Y) = 150\ \frac{2}{5}\ \frac{3}{5} = 36$
~~18~~		
19	From -3 to 3	
20	~~20~~	
21	~~21~~	
22	~~22~~	
23	~~23~~	
24	1250	Bernoulli process Length n Find n

X	Y
55	-3
⋮	⋮
145	3

success: no defect $p = .8$
failure: defect $q = .2$

$$n(.8) \geq 1000$$

$$n \geq \frac{1000}{.8} = \frac{10000}{8} = 1250$$

Problem	Ans	Reason

25

Bernoulli process length $n = 200$

success $p = .2$

$q = 1 - p = .8$

Ans = $C(200,50)(.2)^{50}(.8)^{150} + \cdots + C(200,200)(.2)^{200}(.8)^{0}$

26

Bernoulli $n = 400$

$p = 4/5 = .8$ $q = 1/5 = .2$

Ans = $C(400,0)(.8)^{400}(.2)^{0} + \cdots C(400,60)(.8)^{340}(.2)^{60}$

27

(a)

Bernoulli $n = 100$

$p = 1/2$ $q = 1/2$

$Pr(\#suc \geq 70) =$

$$\frac{C(100,70) + C(100,71) + \cdots C(100,100)}{2^{100}}$$

(b)

$n = 100$, $p = .8$ $q = .2$

$Pr(\#\text{success} < 70)$

$= C(100,0)(.8)^{0}(.2)^{100} + C(100,1)(.8)^{1}(.2)^{99} + \cdots C(100,69)(.8)^{69}(.2)^{31}$

No. 4.R

Date 11

Problem	Ans	Reason
~~28~~		
29	$E(X) = \frac{4900}{64}$	Bernoulli $n = 3$

success: a sale $p = 1/4$ $q = 3/4$

#	X	Pr	X Pr
0	0	$1\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3$	0
1	100	$3\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2$	$\frac{27 \cdot 100}{64}$
2	200	$3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^1$	$\frac{9 \cdot 2 \cdot 100}{64}$
3	400	$1\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^0$	$\frac{4 \cdot 100}{64}$
			$\frac{49 \cdot 100}{64}$

$$Var(X) = \left(0 - \frac{4900}{64}\right)^2 \frac{27}{64} + \left(100 - \frac{4900}{64}\right)^2 \frac{27}{64}$$

$$+ \left(200 - \frac{4900}{64}\right)^2 \frac{9}{64} + \left(400 - \frac{4900}{64}\right) \frac{1}{64}$$

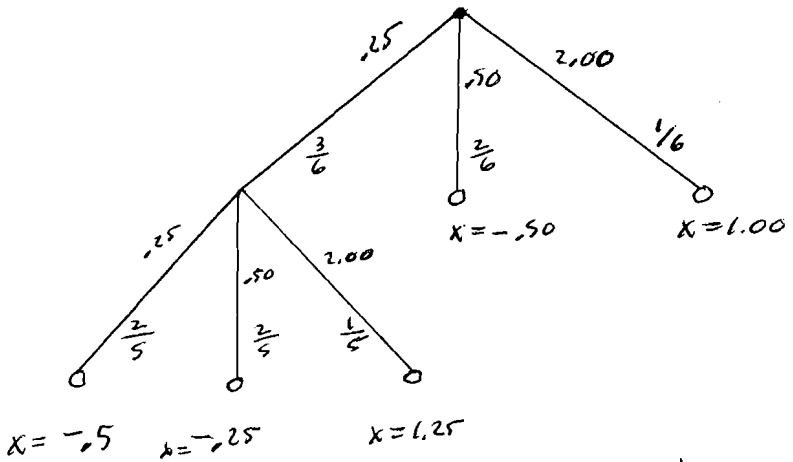
$$STD = \sqrt{Var(X)}$$

No. 4, R

Date 12

Problem	Ans	Reason
30	$-\frac{1}{40} = -.025$ ie $-2\frac{1}{2}$ ¢	Fish value 6 fish

.25	.50	2.00
3	2	1



X	Pr	X Pr
-.50	$\frac{2}{5}\frac{3}{6} + \frac{2}{6}\cdot\frac{6}{30}$	$-\frac{8}{30}$
-.25	$\frac{3}{6}\frac{2}{5}$	$-\frac{1}{20}$
1.00	$\frac{1}{6}$	$\frac{1}{6}$
1.25	$\frac{3}{6}\frac{1}{5}$	$\frac{1}{8}$

