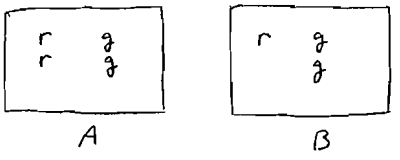
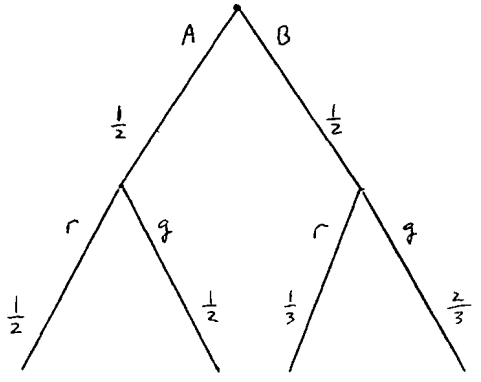
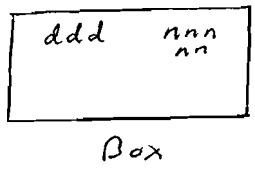
Section 3.4 HW Solutions

Problem	Ans	Reason
1	(a) $\frac{1}{2}$ (b) $\frac{17}{24}$ (c) $\frac{2}{3}$ (d) $\frac{8}{17}$	$Pr(b) = \frac{1}{2}\frac{2}{3} + \frac{1}{2}\frac{3}{4} = \frac{1}{2}\left(\frac{2}{3}+\frac{3}{4}\right) = \frac{1}{2}\frac{17}{3\cdot4} = \frac{17}{24}$ $Pr(B\mid b) = \frac{Pr(b\cap B)}{Pr(b)} = \frac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{17}{24}} = \frac{8}{17}$
2	(a) .26 (b) $\frac{6}{13}$ (c) $\frac{7}{13}$ (d) .3	$Pr(A) = (.3)(.4) + (.7)(.2)$ $= .12 + .14$ $= .26$ $Pr(X\mid A) = \frac{Pr(X\cap A)}{Pr(A)} = \frac{(.3)(.4)}{.26} = \frac{.12}{.26} = \frac{6}{13}$ $Pr(Y\mid B) = \frac{Pr(Y\cap B)}{Pr(B)} = \frac{(.7)(.3)}{(.7)(.3)+(.3)(.6)}$ $= \frac{21}{21+18} = \frac{21}{39} = \frac{7\cdot3}{13\cdot3} = \frac{7}{13}$
3	4/7	  $Pr(B\mid g) = \frac{Pr(B\cap g)}{Pr(g)}$ $= \frac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{2}}$ $= \frac{\frac{2}{3}}{\frac{2}{3}+\frac{1}{2}}$ $= \frac{\frac{2}{3}}{7/6} = \frac{2}{3}\cdot\frac{6}{7} = 4/7$

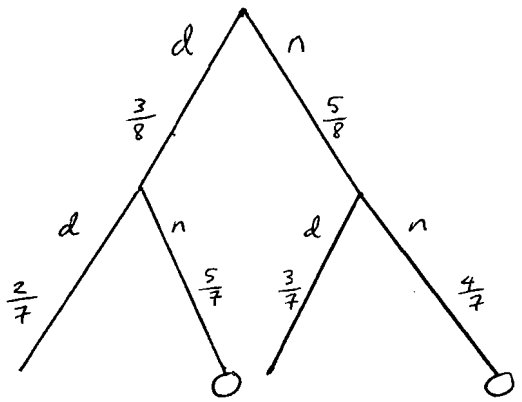
Problem	Ans	Reason
4	(a) 3/5	
	(b) 18/53	$Pr(Y/B) = Pr(Y \cap B) / Pr(B)$
	(c) 33/53	$= \dfrac{\frac{1}{5} \cdot \frac{3}{5}}{\frac{2}{5}\frac{1}{4} + \frac{1}{5}\frac{3}{5} + \frac{2}{5}\frac{1}{3}}$
		$= \dfrac{3/5}{\frac{2}{4} + \frac{3}{5} + \frac{2}{3}}$
		$= \dfrac{3/5}{53/30}$
		$= \frac{3}{5} \cdot \frac{30}{53} = \frac{3 \cdot 6}{53} = \frac{18}{53}$
		$Pr(X \text{ or } Y / B) = Pr(X/B) + Pr(Y/B)$
		$Pr(X/B) = \dfrac{\frac{2}{5}\frac{1}{4}}{\frac{1}{5} \cdot 53/30}$
		$= \dfrac{\frac{1}{2}}{53/30} = \frac{1}{2}\frac{30}{53} = \frac{15}{53}$
		$\frac{18}{53} + \frac{15}{53} = \frac{33}{53}$
5	(a) 1/5	$\frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5}$
	(b) 1/5	$Pr(Y \mid b \text{ and } B) = \dfrac{Pr(Y \text{ and } b \text{ and } B)}{Pr(b \text{ and } B)}$
		$= \dfrac{\frac{2}{5} \cdot \frac{1}{3} \cdot \frac{3}{5}}{\frac{2}{5}\frac{1}{3}\frac{3}{5} + \frac{3}{5}\frac{2}{3}\frac{4}{5}}$
		$= \dfrac{6}{6 + 24} = \frac{6}{30} = \frac{1}{5}$

