Section 4.2 HW Solutions

Problem	Ans	Reason
1	3.5	

X=1	X=3	X=8	S
.5	.2	.3	

partition of sample space S showing probabilities

$$E(x) = (.5)1 + (.2)3 + (.3)8$$
$$= .5 + .6 + 2.4$$
$$= 3.5$$

2	1.5

X=-1	X=0	X=2	X=5	S
.2	.4	.1	.3	

Partition of sample space S showing probabilities

$$E(x) = (.2)(-1) + (.4)0 + (.1)2 + (.3)5$$
$$= -.2 + 0 + .2 + 1.5$$
$$= 1.5$$

3	13/3

X	Pr(x)	X Pr(x)
1	$\frac{1}{21}$	$\frac{1}{21}$
2	$\frac{2}{21}$	$\frac{4}{21}$
3	$\frac{3}{21}$	$\frac{9}{21}$
4	$\frac{4}{21}$	$\frac{16}{21}$
5	$\frac{5}{21}$	$\frac{25}{21}$
6	$\frac{6}{21}$	$\frac{36}{21}$
		$\frac{91}{21} = E(x)$

$$\frac{91}{21} = \frac{70+21}{21} = \frac{7\cdot 10 + 7\cdot 3}{7\cdot 3} = \frac{13}{3}$$

Problem	Ans	Reason

4 — Ans: 1

X	Pr(x)	x Pr(X)
−1	$\frac{1}{21}$	$\frac{-1}{21}$
2	$\frac{2}{21}$	$\frac{4}{21}$
−3	$\frac{3}{21}$	$\frac{-9}{21}$
4	$\frac{4}{21}$	$\frac{16}{21}$
−5	$\frac{5}{21}$	$\frac{-25}{21}$
6	$\frac{6}{21}$	$\frac{36}{21}$
		1 = E(x)

5 — Ans: −1.1

X	Pr(x)	x Pr(x)
−5	.4	−2.0
−1	.2	−0.2
2	.3	.6
5	.1	.5
		−1.1 = E(x)

6

X	Pr(x)	x Pr(x)
−10	.2	−2
−1	.3	−.3
2	.4	.8
10	.1	1
		−.5 = E(x)

$\frac{-2}{.2} = \frac{-20}{2} = -10$

$(-1)(.3) = -.3$

$1 - .1 - .2 - .3 = .4$

$10(.1) = 1$

$-2 - .3 + .8 + 1 = -.5$

Problem | Ans | Reason

7

X	Pr(x)	x Pr(x)
-1	.3	-.3
-2	.2	-.4
3/2	.2	.3
2	.3	.6
		.2 = E(x)

$\frac{-.3}{-1} = .3$

$\frac{-.4}{-2} = \frac{.4}{2} = .2$

$.3/.2 = 3/2$

8

X	Pr(x)	x Pr(x)
-2	.1	-.2
1	.2	.2
2	.3	.6
3	.1	.3
4	.3	1.2
		2.1 = E(x)

$1 - .2 - .3 - .1 - .3 = .1$

$2.1 - .2 - .6 - .3 - 1.2 = -.2$

$.6/.3 = 6/3 = 2$

$.3/3 = .1$

$1.2/4 = .3$

9 15¢

5

n n n	d d	q q q q
x = 5	x = 10	x = 25

drawer

X	Pr(x)	x Pr(x)
5	$\frac{3}{9}$	$\frac{15}{9}$
10	$\frac{2}{9}$	$\frac{20}{9}$
25	$\frac{4}{9}$	$\frac{100}{9}$
		$\frac{135}{9}$

$\begin{array}{r} 100 \\ 15 \\ 20 \\ \hline 135 \end{array}$

$\frac{135}{9} = 15$

Problem | Ans | Reason

10 | .4

X	Pr(x)	x Pr(x)
1	$1-a-.3$	$1-a-.3$
2	.3	.6
5	a	$5a$
		$2.9 = E(x)$

$2(.3) = .6$

$1 - a - .3 + .6 + 5a = 2.9$

$4a = 1.6$

$a = .4$

11 | $\frac{4}{7}$

b b	g g
b b	g

Box of 7 balls
choose 4

Events

# blue	# green	X	Pr(x)	x Pr(x)
4	0	4	$\frac{1}{C(7,4)}$	$\frac{4}{35}$
3	1	2	$\frac{C(4,3)C(3,1)}{C(7,4)}$	$\frac{24}{35}$
2	2	0	$\frac{C(4,2)C(3,2)}{C(7,4)}$	$\frac{0}{35}$
1	3	-2	$\frac{C(4,1)}{C(7,4)}$	$\frac{-8}{35}$
				$E(x) = \frac{20}{35}$

