Section 4.1 HW Solutions

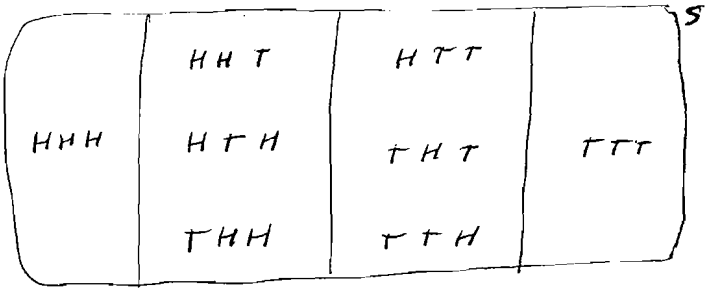
Problem

1

val X	Pr
3	$(.4)^3$
4	$3(.4)^2(.6)$
5	$3(.4)(.6)^2$
6	$(.6)^3$

Sample space S is Cartesian product

$S = \{H, T\} \times \{H, T\} \times \{H, T\}$



#H	3	2	1	0
#T	0	1	2	3
X	3	4	5	6
Pr	$(.4)^3$	$3(.4)^2(.6)$	$3(.4)(.6)^2$	$(.6)^3$

2

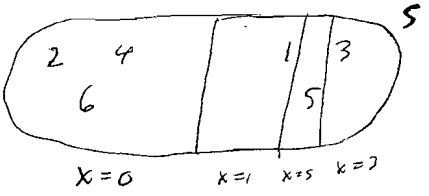
X	Pr
9	$\frac{1}{8}$
4	$\frac{3}{8}$
-1	$\frac{3}{8}$
-6	$\frac{1}{8}$

#H	3	2	1	0
#T	0	1	2	3
X	9	4	-1	-6
Pr	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

3

X	Pr
0	$\frac{3}{6}$
1	$\frac{1}{6}$
3	$\frac{1}{6}$
5	$\frac{1}{6}$

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

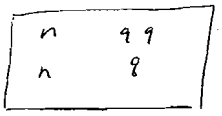


Problem | Ans | Reason

4

X	Pr
10	$\frac{1}{10}$
30	$\frac{6}{10}$
50	$\frac{3}{10}$

$Pr[X=30] = \frac{6}{10}$

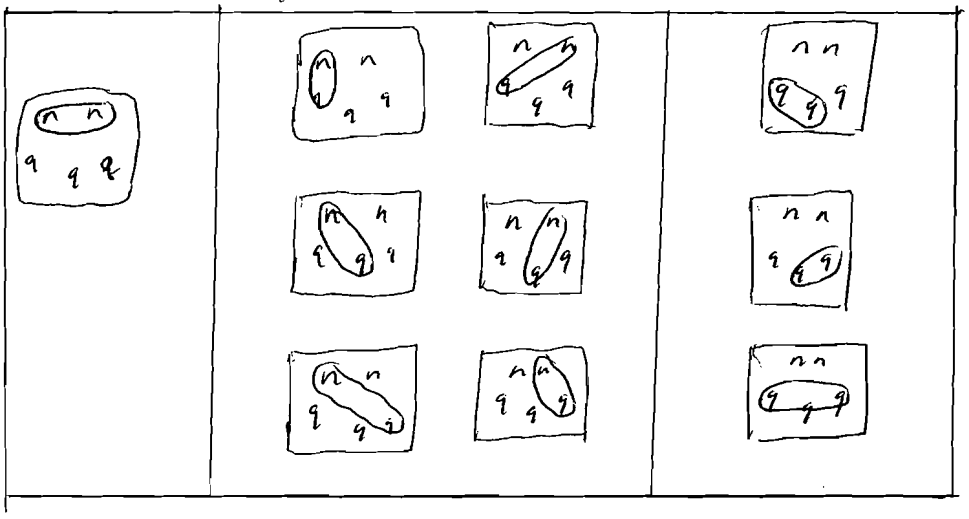


Box

Sample space S: 2-element subsets

$n(S) = C(5,2) = 10$

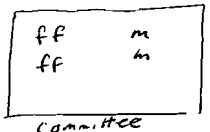
S:



X	10	30	50
sized event	1	6	3
Pr	$\frac{1}{10}$	$\frac{6}{10}$	$\frac{3}{10}$

5

X	Pr
0	$\frac{1}{15}$
1	$\frac{8}{15}$
2	$\frac{6}{15}$



committee

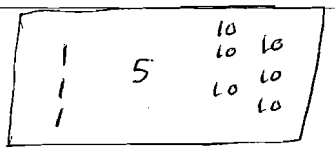
S: 2-element subsets $n(S) = C(6,2) = 15$

