Section 3.R HW Solutions

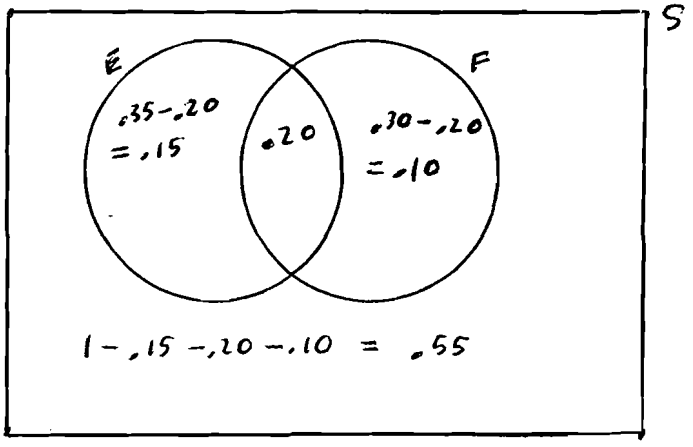
No. 3.R
Date 1

Problem	Ans
1	(a) .54
	(b) 1
	(c) .23
	(d) .27

Reason

Outcome	θ_1	θ_2	θ_3	θ_4	θ_5	
wt	.09	.23	.41	.14	.13	
E_1	•	•		•		
E_2			•	•	•	
E_3		•	•			
E_1'			•		•	.41+.13=.54
$E_1 \cup E_2$	•	•	•	•	•	
$E_1 \cap E_3$		•				
E_3'	•			•	•	
$E_2 \cap E_3'$				•	•	.14+.13=.27