$C(7,4) = C(7,3) = \frac{7\ 6\ 5}{3\ 2\ 1} = 35$

$\frac{20}{35} = \frac{4.5}{7.5} = \frac{4}{7}$

No. 4.2

Date 5

Problem	Ans	Reason
12	23.5	$20(.3)$ = Expected number of hits against LH pitcher $70(.25)$ = " " RH Total number of expected hits is $20(.3) + 70(.25)$ $= 6 + 17.5 = 23.5$
13	4	$E(x) = 10 \times \frac{2}{5} = 4$
14	3/2	S: $x=-1$: 1, 3.5; $x=2$: 2; $x=4$: 4; $x=6$: 6

x	$Pr(x)$	$x \cdot Pr(x)$
-1	$\frac{1}{2}$	$\frac{-1}{2}$
2	$\frac{1}{6}$	$\frac{2}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$
		$E(x) = \frac{9}{6} = 3/2$

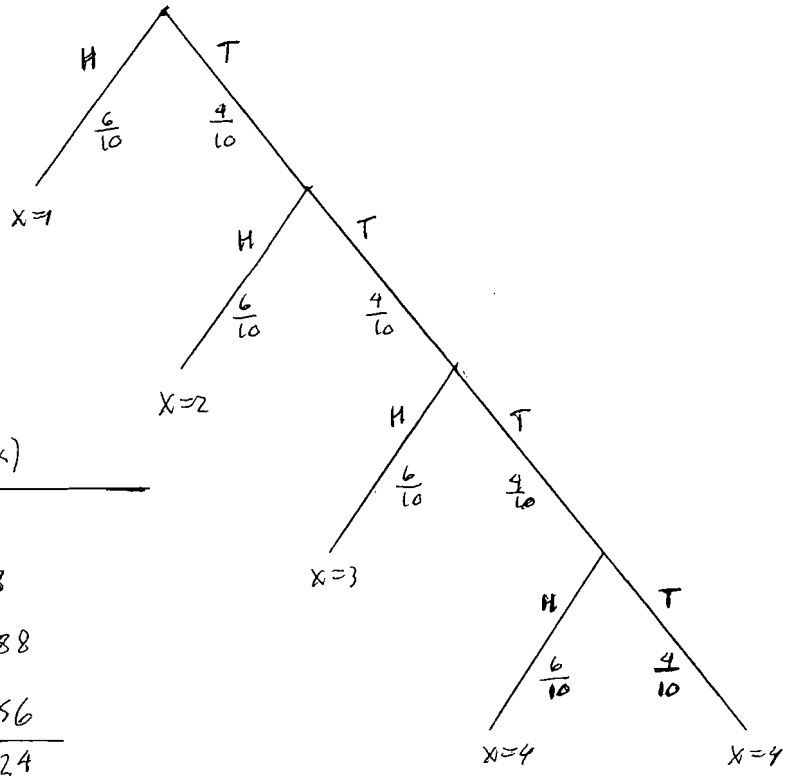
15	7/6	$\frac{1}{2}\left(2 \cdot \frac{1}{2}\right) + \frac{1}{2}\left(2 \cdot \frac{2}{3}\right)$ $= \frac{1}{2} + \frac{2}{3}$ $= \frac{7}{6}$

Problem	Ans	Reason
16	.5 $	red 7, green 3 (Box) — 10 balls; choose 2

Events	#red	#green	X	Pr(x)	x Pr(x)
	2	0	3 − 1.5	$\frac{C(7,2)}{C(10,2)}$	$\frac{7}{10}$
	1	1	0 − 1.5	$\frac{7\cdot 3}{C(10,2)}$	$\frac{-7}{10}$
	0	2	9 − 1.5	$\frac{C(3,2)}{C(10,2)}$	$\frac{1}{2}$