X	0	1	2
sized event	1	$4 \cdot 2 = 8$	$C(4,2) = 6$
Pr	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{6}{15}$

Problem | Ans | Reason

6

X	Pr
2	$\frac{3}{45}$
6	$\frac{3}{45}$
11	$\frac{18}{45}$
15	$\frac{6}{45}$
20	$\frac{15}{45}$

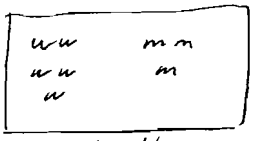


choose 2

Event E	n(E)	Pr(E)	X
1,1	3	$\frac{3}{45}$	2
1,5	3	$\frac{3}{45}$	6
1,10	18	$\frac{18}{45}$	11
5,5	0	0	10
5,10	6	$\frac{6}{45}$	15
10,10	15	$\frac{15}{45}$	20

7

$\frac{30}{56} = \frac{15}{28}$



8 people

standby

Choose 3

S: 3-el subsets of set of 8 people

$n(S) = C(8,3) = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

Event E: 2 women

$n(E) = C(5,2) \cdot C(3,1) = 10 \cdot 3 = 30$

$Pr(E) = \frac{30}{56}$

Problem | Ans | Reason

8

X	Pr
0	.1296
1	.1536
4	.7168

$S = \{H,T\} \times \{H,T\} \times \{H,T\} \times \{H,T\}$

Event E	X	n(E)	Pr(E)
4 Heads	4	1	$1(.6)^4(.4)^0$
3 Heads	3	4	$4(.6)^3(.4)^1$
2 Heads	2	6	$6(.6)^2(.4)^2$
1 Heads	1	4	$4(.6)^1(.4)^3$
0 Heads	0	1	$1(.6)^0(.4)^4$

$(.6)^4 = .1296$
$4(.6)(.4)^3 = .1536$

9

X	Pr
0	.008
1	.096
2	.384
3	.512

$S = \{I,N\} \times \{I,N\} \times \{I,N\}$ I = item

$p(I) = .8$ $p(N) = .2$

Event	X	n(E)	Pr(E)
3 I's	3	1	$(.8)^3(.2)^0\ 1$
2 I's	2	3	$(.8)^2(.2)^1\ 3$
1 I	1	3	$(.8)^1(.2)^2\ 3$
0 I's	0	1	$(.8)^0(.2)^3\ 1$

$(.2)^3 = .008$ $3(.8)(.2)^2 = .096$
$3(.8)^2(.2) = .384$ $(.8)^3 = .512$

Problem	Ans	Reason

10 — Ans: $1/9$

X: (green dice across, red dice down)

red \ green	1	2	3	4	5	6
1	1	2	3	4	(5)	6
2	3	4	(5)	6	7	8
3	4	(5)	6	7	8	9
4	(5)	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Sample space $S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

$n(S) = 6^2 = 36$ Event E: $X = 5$

$n(E) = 4$ $Pr(E) = 4/36 = 1/9$

11 — Ans:

X	Pr
4	$1/3$
6	$1/3$
8	$1/3$

[1] [3] [3] [5]

choose 2

Sample space S: 2-el subsets of set of 4 cards

$n(S) = 6$

	Event E	X	n(E)	Pr(E)
	1,1	2	0	0
→	1,3	4	2	$2/6$
→	1,5	6	1	$1/6$
→	3,3	6	1	$1/6$
→	3,5	8	2	$2/6$
	5,5	10	0	0

Problem | Ans | Reason

12

X	Pr
0	$C(48,2)/C(52,2)$
1	$4\cdot 48/C(52,2)$
2	$6/C(52,2)$

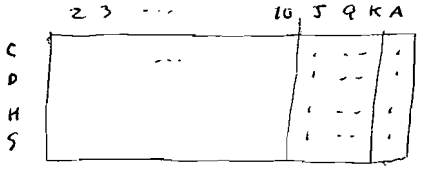
52-card deck

Sample space S: 2-element subsets

Event E	X	$n(E)$	$Pr(E)$
0 aces	0	$C(48,2)$	$C(48,2)/C(52,2)$
1 ace	1	$4\cdot 48$	$4\cdot 48/C(52,2)$
2 aces	2	$C(4,2)$	$6/C(52,2)$