2	(a) .65
	(b) .45
	(c) .15



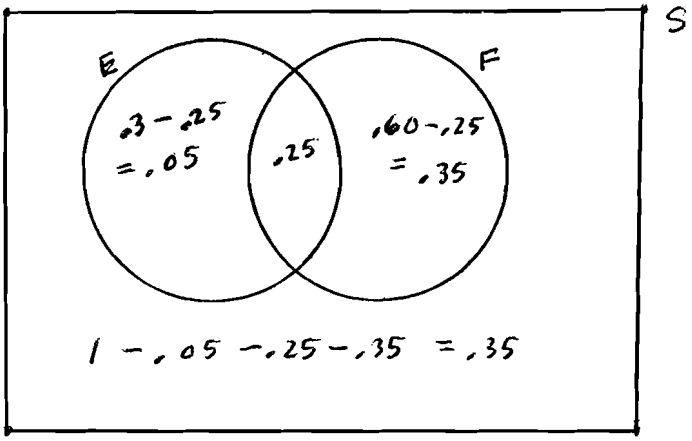
$Pr(E') = .10 + .55 = .65$

$Pr(E \cup F) = .15 + .20 + .10 = .45$

$Pr(E \cap F') = .15$

Problem

3 (a) .7
(b) .65
(c) .05
(d) .35



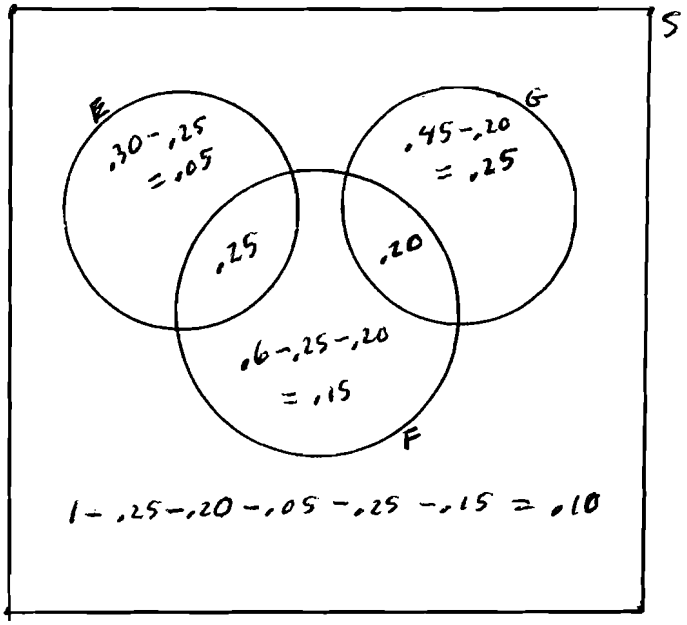
$Pr(E') = .35 + .35 = .70$

$Pr(E \cup F) = .05 + .25 + .35 = .65$

$Pr(E \cap F') = .05$

$Pr(E' \cap F') = .35$

4 (a) .75
(b) .55
(c) .30
(d) .25



$Pr(E \cup G) = .3 + .45 = .75$

$Pr(E \cup G') = Pr(G') = .55$

$Pr(E \cap G') = Pr(E) = .30$

$Pr(G \cap F') = .25$

Problem	Ans	Reason
5	$\frac{1}{3}$	Abbreviate $w = Pr[\theta_2]$

Outcome	θ_1	θ_2	θ_3
wt	$2w$	w	$3w$

$1 = 2w + w + 3w = 6w$

$w = 1/6$

Outcome	θ_1	θ_2	θ_3
wt	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$

6 $Pr(E_1) = 2/9$

$Pr(E_2) = 1/9$

$Pr(E_3) = 2/3$

Abbreviate $w = Pr[E_2]$

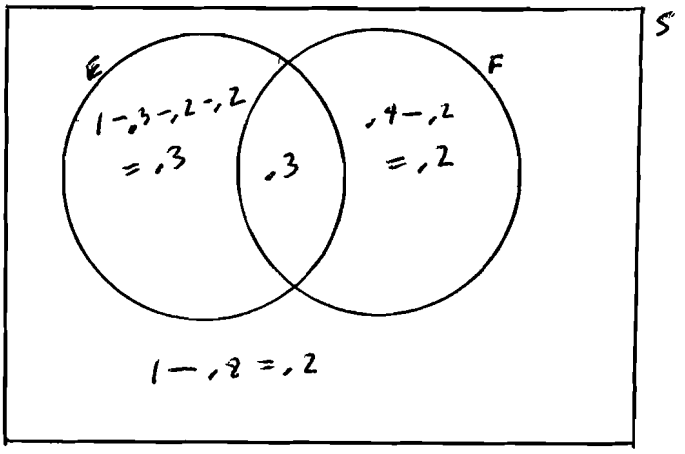
Event	E_1	E_2	E_3
wt	$2w$	w	$6w$

$1 = 2w + w + 6w = 9w$

$w = 1/9$

Event	E_1	E_2	E_3
wt	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{6}{9} = \frac{2}{3}$