$E(x) = \frac{1}{2}$

17 1.624



X	Pr(x)	x Pr(x)
1	$\frac{6}{10}$	.6
2	$\frac{4}{10}\frac{6}{10}$	.48
3	$\left(\frac{4}{10}\right)^2\frac{6}{10}$	.288
4	$\left(\frac{4}{10}\right)^3$	.256
		1.624

Problem	Ans	Reason
18	42.6	$32(.6) + 78(.3)$ $= 19.2 + 23.4$ $= 42.6$
19	-9.3 \$	

W	Pr(W)	W Pr(W)
10,000	.00002	.2
100,000	.000004	.4
1,000,000	.0000001	.1
0		0
		E(w) = .7

$X = W - 10$

$E(x) = E(w) - 10 = .7 - 10 = -9.3$

20 var $= \frac{55}{64}$

SD $= \frac{\sqrt{55}}{8}$

# heads	X	Pr	X Pr
0	0	$\frac{1}{32}$	0
1	0	$\frac{5}{32}$	0
2	0	$\frac{10}{32}$	0
3	1	$\frac{10}{32}$	$\frac{10}{32}$
4	1	$\frac{5}{32}$	$\frac{5}{32}$
5	5	$\frac{1}{32}$	$\frac{5}{32}$
			$\frac{20}{32} = \frac{5 \cdot 4}{8 \cdot 4} = \frac{5}{8}$

$$\text{Var}(X) = \left(0 - \tfrac{5}{8}\right)^2 \left(\tfrac{1}{32} + \tfrac{5}{32} + \tfrac{10}{32}\right) + \left(1 - \tfrac{5}{8}\right)^2 \left(\tfrac{10}{32} + \tfrac{5}{32}\right) + \left(5 - \tfrac{5}{8}\right)^2 \tfrac{1}{32} = \frac{55}{64}$$

Problem	Ans	Reason

21 $\frac{81}{6} = 13.5$

III	5	TT
II	5	T

T = 10

Joe's stash

choose 3

#1's	#5's	#T's	X	Pr(x)	x Pr(x)
3	0	0	3	$\frac{C(5,3)}{C(10,3)}$	$\frac{30}{120}$
2	1	0	7	$\frac{C(5,2)C(2,1)}{C(10,3)}$	$\frac{140}{120}$
2	0	1	12	$\frac{C(5,2)C(3,1)}{C(10,3)}$	$\frac{360}{120}$
1	2	0	11	$\frac{C(5,1)C(2,2)}{C(10,3)}$	$\frac{55}{120}$
1	1	1	16	$\frac{C(5,1)C(2,1)C(3,1)}{C(10,3)}$	$\frac{480}{120}$
1	0	2	21	$\frac{C(5,1)C(3,2)}{C(10,3)}$	$\frac{315}{120}$
0	2	1	20	$\frac{C(2,2)C(3,1)}{C(10,3)}$	$\frac{60}{120}$
0	1	2	25	$\frac{C(2,1)C(3,2)}{C(10,3)}$	$\frac{150}{120}$
0	0	3	30	$\frac{C(3,3)}{C(10,3)}$	$\frac{30}{120}$

Events

$E(x) = \frac{81}{6}$

$C(10,3) = 120$

22 Carnival gains .14 $

white	red	blue
90	9	1
x = -.3	x = 1 - .6	x = 10 - .6

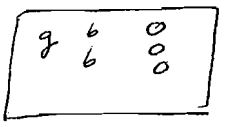
outcome	X	Pr	X Pr
white	-.3	$\frac{90}{100}$	$\frac{-27}{100}$
red	.4	$\frac{9}{100}$	$\frac{3.6}{100}$
blue	9.4	$\frac{1}{100}$	$\frac{9.4}{100}$

$\frac{-14}{100}$

You lose .14 $

Carnival gains .14 $

Problem	Ans	Reason
23	$E(x) = 12$	
	$SD = \sqrt{97.6}$	



Bag 1 6 balls

choose 2

value:

g	b	o
23	5	1

#g	#b	#o	X	Pr	X Pr
0	2	0	10	$\frac{1}{15}$	$\frac{10}{15}$
0	1	1	6	$\frac{6}{15}$	$\frac{36}{15}$
0	0	2	2	$\frac{3}{15}$	$\frac{6}{15}$
1	1	0	28	$\frac{2}{15}$	$\frac{56}{15}$
1	0	1	24	$\frac{3}{15}$	$\frac{72}{15}$