13

X	Pr
0	$C(36,3)/C(52,3)$
2	$C(12,1)\ C(36,2)/C(52,3)$
4	$C(12,2)\ C(36,1)/C(52,3)$
5	$C(4,1)\ C(36,2)/$
6	$C(12,3)/C(52,3)$
7	$C(4,1)C(12,1)C(36,1)/C(52,3)$
9	$C(4,1)\ C(12,2)/C(52,3)$
10	$C(4,2)C(36,1)/C(52,3)$
12	$C(4,2)C(12,1)/C(52,3)$
15	$C(4,3)/C(52,3)$



Choose 3

$n(S) = C(52,3)$

face cards besides Ace

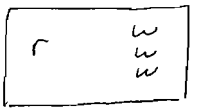
Events	0	1	2	3
0	$z=0$ $C(36,3)$	$z=2$ $12\,C(36,2)$	$z=4$ $C(12,2)C(36,1)$	$z=6$ $C(12,3)$
1	$z=5$ $4\cdot C(36,2)$	$z=7$ $4\cdot 12\cdot C(36,1)$	$z=9$ $4\,C(12,2)$	\
2	$z=10$ $6\,C(36,1)$	$z=12$ $6\cdot 12$	\	\
3	$z=19$ 4	\	\	\

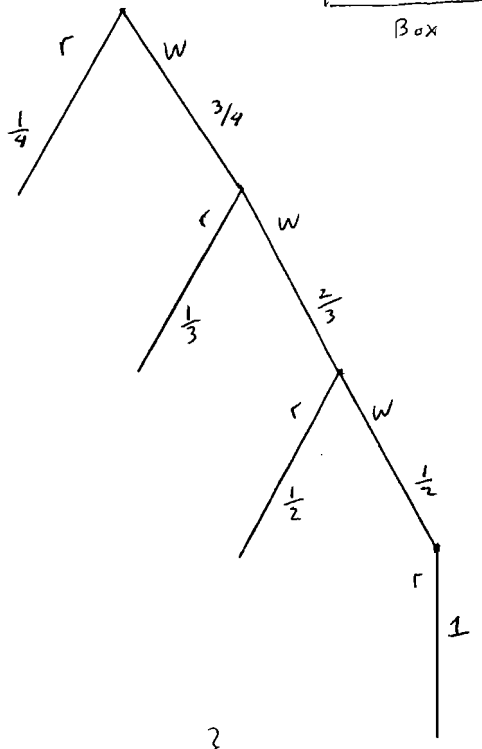
aces

Problem	Ans	Reason

14

X	Pr
1	$\frac{1}{4}$
2	$\frac{1}{4}$
3	$\frac{1}{4}$
4	$\frac{1}{4}$





$S = \{ r, \ wr, \ wwr, \ wwwr \}$

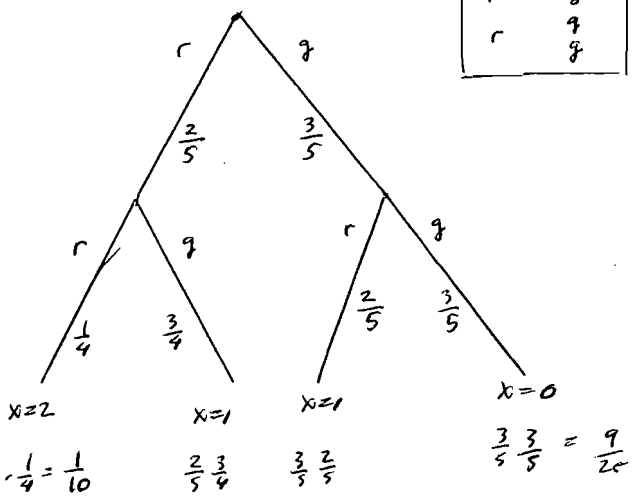
Event	r	wr	wwr	wwwr
X	1	2	3	4
Pr	$\frac{1}{4}$	$\frac{3}{4}\frac{1}{3}$	$\frac{3}{4}\frac{2}{3}\frac{1}{2}$	$\frac{3}{4}\frac{2}{3}\frac{1}{2}1$

Problem	Ans	Reason

15

X	Pr
0	$9/25$
1	$27/50$
2	$1/10$

r	g
r	g



Pr $\frac{2}{5}\cdot\frac{1}{4}=\frac{1}{10}$ $\frac{2}{5}\frac{3}{4}$ $\frac{3}{5}\frac{2}{5}$ $\frac{3}{5}\frac{3}{5}=\frac{9}{25}$