Problem	Ans	Reason
7	(a) .6 (b) .5 (c) .6	

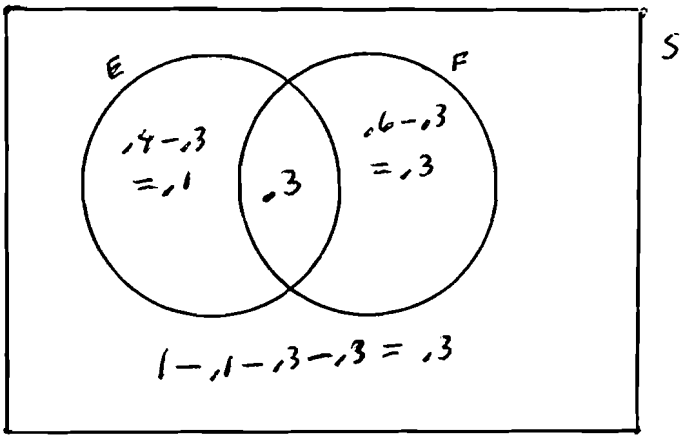


$Pr(E) = 1 - Pr(E') = 1 - .4 = .6$

$Pr(F) = .3 + .2 = .5$

$Pr(E|F) = Pr(E \cap F) / Pr(F) = .3/.5 = 3/5 = .6$

8 $\frac{1}{4}$



$Pr(F) = 1 - Pr(F') = 1 - .4 = .6$

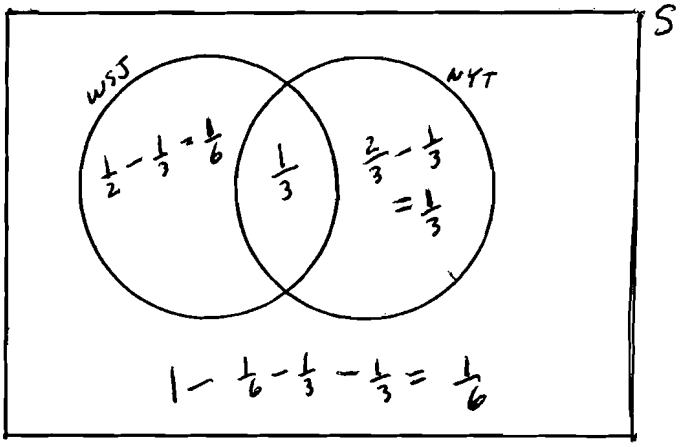
$.5 = Pr(E|F) = Pr(E \cap F) / Pr(F) = \frac{Pr(E \cap F)}{.6}$

$Pr(E \cap F) = (.5)\cdot(.6) = .3$

$Pr(E|F') = Pr(E \cap F') / Pr(F')$

$= \frac{.1}{.4} = \frac{1}{4}$

Problem	Ans	Reason
9	1/2	



$Pr(WSJ) = \frac{1}{2}$ $\quad Pr(NYT) = \frac{2}{3}$

$Pr(WSJ \cap NYT) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

$\frac{1}{6} + \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

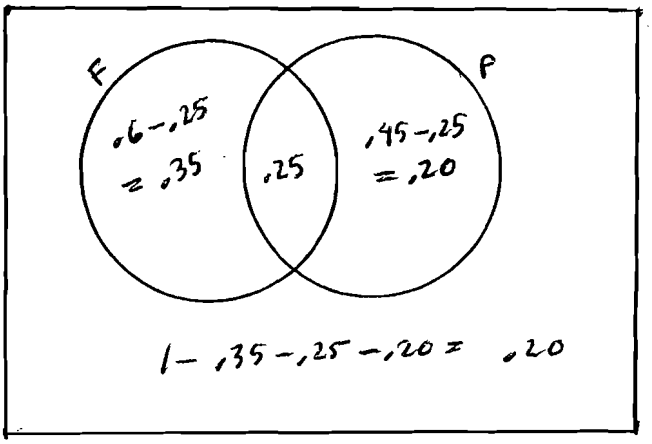
10	(a) Indep	$\underbrace{Pr(E \cap F)}_{.12} = \underbrace{Pr(E)}_{.3}\ \underbrace{Pr(F)}_{.4}$
	(b) Not indep	$\underbrace{Pr(E \cap G)}_{.1} \neq \underbrace{Pr(E)}_{.3}\ \underbrace{Pr(G)}_{.5}$
	(c) Indep	$\underbrace{Pr(F \cap G)}_{.2} = \underbrace{Pr(F)}_{.4}\ \underbrace{Pr(G)}_{.5}$

$Pr(F \cap G) = Pr(F) + Pr(G) - Pr(F \cup G)$

$= .4 + .5 - .7$

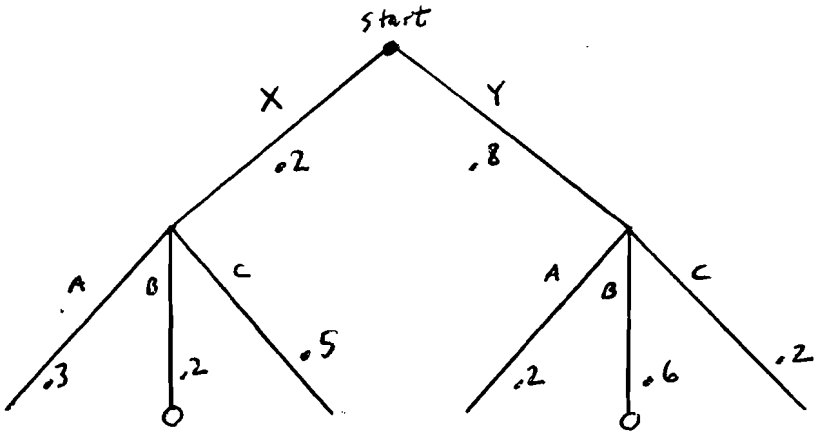
$= .2$

Problem	Ans	Reason
11	5/12	



$$Pr(P \mid F) = \frac{Pr(P \cap F)}{Pr(F)} = \frac{.25}{.60} = \frac{25}{60} = \frac{5}{12}$$