$E(X) = \frac{180}{15} = 12$

$$\text{Var} = (10-12)^2 \frac{1}{15} + (6-12)^2 \frac{6}{15} +$$

$$(2-12)^2 \frac{3}{15} + (28-12)^2 \frac{2}{15} + (24-12)^2 \frac{3}{15}$$

$$= 97.6$$

Problem	Ans	Reason
24	$\frac{105}{56}$	wwww gg www g Cage choose 3 8 mice

#w	X	Pr	X Pr
3	3	$\frac{C(5,3)C(3,0)}{C(8,3)}$	$\frac{30}{56}$
2	2	$\frac{C(5,2)C(3,1)}{C(8,3)}$	$\frac{60}{56}$
1	1	$\frac{C(5,1)C(3,2)}{C(8,3)}$	$\frac{15}{56}$
0	0	$\frac{C(5,0)C(3,3)}{C(8,3)}$	$\frac{0}{56}$

$E(x) = \frac{105}{56}$

25 $7/5 = 1.4$

$2\left(\frac{1}{2}\right) + 2\left(\frac{1}{5}\right) = \frac{7}{5}$

26 Expected # success = 100

$SD = \sqrt{99}$

$np = 1000\ (.1) = 100$

$npq = 1000\ (.1)\ (.99)$

$= 99$

Problem	Ans	Reason

27

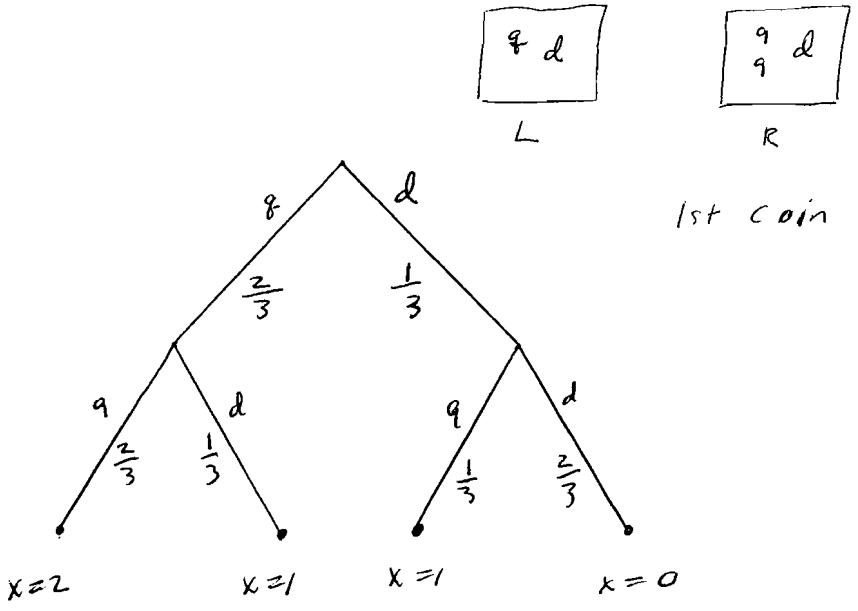
# def	X	Pr
0	100	$C(10,0)(.9)^{10}(.1)^0$
1	80	$C(10,1)(.9)^9(.1)^1$
2	80	$C(10,2)(.9)^8(.1)^2$
3	50	$C(10,3)(.9)^7(.1)^3$
4	50	$C(10,4)(.9)^6(.1)^4$
5	0	$C(10,5)(.9)^5(.1)^5$
6	0	$C(10,6)(.9)^4(.1)^6$
7	0	$C(10,7)(.9)^3(.1)^7$
8	0	$C(10,8)(.9)^2(.1)^8$
9	0	$C(10,9)(.9)^1(.1)^9$
10	0	$C(10,10)(.9)^0(.1)^{10}$

$p = .9 \quad q = .1$

$$E(X) = 100(.9)^{10} + 80 \cdot C(10,1)(.9)^9(.1)^1 + 80\,C(10,2)(.9)^8(.1)^2 + 50\,C(10,3)(.9)^7(.1)^3 + 50\,C(10,4)(.9)^6(.1)^4$$