16

X	Pr
0	$C(9,5)/C(12,5)$
1	$3C(9,4)/C(12,5)$
2	$3C(9,3)/C(12,5)$
3	$C(9,2)/C(12,5)$

work	dont work
9	3

Box

Choose 5

12 calculators

$n(S)=C(12,5)$

Event E	X	n(E)
X=0	0	$C(9,5)$
X=1	1	$C(3,1)C(9,4)$
X=2	2	$C(3,2)C(9,3)$
X=3	3	$C(3,3)C(9,2)$

17

X	Pr
0	$1/10$
1	$6/10$
2	$3/10$

sss	nn

choose 3

$n(S)=C(5,3)=10$

Event E	X	n(E)
X=0	0	1
X=1	1	$2\cdot3=6$
X=2	2	$1\cdot3=3$

Problem | Ans | Reason

18

X	Pr
3	$C(10,3)/C(20,3)$
4	$C(10,2)C(10,1)/C(20,3)$
5	$C(10,1)C(10,2)/C(20,3)$
6	$C(10,3)/C(20,3)$

even balls	odd balls
2, 4, 6 ··· 20	1, 3, 5 ··· 19
(10)	(10)

choose 3

$n(S) = C(20,3)$

Event E	X	n(E)
# odd balls = 0	3	$C(10,3)$
# ··· 1	2+2=4	$C(10,1)C(10,2)$
# ·· 2	1+4=5	$C(10,2)C(10,1)$
# ·· 3	0+6=6	$C(10,3)$

19

X	
0	$C(48,5)/C(52,5)$
1	$4C(48,4)/C(52,5)$
2	$6C(48,3)/C(52,5)$
3	$4C(48,2)/C(52,5)$
4	$C(48,1)/C(52,5)$

2 3 ·· Q K A

C
D
H
S

choose 5

$n(S) = C(52,5)$

Event E	X	n(E)
X = 0	0	$C(48,5)$
X = 1	1	$C(4,1)C(48,4)$
X = 2	2	$C(4,2)C(48,3)$
X = 3	3	$C(4,3)C(48,2)$
X = 4	4	$C(4,4)C(48,1)$

20

X	Pr
0	1/56
1	15/56
2	30/56
3	10/56

Slow	Acc	Fast
1	5	2

8 stopwatches

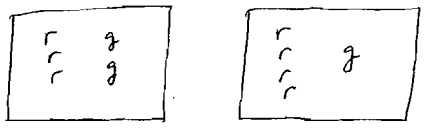
choose 3

$n(S) = C(8,3) = 56$

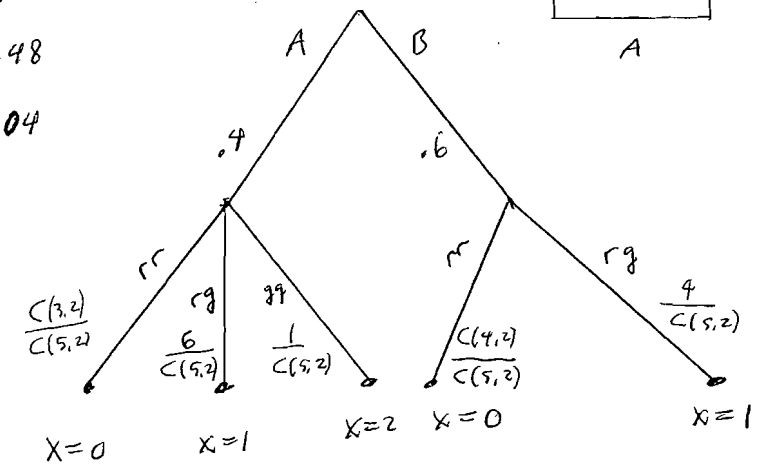
Event E	X	n(E)
X = 0	0	1
X = 1	1	$5 \quad C(3,2) = 15$
X = 2	2	$C(5,2)C(3,1) = 30$
X = 3	3	$C(5,3) = 10$