12	1/13	



$$Pr(X \mid B) = \frac{Pr(X \cap B)}{Pr(B)} = \frac{(.2)(.2)}{(.2)(.2) + (.8)(.6)}$$

$$= \frac{4}{4 + 48} = \frac{4}{52} = \frac{1}{13}$$

No. 3.R

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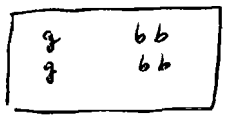
Problem	Ans	Reason
13	7/8	 6 balls jar

Bernoulli trial

Success: chose green ball

$p = 2/6 = 1/3$

$q = 4/6 = 2/3$

$$\Pr(\text{at least 2 blue out of 4} \mid \text{at least one } g, b)$$

$$= \Pr(0 \text{ or } 1 \text{ or } 2 \text{ out of } 4 \mid 1 \text{ or } 2 \text{ or } 3 \text{ out of } 4)$$

$$= \Pr(1 \text{ or } 2 \text{ out of } 4) / \Pr(1 \text{ or } 2 \text{ or } 3 \text{ out of } 4)$$

$$= \frac{C(4,1)pq^3 + C(4,2)p^2q^2}{C(4,1)pq^3 + C(4,2)p^2q^2 + C(4,3)p^3q}$$

$$= \frac{4\,\frac{1}{3}\left(\frac{2}{3}\right)^3 + 6\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2}{4\,\frac{1}{3}\left(\frac{2}{3}\right)^3 + 6\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3\frac{2}{3}} = \frac{4\cdot 8 + 6\cdot 4}{4\cdot 8 + 6\cdot 4 + 4\cdot 2}$$

$$= \frac{8+6}{8+6+2} = \frac{14}{16} = \frac{7}{8}$$

14 (a) 5/12

$$\Pr(E/F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{.25}{.60} = \frac{25}{60} = \frac{5\cdot 5}{5\cdot 12} = 5/12$$

(b) 1/4

$$\Pr(E \cap F) = \Pr(E) - \Pr(E \cap F')$$

$$= .40 - .25$$

$$= .15$$

$$\Pr(E/F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{.15}{.60} = \frac{15}{60} = \frac{1}{4}$$