Problem	Ans	Reason
28	1.6	$4(.4) = 1.6$
29	-2	$E(3 \text{ times } \#\text{success} - \#\text{failure})$ $= 3\,E(\#\text{success}) - E(\#\text{failure})$ $= 3 \quad 10(.2) \quad - \quad 10(.8)$ $= \quad 6 - 8$ $= -2$
30	$E(x) = 4$ $Var(x) = 8/3$	1 2 3 4 5 6 L L L L H H success: get H $\quad p = 1/3$ $\quad q = 2/3$ $E(x) = 12 \cdot \frac{1}{3} = 4$ $Var(x) = 12\,pq$ $= 12 \quad \frac{1}{3} \quad \frac{2}{3} = 8/3$

Problem	Ans	Reason
31	$\frac{11}{9}$	

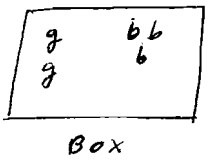


$$E(x) = 2\left(\frac{2}{3}\,\frac{2}{3}\right) + 1\left(\frac{2}{3}\,\frac{1}{3} + \frac{1}{3}\,\frac{1}{3}\right) + 0\left(\frac{1}{3}\,\frac{2}{3}\right)$$

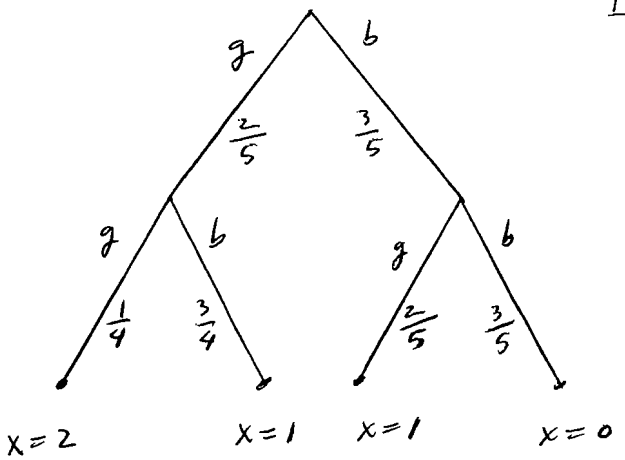
$$= \frac{8 + 2 + 1 + 0}{9} = \frac{11}{9}$$

Problem	Ans	Reason

32



5 balls



X	Pr	X Pr
0	$\frac{3}{5}\frac{3}{5} = \frac{9}{25}$	0
1	$\frac{2}{5}\frac{3}{4} + \frac{3}{5}\frac{2}{5} = \frac{27}{50}$	$\frac{27}{50}$
2	$\frac{2}{5}\frac{1}{4} = \frac{1}{10}$	$\frac{2}{10}$
		$E(X) = \frac{37}{50}$