Problem	Ans	Reason
21		

X	Pr
0	.48
1	.48
2	.04



A B



$$\frac{(.4)\,C(3,2)}{C(5,2)} = \frac{4}{10}\,\frac{3}{10} = \frac{12}{100} = .12$$

$$.4\,\frac{6}{C(5,2)} = \frac{4}{10}\cdot\frac{6}{10} = \frac{24}{100} = .24$$

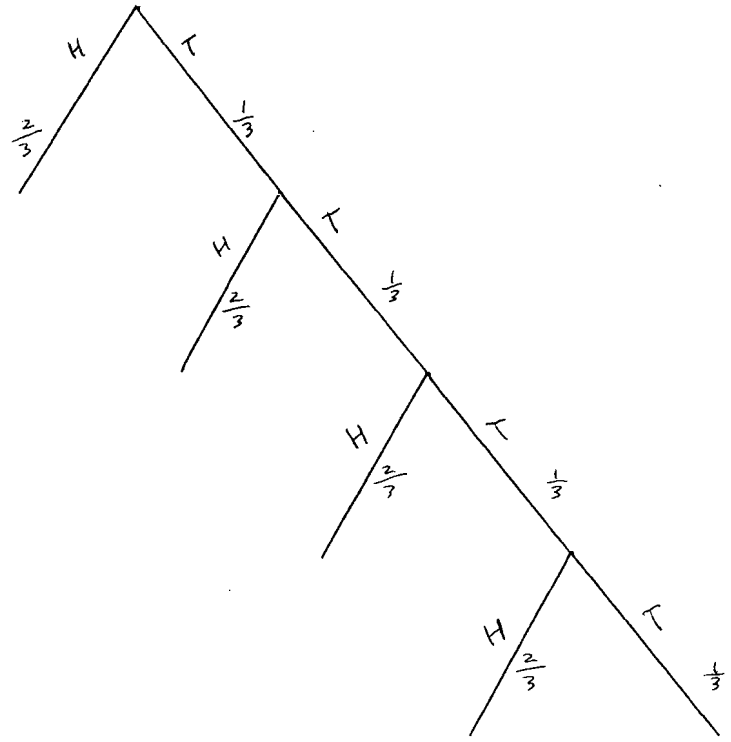
$$\frac{.4}{C(5,2)} = \frac{4}{10}\,\frac{1}{10} = \frac{4}{100} = .04$$

$$\frac{(.6)\,C(4,2)}{C(5,2)} = \frac{6}{10}\,\frac{6}{10} = \frac{36}{100} = .36 \qquad .12 + .36 = .48$$

$$(.6)\,\frac{4}{C(5,2)} = \frac{6}{10}\,\frac{4}{10} = \frac{24}{100} = .24 \qquad .24 + .24 = .48$$

Problem	Ans	Reason
22		

X	Pr
1	$\frac{2}{3}$
2	$\frac{2}{9}$
3	$\frac{2}{27}$
4	$\frac{1}{27}$



$S = \{ H, TH, TTH, TTTH, TTTT \}$

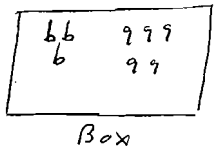
$X=1 \quad X=2 \quad X=3 \quad X=4 \quad X=4$

Pr: $\frac{2}{3} \quad \frac{1}{3}\frac{2}{3} \quad \frac{1}{3}\frac{1}{3}\frac{2}{3} \quad \frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{2}{3} \quad \frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}$