(c) .24 = 6/25

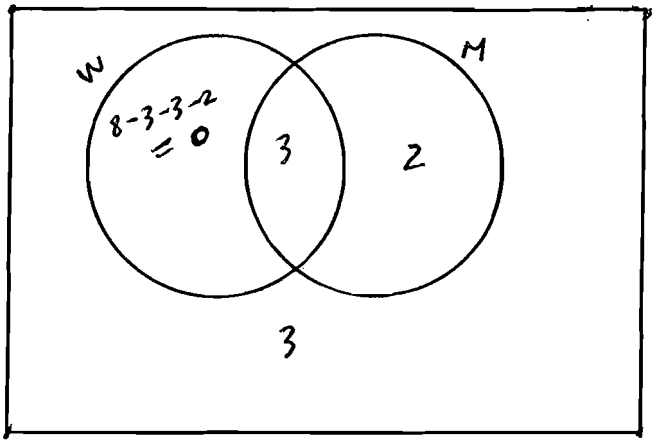
$$\Pr(E \cap F) = \Pr(E)\Pr(F)$$

$$= (.40)(.60)$$

$$= .24$$

Problem	Ans	Reason

15 **1**



Choose 2

Sample space S : 2-element subsets of the set of 8 mice

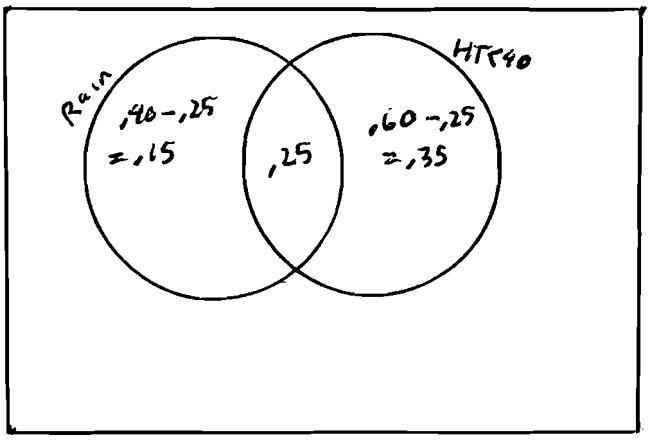
Event A: at least one M

Event B: exactly one W

$Pr(A|B) = Pr(A \cap B) / Pr(B)$

$B \subset A$ so $A \cap B = B$ $\quad Pr(A|B) = 1$

16 **3/8**

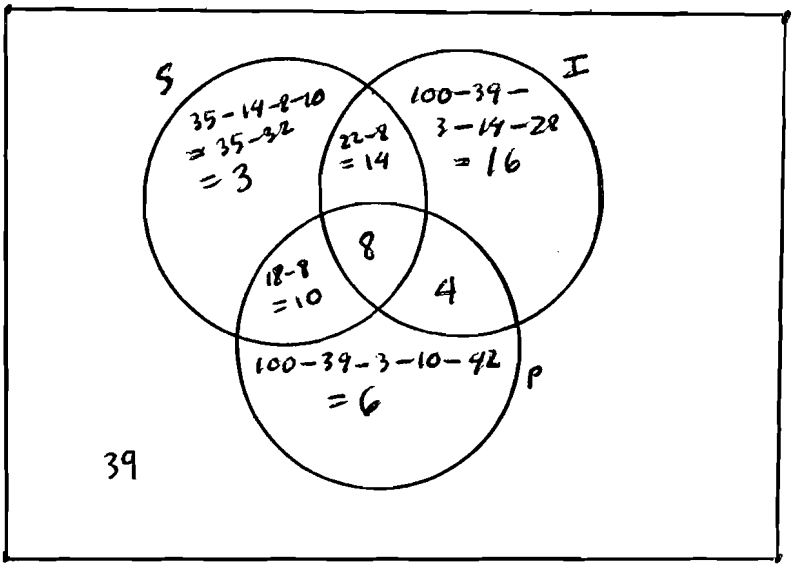


$$Pr[HT \geq 40 \mid Rain] = \frac{Pr(HT \geq 40 \text{ and } Rain)}{Pr(Rain)}$$

$$= \frac{.15}{.40} = \frac{15}{40} = \frac{3 \cdot 5}{8 \cdot 5} = 3/8$$

Problem	Ans	Reason

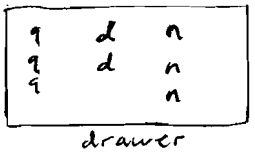
17 $1/4$



$$\Pr[\text{exactly 1 favorable}] = \frac{3+16+6}{100}$$

$$= \frac{25}{100} = 1/4$$

18 $\frac{16}{25}$



8 coins

Choose 2

value	Q	D	N
Q	50	35	30
D		20	15
N			10

$$\Pr(\text{val} \geq 20 \mid \text{at most 1 Q}) = \Pr(\{QD, QN, DD\}) \Big/ (1 - \Pr(QQ))$$

$$= \frac{\frac{3\cdot 2}{C(8,2)} + \frac{3\cdot 3}{C(8,2)} + \frac{1}{C(8,2)}}{1 - \frac{C(3,2)}{C(8,2)}} = \frac{6+9+1}{28-3}$$

