Section 3.5 HW SOLUTIONS

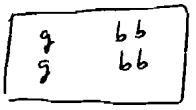
Problem	Ans	Reason
1	(a) .432	$Pr(SFF, FSF, FFS) = PQQ + QPQ + QQP$ $= 3PQ^2$ $= 3(.4)(.6)^2$ $= .432$
	(b) .4096	$Pr(SFFFF, FSFFF, FFSFF, FFFSF, FFFFS)$ $= PQQQQ + QPQQQ + QQPQQ + QQQPQ + QQQQP$ $= 5PQ^4$ $= 5(.2)(.8)^4$ $= .4096$
	(c) $\frac{40}{243}$	$Pr(3 \text{ out of } 5 \text{ success}) = C(5,3)P^3Q^2$ $= 10\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2$ $= \frac{40}{243}$
2	(a) .1536	$Pr(2 \text{ out of } 4 \text{ success}) = C(4,2)P^2Q^2$ $= 6(.8)^2(.2)^2$ $= .1536$
	(b) .0256	$Pr(3 \text{ out of } 4 \text{ success}) = C(4,3)P^3Q$ $= 4(.2)^3(.8)^1$ $= .0256$
	(c) $\frac{64}{125}$	$Pr(0 \text{ out of } 3 \text{ success}) = Q^3$ $= \left(\frac{4}{5}\right)^3$ $= \frac{64}{125}$
3	(a) .31104	$Pr(4 \text{ out of } 6 \text{ success}) = C(6,4)P^4Q^2$ $= 15(.6)^4(.4)^2$ $= .31104$
	(b) .2304	$Pr(3 \text{ out of } 5 \text{ success}) = C(5,3)P^3Q^2$ $= 10(.4)^3(.6)^2$ $= .2304$

Problem	Ans	Reason
4	(a) $\frac{10}{32}$	$Pr(3 \text{ out of } 6) = C(6,3)p^3q^3$ $= 20(.5)^3(.5)^3$ $= 20\left(\frac{1}{2}\right)^6$ $= \frac{10}{2^5} = \frac{10}{32}$
	(b) .3087	$Pr(2 \text{ out of } 5) = C(5,2)(.3)^2(.7)^3$ $= 10(.3)^2(.7)^3$ $= .3087$
5	(a) .09888	$Pr(\text{at least } 3 \text{ out of } 6)$ $= C(6,3)p^3q^3 + C(6,4)p^4q^2 + C(6,5)p^5q + C(6,6)p^6$ $= 20(.2)^3(.8)^3 + 15(.2)^4(.8)^2 + 6(.2)^5(.8) + 1(.2)^6$ $= .09888$
	(b) .05792	$Pr(\text{at most } 2 \text{ out of } 5)$ $= C(5,0)p^0q^5 + C(5,1)pq^4 + C(5,2)p^2q^3$ $= 1(.2)^5 + 5(.8)(.2)^4 + 10(.8)^2(.2)^3$ $= .05792$
6	(a) $\frac{171}{1296}$	Success: 3 is rolled Failure: 1, 2, 4, 5, 6 rolled $p = 1/6 \quad q = 5/6$ $Pr(\text{at least } 2 \text{ out of } 4) =$ $C(4,2)p^2q^2 + C(4,3)p^3q + C(4,4)p^4$ $= 6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 + 4\left(\frac{1}{6}\right)^3\frac{5}{6} + 1\left(\frac{1}{6}\right)^4$ $= \frac{171}{6^4} = \frac{171}{1296}$
	(b) $\frac{11}{16}$	Success: 1, 3, 5 rolled $p = 1/2 \quad q = 1/2$ $Pr(\text{at least } 2 \text{ out of } 4) = 6\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^4 + 1\left(\frac{1}{2}\right)^4$ $= \frac{11}{16}$