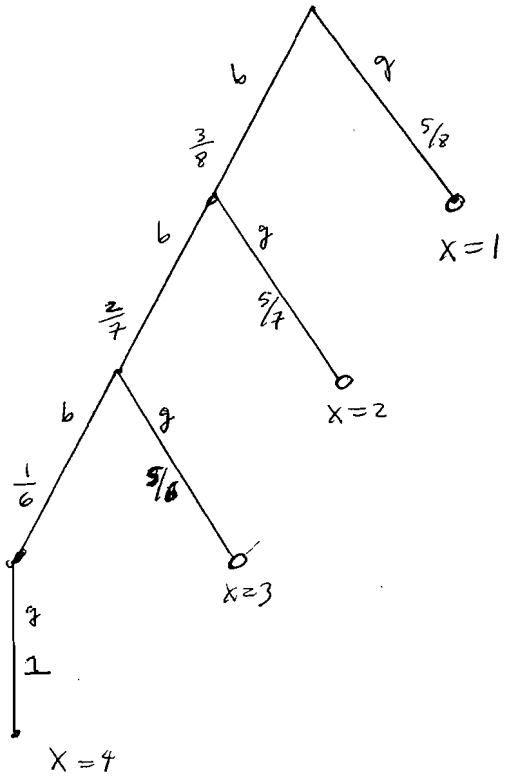
Problem	Ans	Reason

23

X	Pr
1	5/8
2	15/56
3	5/56
4	1/56



8 balls



Outcome	g	bg	bbg	bbbg
X	1	2	3	4
Pr	5/8	3/8 5/7	3/8 2/7 5/6	3/8 2/7 1/6

24

2/15

2 3 4 5 6 7
choose 2

S: 2-el subsets

$n(S) = C(6,2) = 15$

Event E, x = 8

$E = \{62, 53,\}$ $n(E) = 2$

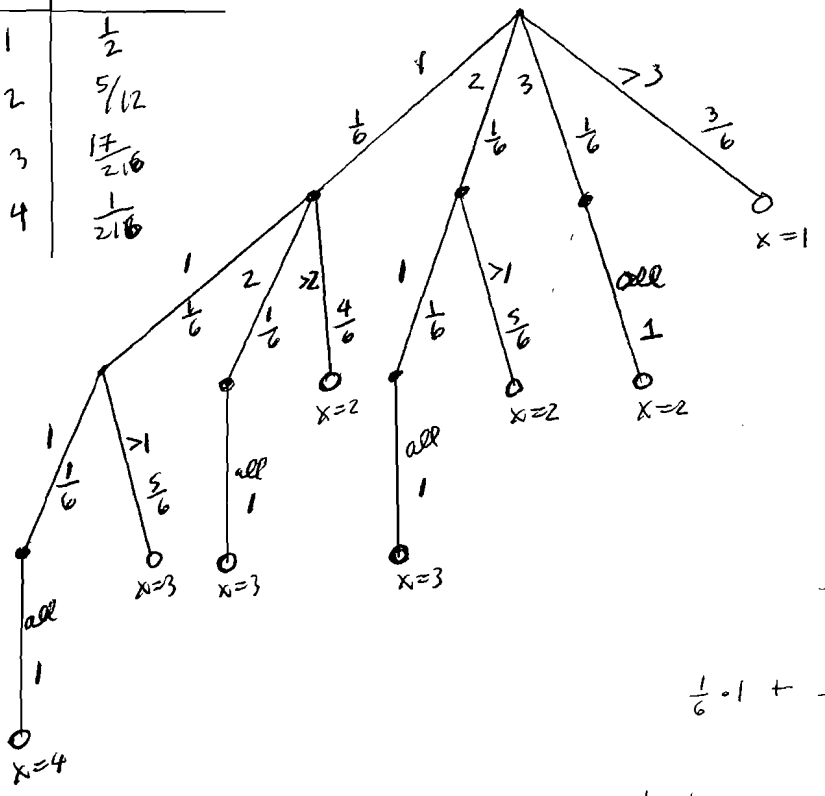
$Pr(E) = 2/15$

No. 4.1
Date 13

Problem	Ans	Reason

25

X	Pr
1	$\frac{1}{2}$
2	$\frac{5}{12}$
3	$\frac{17}{216}$
4	$\frac{1}{216}$



$\frac{3}{6} = \frac{1}{2}$

$\frac{1}{6} \cdot 1 + \frac{1}{6}\frac{5}{6} + \frac{1}{6}\frac{4}{6} = \frac{5}{12}$

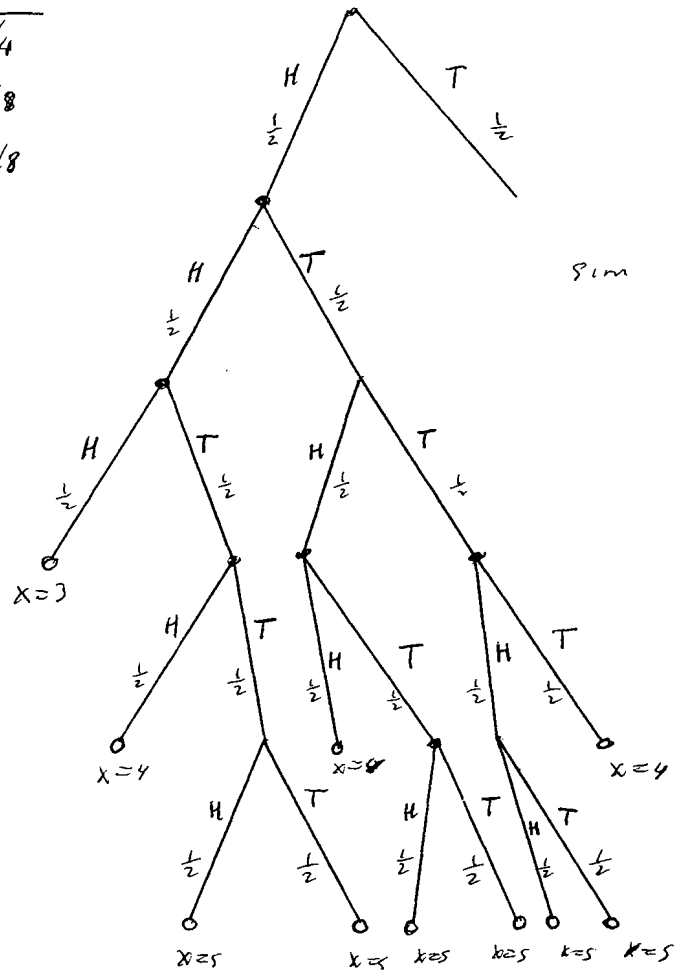
$\frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6}\frac{5}{6} =$