$$E(X) = \frac{-8.4 - 1.6 + 1.20 + 15}{120}$$

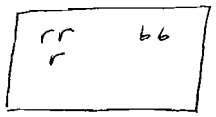
$$= \frac{-3}{120}$$

$$= -1/40$$

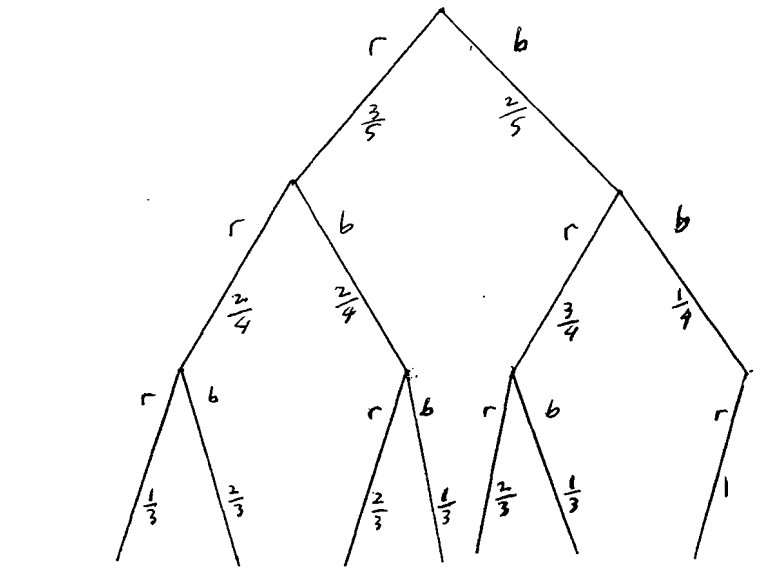
No. 4.R

Date 13

Problem	Ans	Reason
31	3.2	



Box



X	0	0	0	0	5	5	5
Y	2	1	1	0	2	1	1
X+Y	2	1	1	0	7	6	6

$$E(X+Y) = \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}\cdot 2 + \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3}\cdot 1 + \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3}\cdot 1 + \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}\cdot 0 + \frac{2}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot 7 + \frac{2}{5}\cdot\frac{3}{4}\cdot\frac{1}{3}\cdot 6 + \frac{2}{5}\cdot\frac{1}{4}\cdot\frac{3}{3}\cdot 6$$

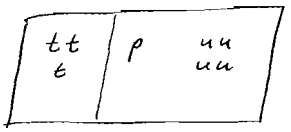
$$= \frac{12 + 12 + 12 + 0 + 84 + 36 + 36}{60} = 3.2$$

No. 4.R

Date 14

Problem	Ans	Reason
32	a)	

X	Pr
0	5/70
1	30/70
2	30/70
3	5/70



8 rats

Cage

chone 4

Random X : # trained rats.

b) $1.5 = 3/2$

c) $\frac{15}{28}$

X	Pr		x Pr
0	$C(5,4)/C(8,4)$	$= 5/70$	0
1	$C(3,1)C(5,3)/C(8,4)$	$= 30/70$	$\frac{30}{70}$
2	$C(3,2)C(5,2)/C(8,4)$	$= 30/70$	$\frac{60}{70}$
3	$C(3,3)C(5,1)/C(8,4)$	$= 5/70$	$\frac{15}{70}$

$\frac{105}{70} = 1.5$

$$\text{Var}(x) = (0 - 3/2)^2 \frac{5}{70} + (1 - 3/2)^2 \frac{30}{70}$$

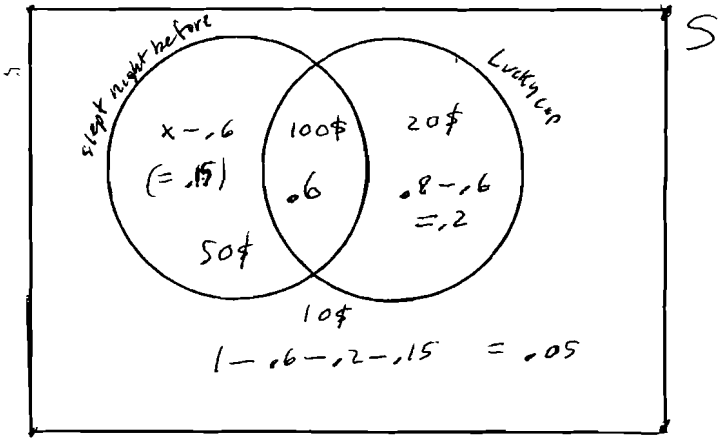
$$+ (2 - 3/2)^2 \frac{30}{70} + (3 - 3/2)^2 \frac{5}{70}$$

$$= \frac{9}{4}\frac{5}{70} + \frac{1}{4}\frac{30}{70}$$

$$+ \frac{1}{4}\frac{30}{70} + \frac{9}{4}\frac{5}{70}$$

$$= \frac{150}{280} = \frac{15}{28}$$

Problem	Ans	Reason
33 ans in bk wrong	$E(x) = 72\$$	



Sample space: set of black jack nights

$$x\ (.8) = .6$$
$$x = .6/.8 = 3/4 = .75$$

$$E(x) = 10\ (.05) + 20\ (.20) + 50\ (.15) + 100\ (.60)$$
$$= .50 + 4.00 + 7.50 + 60$$
$$= 72$$

Problem	Ans	Reason

34 121,560

X	P_r	X P_r
120000 (1.1)	.6	120000 (.66)
120000 (1.05)	.3	120000 (.315)
120000 (.98)	.1	120000 (.098)
		120000 (1.013)
		= 121560

35 764,012.91

expected gain to state from each person is

$$(2-10^6)\frac{1}{C(50,5)} + 2\left(1-\frac{1}{C(50,5)}\right)$$

$$= \frac{2C(50,5)-10^6}{C(50,5)}$$

total expected gain to state is

$$500{,}000\ \frac{2C(50,5)-10^6}{C(50,5)} \doteq 769{,}012.91$$