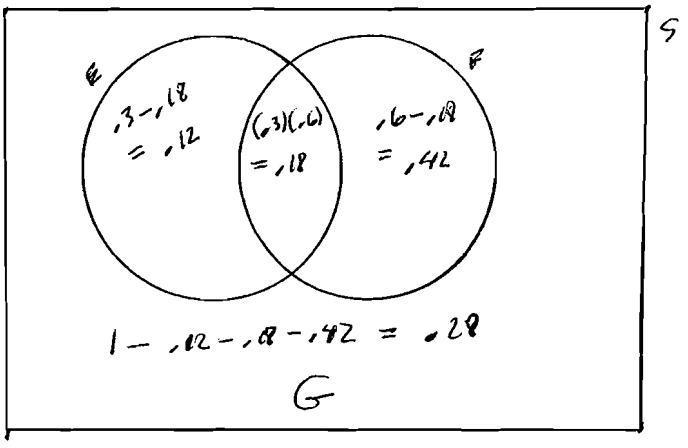
$$= \frac{16}{25}$$

Problem	Ans	Reason
19	$\frac{16320}{3^{10}}$	Bernoulli trial $p = \frac{2}{6} = \frac{1}{3}$ $\quad q = \frac{2}{3}$ $Pr(3 \text{ or } 7 \text{ out of } 10) = C(10,3)\, p^3 q^7 + C(10,7)\, p^7 q^3$ $= 120 \frac{2^3 \cdot 1^7 + 2^7 \cdot 1^3}{3^{10}} = 120 \frac{8 + 128}{3^{10}}$ $= \frac{120 \cdot 136}{3^{10}} = \frac{16320}{3^{10}}$
20	$\frac{7500}{7776}$	$Pr(\text{not all same color}) = 1 - Pr(\text{all red}) - Pr(\text{all white}) - Pr(\text{all blue})$ $Pr(\text{all red}) = \left(\frac{3}{6}\right)^5$ $\quad Pr(\text{all white}) = \left(\frac{1}{6}\right)^5$ $Pr(\text{all blue}) = \left(\frac{2}{6}\right)^5$ $\text{Ans} = 1 - \left(\frac{3}{6}\right)^5 - \left(\frac{1}{6}\right)^5 - \left(\frac{2}{6}\right)^5 = \frac{7500}{7776}$
21	$p = \frac{5}{11}$	Find p such that $C(10,4)\, p^4 (1-p)^6 = C(10,5)\, p^5 (1-p)^5$ $\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1}(1-p) = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} p$ $1 - p = \frac{6}{5} p$ $1 = p + \frac{6}{5} p = \frac{11}{5} p \qquad p = 5/11$

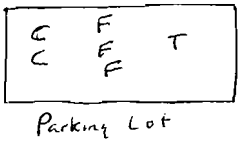
No. 3.R

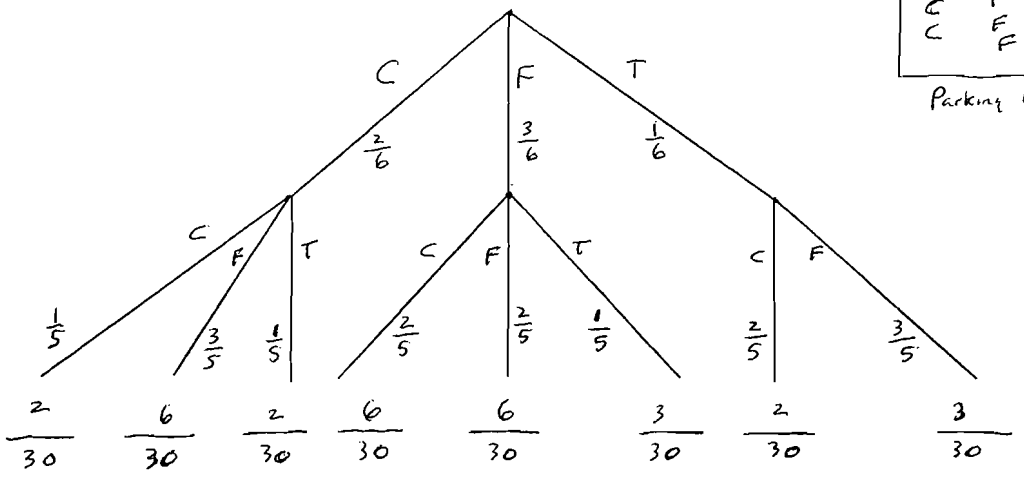
Date 11

Problem	Ans	Reason
22	.28	



23



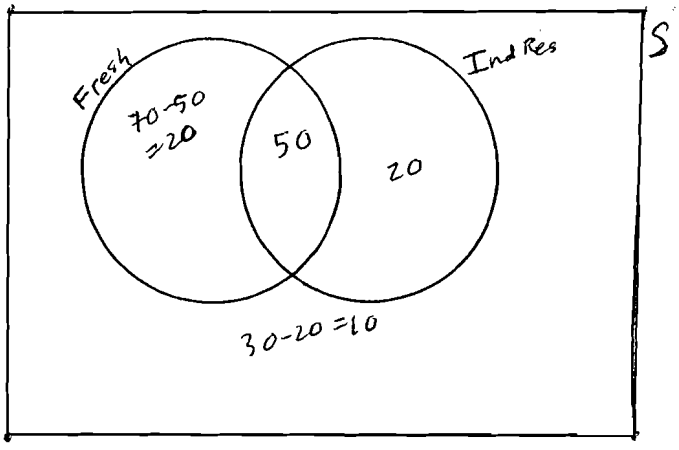


24 (a) $\frac{1}{2}$ $\quad Pr(\text{2nd car is F}) = \frac{6}{30} + \frac{6}{30} + \frac{3}{30} = \frac{15}{30} = \frac{1}{2}$