No. 4.1

Date 14

Problem	Ans	Reason
26		

X	Pr
3	1/4
4	3/8
5	3/8



sim

$\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

$\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{6}{16} = \frac{3}{8}$

$\frac{6 \cdot 2}{32} = \frac{12}{32} = \frac{6}{16} = \frac{3}{8}$

Problem	Ans	Reason

27

X	Pr
0	$1\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^5$
1	$5\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^4$
2	$10\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3$
3	$10\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2$
4	$5\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^1$
5	$1\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^0$

all wheel Dr	No allow drive
4	16

Motor Pool

20 Cars

A: all wheel drive

N: Not all wheel dr

$S = \{A,N\} \times \{A,N\} \times \{A,N\} \times \{A,N\} \times \{AN\}$

$Pr(A) = \frac{1}{5} \quad Pr(N) = 4/5$

$$\frac{4^5}{5^5} = \frac{4^5}{10^5} = \frac{1024 \cdot 32}{10^5}$$

$\approx .32768$

1024
32
2048
3072
32768

28

X	Pr
0	$(.9)^{10} + 10(.1)(.9)^9$
1000	$C(10,2)(.1)^2(.9)^8$
5000	

Event E_i : i sales $\quad Pr(E_i) = (.1)^i C(10,i)(.9)^{10-i}$

Event	E_0	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}
X	0	0	1000	5000	→						

$$C(10,2)(.1)^2(.9)^8 = \frac{10 \cdot 9}{2} \frac{1}{10} \frac{1}{10} \left(\frac{9}{10}\right)^8$$

$$= \left(\frac{9}{10}\right)^9 \frac{1}{2}$$

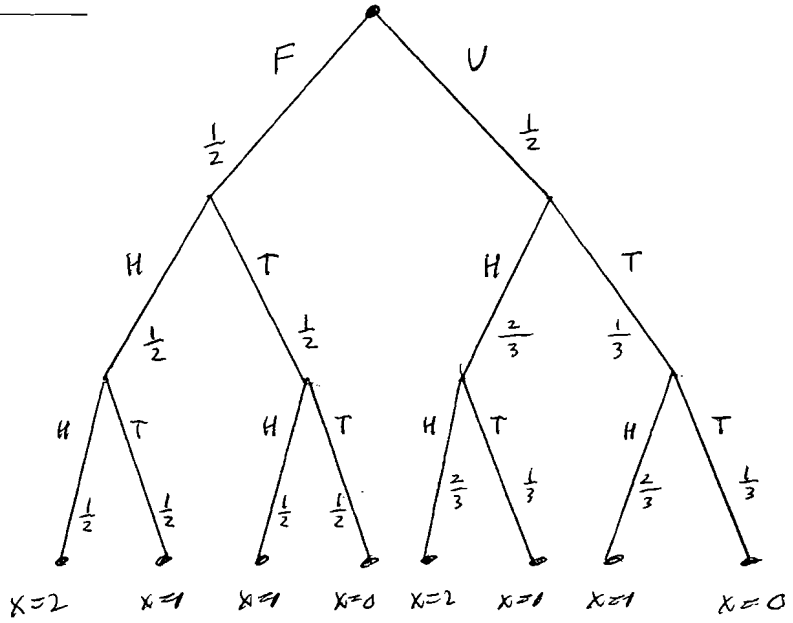
$$1 - (.9)^{10} - 10(.1)(.9)^9 - 45(.1)^2(.9)^8$$