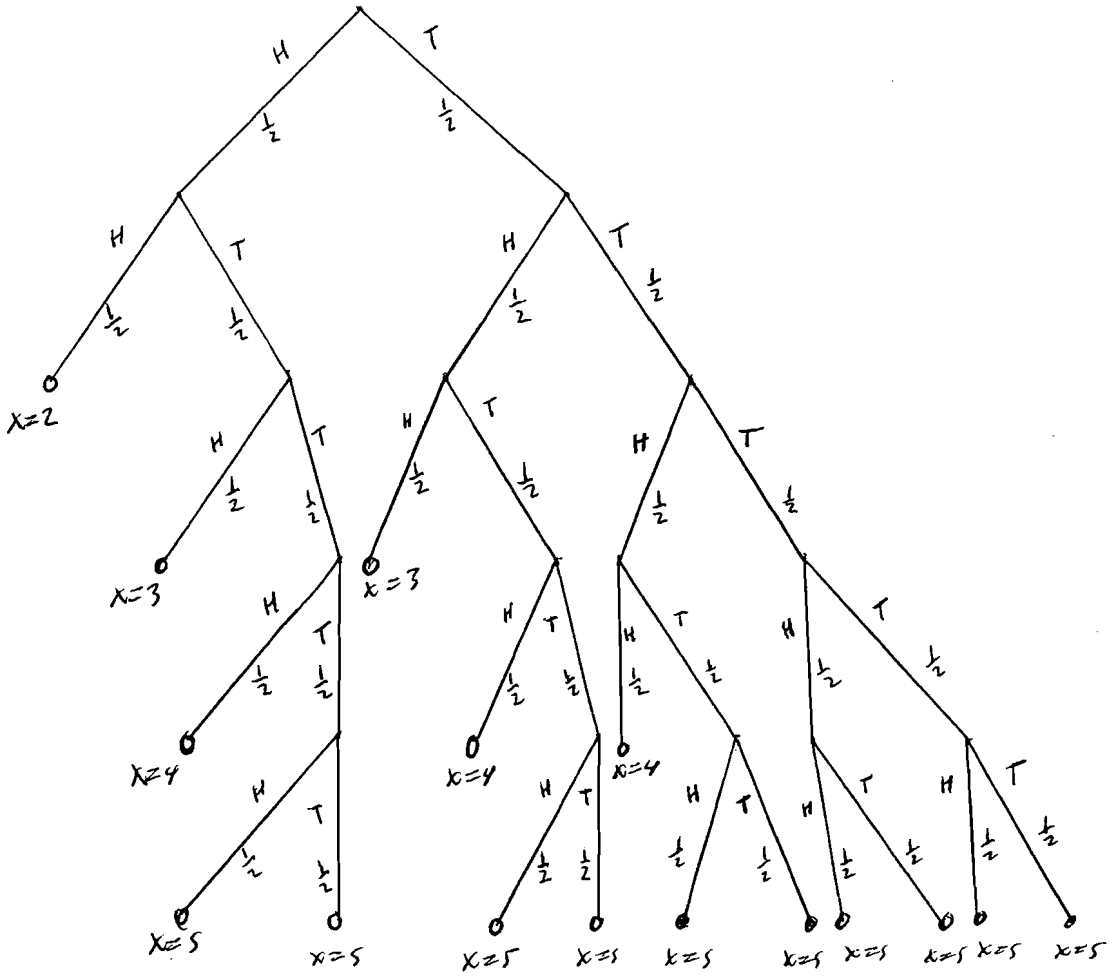
$$Var(x) = \left(0 - \frac{37}{50}\right)^2 \frac{9}{25} + \left(1 - \frac{37}{50}\right)^2 \frac{27}{50} + \left(2 - \frac{37}{50}\right)^2 \frac{2}{10}$$

$$SD = \sqrt{Var(x)}$$

No. 4.2

Date 15

Problem	Ans	Reason
33	$E(X) = \frac{57}{16}$ $Var(X) = \frac{351}{256}$ $SD = \frac{\sqrt{351}}{16}$	



X	Pr	X Pr
2	$\frac{1}{4}$	$\frac{2}{4}$
3	$\frac{1}{4}$	$\frac{3}{4}$
4	$\frac{3}{16}$	$\frac{12}{16}$
5	$\frac{10}{32}$	$\frac{50}{32}$
		$E(X) = \frac{57}{16}$

$$Var(X) = \left(2 - \frac{57}{16}\right)^2 \frac{1}{4} + \left(3 - \frac{57}{16}\right)^2 \frac{1}{4} + \left(4 - \frac{57}{16}\right)^2 \frac{3}{16} + \left(5 - \frac{57}{16}\right)^2 \frac{10}{32}$$

$$= \frac{351}{256} \qquad SD = \frac{\sqrt{351}}{16}$$

Problem	Ans	Reason
34	Var = 37/4 SD = $\frac{\sqrt{37}}{2}$	$Var(x) = (1-3.5)^2(.5) + (3-3.5)^2(.2) + (8-3.5)^2(.3)$ $= (2.5)^2(.5) + (.5)^2(.2) + (4.5)^2(.3)$ $= \frac{(25)^2 5 + 5^2 \times 2 + (45)^2 \times 3}{1000}$ $= \frac{5^2(125 + 2 + 243)}{1000}$ $= \frac{25 \cdot 370}{1000} = \frac{25 \cdot 37}{100} = \frac{37}{4}$
35	Var = 12.69 SD = $\sqrt{12.69}$	$Var(x) = (-5+1.1)^2(.4) + (-1+1.1)^2(.2) + (2+1.1)^2(.3) + (5+1.1)^2(.1)$ $= (3.9)^2(.4) + (.1)^2(.2) + (3.1)^2(.3) + (6.1)^2(.1)$ $= \frac{6084 + 2 + 2883 + 3721}{1000}$ $= \frac{12690}{1000} = 12.69$