Problem	Ans	Reason
7	$\frac{2850}{3125} = .912$	$Pr(1 \le \#\text{success} \le n-1) = 1 - Pr(\text{No success}) - Pr(\text{No Failure})$ $= 1 - p^5 - q^5 = 1 - \left(\frac{2}{5}\right)^5 - \left(\frac{3}{5}\right)^5$ $= \frac{5^5 - 2^5 - 3^5}{5^5} = \frac{2850}{3125}$
8	$\frac{11}{32}$	$Pr(\text{at Least 4 out of 6})$ $= C(6,4)\,p^4 q^2 + C(6,5)\,p^5 q + C(6,6)\,p^6$ $= 15\left(\frac{1}{2}\right)^6 + 6\left(\frac{1}{2}\right)^6 + 1\left(\frac{1}{2}\right)^6$ $= \frac{22}{2^6} = \frac{11}{2^5} = \frac{11}{32}$
9	.576	Success : Heads $p = .4 \quad q = .6$ $Pr(\text{2 out of 5 or 3 out of 5}) = Pr(\text{2 out of 5}) + Pr(\text{3 out of 5})$ $= C(5,2)\,p^2 q^3 + C(5,3)\,p^3 q^2$ $= 10\left(\frac{4}{10}\right)^2\left(\frac{6}{10}\right)^3 + 10\left(\frac{4}{10}\right)^3\left(\frac{6}{10}\right)^2$ $= \frac{4^2 \cdot 6^2 (6+4)}{10^4} = 4^2 6^2 / 10^3 = .576$
10	$\frac{6^2 4^2 1260}{10^6}$	Success : heads $\quad p = .6 \quad q = .4$ $Pr(\text{2 or 3 or 4 out of 6}) =$ $C(6,2)\,p^2 q^4 + C(6,3)\,p^3 q^3 + C(6,4)\,p^4 q^2$ $= p^2 q^2 (15 q^2 + 20 pq + 15 p^2)$ $= \left(\frac{6}{10}\right)^2\left(\frac{4}{10}\right)^2 \left(15\left(\frac{4}{10}\right)^2 + 20\,\frac{6}{10}\,\frac{4}{10} + 15\left(\frac{6}{10}\right)^2\right)$ $= \frac{6^2 4^2}{10^6}(1260)$

Problem	Ans	Reason
11	64/81	Success : heads $p = 2q \qquad p = 2(1-p) = 2-2p$ $p = 2/3 \qquad q = 1/3$ $Pr(\text{at least one } H, T) = 1 - Pr(0 \text{ out of } 4) - Pr(4 \text{ out of } 4)$ $= 1 - p^4 - q^4$ $= 1 - \left(\frac{2}{3}\right)^4 - \left(\frac{1}{3}\right)^4$ $= \frac{3^4 - 2^4 - 1}{3^4} = \frac{64}{81}$
12	(a) 40/243	$p = 2q = 2(1-p)$ $p = 2/3 \qquad q = 1/3$ $Pr(2 \text{ out of } 5) = C(5,2)\, p^2 q^3$ $= 10\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3$ $= \frac{40}{3^5} = \frac{40}{243}$
	(b) 64/81	$Pr(\text{at least 3 out of } 5) =$ $C(5,3)\, p^3 q^2 + C(5,4)\, p^4 q + C(5,5)\, p^5$ $= 10\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 + 5\left(\frac{2}{3}\right)^4\frac{1}{3} \quad 1\left(\frac{2}{3}\right)^5$ $= \frac{80 + 80 + 32}{3^5} = \frac{192}{243} = \frac{64}{81}$
13	(a) .0768	Success : eats breakfast $p = .6, \quad q = .4$ $Pr(1 \text{ out of } 5) = C(5,1)\, p\, q^4$ $= 5\,\frac{6}{10}\left[\frac{4}{10}\right]^4 = \frac{768}{10000}$
	(b) .07776	$Pr(5 \text{ out of } 5) = C(5,5)\, p^5 = 1\left(\frac{6}{10}\right)^5 = \frac{7776}{100000}$
	(c) .08704	$Pr(0 \text{ or } 1 \text{ out of } 5) = C(5,0)\, q^5 + C(5,1)\, pq^4$ $= \left(\frac{4}{10}\right)^5 + 5\,\frac{6}{10}\left(\frac{4}{10}\right)^4 = \frac{8704}{10^5}$