(b) $\frac{2}{5}$ $\quad \frac{\frac{6}{30} + \frac{2}{30}}{\frac{1}{2} + \frac{1}{6}} = \frac{\frac{8}{30}}{\frac{4}{6}} = \frac{8}{30} \cdot \frac{6}{4} = \frac{2}{5}$

Problem	Ans	Reason

25 $\frac{2}{3}$



$$Pr[F \mid \text{not } IR] = \frac{Pr[F \cap (IR)']}{Pr(IR')}$$

$$= \frac{20}{20+10} = \frac{20}{30} = \frac{2}{3}$$

26 (a) $1/9$
(b) $3/5$
(c) 1

	1	2	3	4	5	6
1	·	E	·	E	·	E
2	E	E	E	E	E	E
3	·	E	·	E	·	E
4	E	E	E	E	E	E
5	·	E	·	E	·	E
6	E	E	E	E	E	E

event E: at least one even

event O: sum is 8 event Δ: sum is 7

$Pr(O \mid E) = Pr(O \cap E)/Pr(E) = 3/27 = 1/9$

$Pr(E \mid O) = Pr(O \cap E)/Pr(O) = 3/5$

$Pr(E \mid \Delta) = Pr(E \cap \Delta)/Pr(\Delta) = \frac{Pr(\Delta)}{Pr(\Delta)} = 1$

Problem	Ans		Reason
27	(a)	3/5	_ _ _ ↑ each of 1, 2, 4, 6, 9 eq Likely; 3 out of 5 are even
	(b)	3/5	
	(c)	7/12	_ _ _ ↑ each of 1, 2, 4, 6, 9 eq Likely
	(d)	7/12	