Problem	Ans	Reason

6 $3/7$



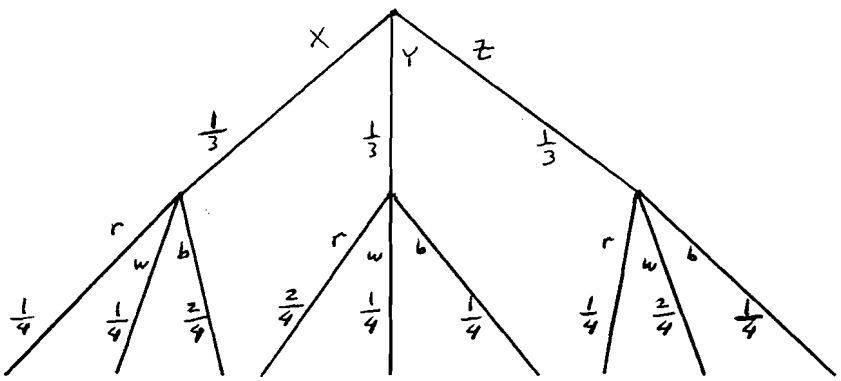
8 coins



$$Pr[\text{1st dime} \mid \text{2nd is picked}] = \frac{\frac{3}{8}\frac{5}{7}}{\frac{3}{8}\frac{5}{7} + \frac{5}{8}\frac{4}{7}} = \frac{15}{15+20}$$

$$= \frac{15}{35} = \frac{3\cdot 5}{7\cdot 5} = \frac{3}{7}$$

7 $\frac{1}{2}$, $\frac{1}{4}$



$$Pr(X \mid b) = \frac{Pr(X \cap b)}{Pr(b)} = \frac{\frac{1}{3}\frac{2}{4}}{\frac{1}{3}\frac{2}{4} + \frac{1}{3}\frac{1}{4} + \frac{1}{3}\frac{1}{4}}$$

$$= \frac{2}{2+1+1} = \frac{2}{4} = 1/2$$

$$Pr(Y \mid b\,a\,w) = \frac{\frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}\right)}{\frac{1}{3}\left(\frac{1}{4}+\frac{2}{4}\right) + \frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}\right) + \frac{1}{3}\left(\frac{2}{4}+\frac{1}{4}\right)}$$

$$= \frac{2}{3+2+3} = \frac{2}{8} = \frac{1}{4}$$

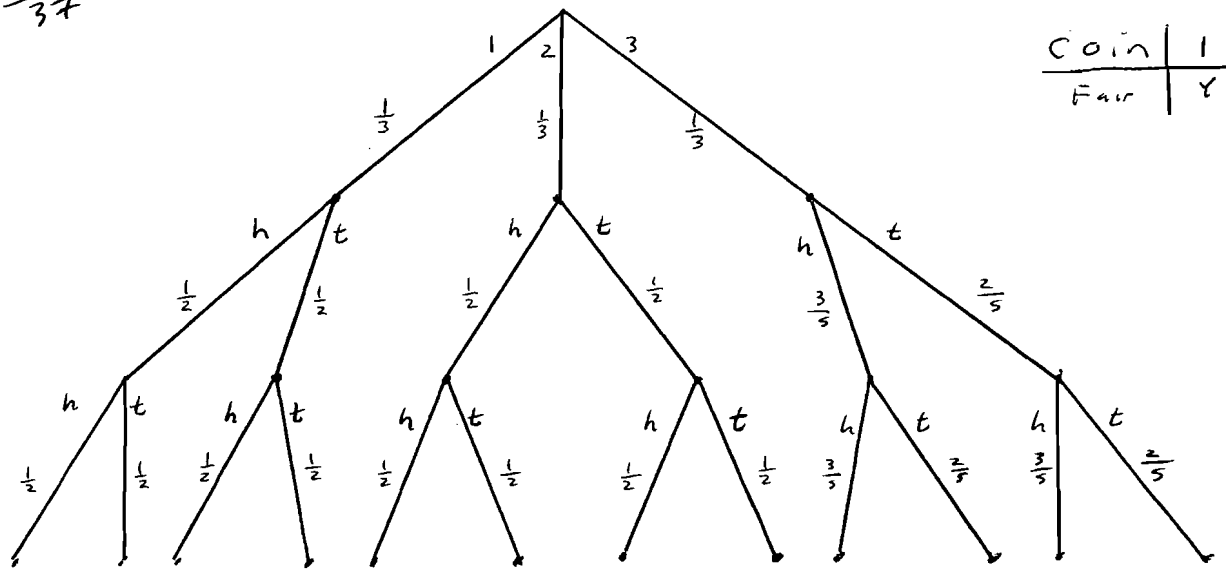
No. 3.4

Date 4

Problem	Ans	Reason

8 $\frac{25}{37}$

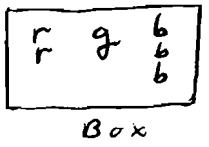
Coin	1	2	3
Fair	Y	Y	N



$$Pr(c1 \text{ or } c2 \mid ht) = \frac{\frac{1}{3}\frac{1}{2}\frac{1}{2} + \frac{1}{3}\frac{1}{2}\frac{1}{2}}{\frac{1}{3}\frac{1}{2}\frac{1}{2} + \frac{1}{3}\frac{1}{2}\frac{1}{2} + \frac{1}{3}\frac{3}{5}\frac{2}{5}}$$