Problem	Ans	Reason
14	$1/32$	Sample space S = cartesian product $\{C, I\} \times \{C, I\} \times \{C, I\} \times \{C, I\} \times \{C, I\}$ $n(S) = 32$ Each outcome eq. likely Event $E = \{CCIII\}$ $n(E) = 1$. $Pr(E) = 1/32$
15	$\frac{190}{2^{20}}$	$p = 1/2$ $q = 1/2$ $Pr(18 \text{ out of } 20) = C(20, 18)\, p^{18} q^2$ $= C(20, 2) \left(\frac{1}{2}\right)^{20}$ $= 190/2^{20}$
16	$56/1024$	$p = 1/2$ $q = 1/2$ $Pr(\text{at least 8 out of 10}) =$ $C(10, 8)\, p^8 q^2 + C(10, 9)\, p^9 q + C(10, 10)\, p^{10}$ $= (45 + 10 + 1) \frac{1}{2^{10}}$ $= \frac{56}{2^{10}} = \frac{56}{1024}$
17	$28 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^6$	---- $p = \frac{1}{4}$ $q = 3/4$ $Pr(2 \text{ out of } 8) = C(8, 2)\, p^2 q^6$ $= 28 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^6$
18	$\frac{25}{4^8}$	$Pr(7 \text{ or } 8 \text{ out of } 8) =$ $C(8, 7)\, p^7 q + C(8, 8)\, p^8$ $= 8 \left(\frac{1}{4}\right)^7 \frac{3}{4} + \left(\frac{1}{4}\right)^8$ $= \frac{25}{4^8}$

Problem	Ans	Reason
19	.271	Success : car starts $p = 9/10$ Pr(at least one failure out of 3) $= 1 - Pr(3 \text{ out of } 3)$ $= 1 - \left(\frac{9}{10}\right)^3 = \frac{10^3 - 9^3}{10^3} = \frac{1000-729}{1000}$ $= 271/1000$
20	$\frac{2}{3}$	 6 balls jar Success : green $p = \frac{2}{6} = 1/3$ $\quad q = 4/6 = \frac{2}{3}$ $Pr[1 \text{ green out of } 3 \mid \text{at least green out of } 3]$ $= Pr[1 \text{ out of } 3] / Pr[\text{at least 2 out of } 3]$ $= \frac{C(3,1)\frac{1}{3}\left(\frac{2}{3}\right)^2}{C(3,1)\frac{1}{3}\left(\frac{2}{3}\right)^2 + C(3,2)\left(\frac{1}{3}\right)^2\frac{2}{3}} = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$
21	6 lines	Case : 4 Lines $\quad p = .7 \quad q = .3$ $Pr(4 \text{ out of } 4) = C(4,4)p^4 = (.7)^4 = .2401$ Case 5 lines $\quad p = .6 \quad q = .4$ $Pr(4 \text{ or } 5 \text{ out of } 5) = C(5,4)p^4q + C(5,5)p^5$ $= 5(.6)^4(.4) + 1(.6)^5$ $= (.6)^4(2.6) = .33696$ Case 6 lines $\quad p = .5 \quad q = .5$ $Pr(4 \text{ or } 5 \text{ or } 6 \text{ out of } 6) = C(6,4)p^4q^2 + C(6,5)p^5q + C(6,6)p^6$ $= (15 + 6 + 1)\left(\frac{1}{2}\right)^6 = \frac{22}{26} = .34375$ $.2401 < .33696 < .34375$

Problem	Ans	Reason
22	.657	success : acceptable widget $p = 7/10 \quad q = 3/10$ Pr(at least failure out of 3) $= 1 - Pr(3 \text{ out of } 3)$ $= 1 - \left(\frac{7}{10}\right)^3 = \frac{10^3 - 7^3}{10^3} = \frac{1000 - 343}{1000} = .657$
23	3	$p = \frac{1}{3} \quad q = \frac{2}{3}$ Find n such that Pr (at least one out of n) $\geq .7$ $1 - Pr(0 \text{ out of } n)$ $1 - \left(\frac{2}{3}\right)^n$ Require: $1 - .7 \geq \left(\frac{2}{3}\right)^n$ ie $.3 \geq \left(\frac{2}{3}\right)^n$
24	5	$p = 1/4$ Find n s.t $1 - \left(\frac{3}{4}\right)^n \geq .75$ $\frac{1}{4} \geq \left(\frac{3}{4}\right)^n$

n	$(2/3)^n$
1	$2/3 = .66\ldots$
2	$4/9 = .44\ldots$
3	$\frac{8}{27} < .3$

n	$(3/4)^n$
1	$3/4$
2	$\frac{9}{16} > \frac{1}{2}$
3	$\frac{27}{64} > 1/3$
4	$\frac{81}{256} > 1/4$
5	$\frac{243}{1024} < 1/4$