$Pr(\text{even} / <500) = Pr(\text{even and } <500) / Pr(<500)$

$= n(\text{even and } <500) / n(<500)$

even and <500

	1	2	4	6	9
1		•	•	•	
2			•	•	
4		•		•	
6					
9					

$n(\text{even and } <500) = 7 \cdot 3 = 21$

$n(<500) = 3 \cdot 4 \cdot 3 = 36$

$21/36 = 7/12$

$Pr(<500 / E) = Pr(E \text{ and } <500) / Pr(E)$

$= n(E \text{ and } <500) / n(E)$

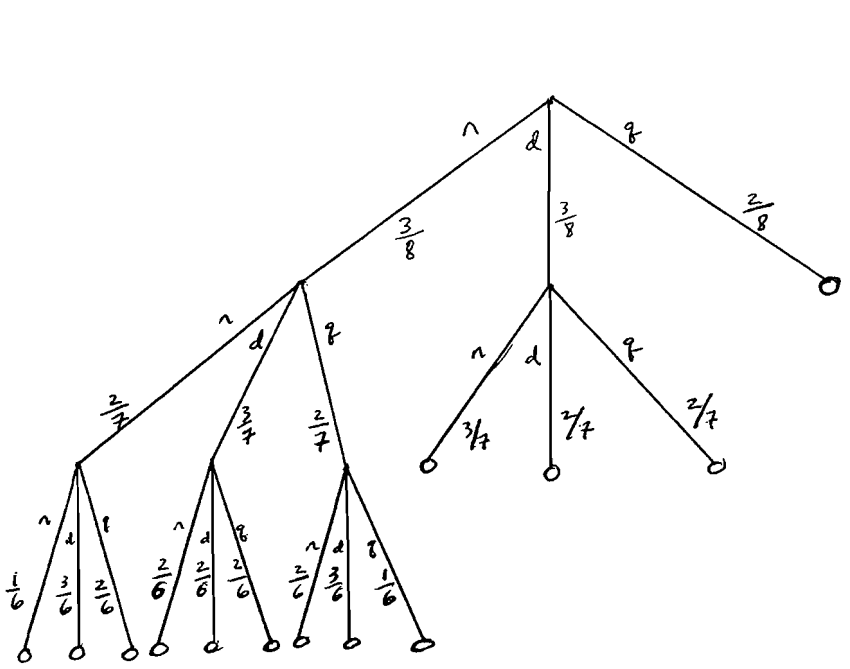
$n(E) = 3 \cdot 4 \cdot 3 = 36$

No. 3.R

Date 14

Problem	Ans	Reason

28



(a) $\frac{15}{28}$ $\quad \frac{3}{8} + \frac{3}{8} \cdot \frac{3}{7} = \frac{3}{8} \cdot \frac{10}{7} = \frac{3}{4} \cdot \frac{5}{7} = \frac{15}{28}$

(b) 15/56 $\quad \frac{3}{8} \cdot \frac{3}{7} + \frac{3}{8} \cdot \frac{2}{7} = \frac{3}{8} \cdot \frac{5}{7} = \frac{15}{56}$

(c) 3/5 $\quad \frac{\frac{3}{8} \cdot \frac{3}{7}}{\frac{3}{8} \cdot \frac{3}{7} + \frac{3}{8} \cdot \frac{2}{7}} = \frac{9}{9+6} = \frac{9}{15} = 3/5$

No. 3.R

Date 15

Problem	Ans	Reason
29	(a) 1/2	g b r r
	(b) 1/3	_ _ _ ↑ each ball eq likely to be 3rd ball picked

R _ _ _ _ R _ _ _ R

↖ ↑ ↗ above 3 events equally likely

30

Bernoulli

$p = 1/4 \qquad q = 3/4$

$$\frac{C(10,7)p^7q^3 + C(10,8)p^8q^2 + C(10,9)p^9q + C(10,10)p^{10}}{1 - C(10,0)p^0q^{10} - C(10,1)p^1q^9}$$

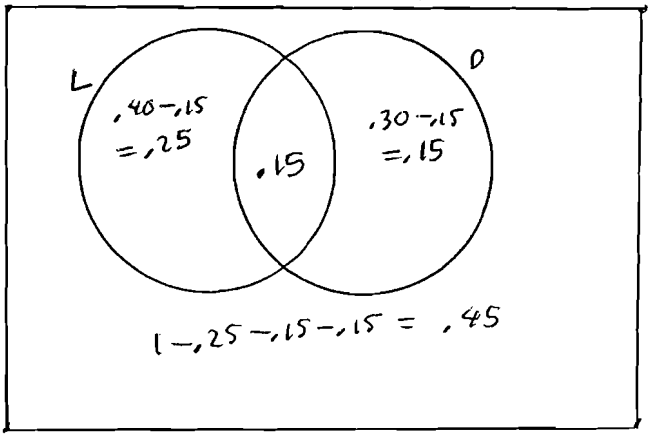
$$= \frac{120\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3 + 45\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 + 10\left(\frac{1}{4}\right)^9\frac{3}{4} + \left(\frac{1}{4}\right)^{10}}{1 - \left(\frac{3}{4}\right)^{10} - 10\,\frac{1}{4}\left(\frac{3}{4}\right)^9}$$

31

$p = .6 \qquad q = .4$

$$\frac{C(10,6)p^6q^4 + C(10,7)p^7q^3 + C(10,8)p^8q^2 + C(10,9)p^9q}{1 - p^{10} - q^{10}}$$

Problem	Ans	Reason
32	(a) .45 (b) 1/2 (c) 5/8	



$$Pr(L|D) = Pr(L \cap D)/Pr(D) = \frac{.15}{.30} = \frac{15}{30} = \frac{1}{2}$$

$$Pr(L/\ L \cap D \text{ but not both}) = \frac{Pr(L \cap D')}{Pr(L \cap D') + Pr(L' \cap D)} = \frac{.25}{.25+.15} = \frac{25}{25+15} = \frac{25}{40} = \frac{5\cdot5}{8\cdot5} = 5/8$$

33 (a) $C(5,2)(.02)^2(.98)^3$

(b) $(.02)^2$

34 $$\frac{C(5,2)(.02)^2(.98)^3}{1-(.98)^5 - 5(.02)(.98)^4}$$

35 $(.98)^3$

FP ___ ___ ___ FP

↖ ↑ ↑

Not False Pos