No. 9.1

Date 16

Problem	Ans	Reason
29		

X	Pr
0	$13/72$
1	$\frac{17}{36}$
2	$\frac{25}{72}$



$$\frac{1}{2}\frac{1}{3}\frac{1}{3} + \frac{1}{2}\frac{1}{2}\frac{1}{2} = \frac{1}{2}\left(\frac{1}{9}+\frac{1}{4}\right) = \frac{1}{2}\ \frac{13}{36} = \frac{13}{72}$$

$$\frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{2}{3}\frac{1}{3} + \frac{1}{2}\frac{1}{3}\frac{2}{3} = \frac{1}{4} + \frac{2}{9} = \frac{17}{36}$$

$$\frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{2}{3}\frac{2}{3} = \frac{1}{8} + \frac{2}{9} = \frac{25}{72}$$

No. 4.1

Date 17

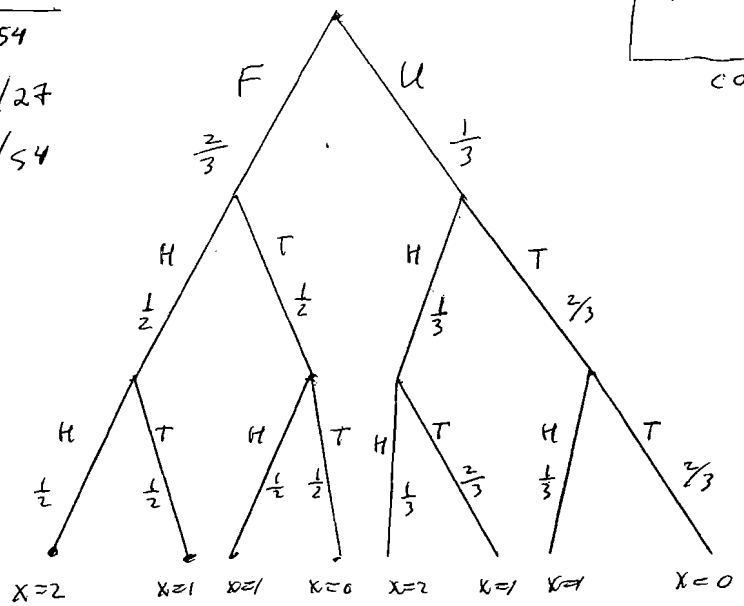
Problem	Ans	Reason
30		

X	Pr
0	17/54
1	13/27
2	11/54

FF U

coins

U: $Pr(H) = \frac{1}{3}$



$$\frac{1}{3}\frac{2}{3}\frac{2}{3} + \frac{2}{3}\frac{1}{2}\frac{1}{2} = \frac{1}{3}\left(\frac{4}{9} + \frac{1}{2}\right) = \frac{1}{3}\frac{17}{18} = \frac{17}{54}$$

$$\frac{2}{3}\frac{1}{2}\frac{1}{2} + \frac{2}{3}\frac{1}{2}\frac{1}{2} + \frac{1}{3}\frac{1}{3}\frac{2}{3} + \frac{1}{3}\frac{2}{3}\frac{1}{3} = \frac{1}{3} + \frac{1}{3}\frac{4}{9}$$

$$= \frac{1}{3}\left(1 + \frac{4}{9}\right) = \frac{1}{3}\frac{13}{9}$$

$$\frac{2}{3}\frac{1}{2}\frac{1}{2} + \frac{1}{3}\frac{1}{3}\frac{1}{3} = \frac{1}{6} + \frac{1}{27} = \frac{1}{3}\left(\frac{1}{2} + \frac{1}{9}\right)$$

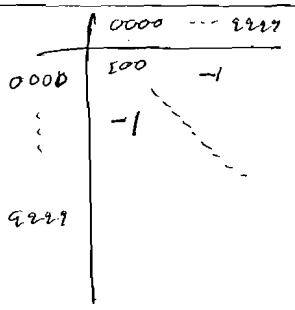
$$= \frac{1}{3}\frac{11}{18} = \frac{11}{54}$$

17
26
11
54

Problem	Ans	Reason

31

X	
500	$\frac{1}{10^4}$
-1	$1-\frac{1}{10^4}$



$S = \{0000, \cdots, 9999\} \times \{0000, \cdots, 9999\}$

$n(S) = 10^8$

Event E: same

$n(E) = 10^4$

$n(E') = 10^8 - 10^4$

$Pr(E) = 10^4/10^8 = \frac{1}{10^4}$

$Pr(E') = 1 - \frac{1}{10^4}$

32

X	Pr
999,999	$\left(\frac{1}{45}\right)^6$
-1	$1-\left(\frac{1}{45}\right)^6$

33

skip