Problem	Ans	Reason

25 — **29**

Success : sell a car

$p = 1/10$

Find n such that

$Pr(\text{at least one out of } n) \geq .95$

Require $\quad 1 - \left(\frac{9}{10}\right)^n \geq .95$

ie $\quad .05 \geq \left(\frac{9}{10}\right)^n$ (tedious to show $n=29$)

n	2	4	6	12	18	24	30
$(9/10)^n$	$\frac{81}{100} \sim \frac{4}{5}$	$\sim\frac{16}{25}$	$\sim\frac{64}{125}\sim\frac{1}{2}$	$\sim\frac{1}{4}$	$\sim\frac{1}{8}$	$\sim\frac{1}{16}$	$\sim\frac{1}{32}$

26 — $\frac{8}{27}$

Success : even

$p = 1/3 \qquad q = 2/3$

$Pr(2 \text{ out of } 4) = C(4,2)\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$

$= 6 \cdot \frac{4}{81} = \frac{24}{81} = \frac{8}{27}$

27 — $\frac{176}{1024} = \frac{11}{64}$

Success : one

$p = 1/2$

$Pr(\text{at least 7 out of } 10) =$

$\left(C(10,7) + C(10,8) + C(10,9) + C(10,10)\right)\left(\frac{1}{2}\right)^{10}$

$C(10,7) = C(10,3) = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

$C(10,8) = C(10,2) = \frac{10 \cdot 9}{2} = 45$

$C(10,9) = 10$

$C(10,10) = 1$

$120 + 45 + 10 + 1 = 176$

Problem	Ans	Reason
28	$\frac{1}{14}$	E E E O $\Pr(EEEO \mid \text{at least one } E, \text{at least one } O)$ $= \frac{\Pr(EEEO)}{1-\Pr(\text{all } E)-\Pr(\text{all } O)}$ $= \frac{(\frac{1}{2})^4}{1-(\frac{1}{2})^4-(\frac{1}{2})^4} = \frac{1}{2^4-1-1} = \frac{1}{14}$
29	$\frac{376}{1024} = \frac{47}{128}$	success : remember $p = 3/4 \quad q = 1/4$ $\Pr(\text{at least 2 failures})$ $= 1-\Pr(\text{0 or 1 failures})$ $= 1 - C(5,0)(\frac{3}{4})^5 - C(5,1)(\frac{3}{4})^4\frac{1}{4}$ $= 1 - (\frac{3}{4})^5 - 5\frac{3^4}{4^5} = \frac{4^5-3^5-5\cdot 3^4}{4^5}$ $= \frac{1024-243-405}{4^5} = \frac{376}{4^5}$
30	$\frac{1}{5^{10}}$	Success : answers match. $p = 1/5$ $\Pr(10 \text{ out of } 10) = (\frac{1}{5})^{10}$
31	$\left[\frac{210 \cdot 4^6}{5^{10}}\right]^2$	$\Pr[\text{both students get 4 out of 10}]$ $= \Pr[\text{student 1 gets 4 out of 10}]\Pr[\text{student 2 gets 4 out of 10}]$ $= \left[C(10,4)(\frac{1}{5})^4(\frac{4}{5})^6\right]^2$ $= \left[\frac{C(10,4)4^6}{5^{10}}\right]^2 = \left[\frac{210 \cdot 4^6}{5^{10}}\right]^2$

Problem	Ans	Reason
32	$\frac{210 \cdot 4^{12}}{5^{20}}$	$C(10,4)\left[\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6\right]^2$
33		Success: prefer kayak $p = .4 \quad q = .6$ For $0 \le i \le 20$ $Pr(i \text{ out of } 20) = C(20,i)(.4)^i(.6)^{20-i}$ $Pr(\text{at most 7 out of } 20) = \sum_{i=0}^{7} C(20,i)(.4)^i(.6)^{20-i}$
34 skip		Find min n such that $.9 \le \sum_{i=0}^{n} C(20,i)(.4)^i(.6)^{20-i}$ (trial and error)
35		Only possible problem: person who desires kayak can't get one $Pr(\text{all satisfied}) = Pr(\text{at most 7 prefer kayak out of } 20)$ $= \sum_{i=0}^{7} C(20,i)(.3)^i(.7)^{20-i}$