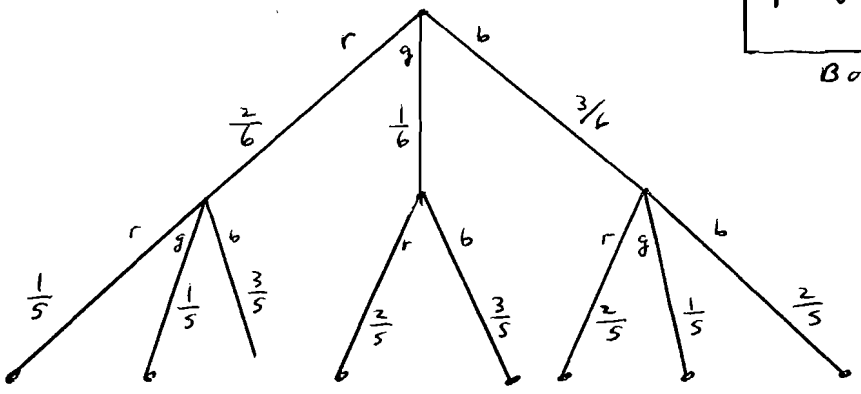
$$= \frac{25}{37}$$

9 3/5



Box

6 balls



$$Pr[\text{1st blue} \mid \text{2nd red}] = \frac{\frac{3}{6}\frac{2}{5}}{\frac{2}{6}\frac{1}{5} + \frac{1}{6}\frac{2}{5} + \frac{3}{6}\frac{2}{5}}$$

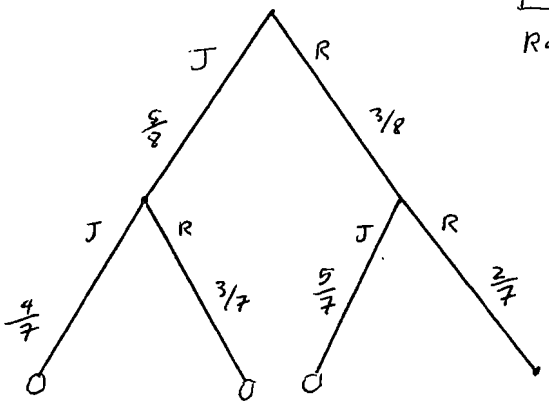
$$= \frac{6}{2+2+6} = \frac{6}{10} = \frac{3}{5}$$

Problem	Ans	Reason

10 $\frac{7}{10}$

Jazz	Rock
5	3

Rack of CD's

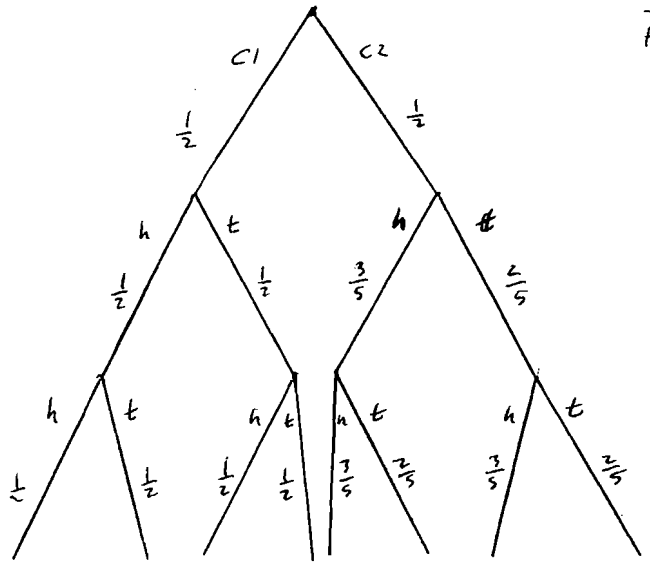


$$Pr[\text{1st is Jazz} \mid \text{at least one Jazz}]$$

$$= \frac{\frac{5}{8}\frac{4}{7} + \frac{5}{8}\frac{3}{7}}{\frac{5}{8}\frac{4}{7} + \frac{5}{8}\frac{3}{7} + \frac{3}{8}\frac{5}{7}} = \frac{4+3}{4+3+3} = \frac{7}{10}$$

11 $\frac{25}{61}$

Coin	1	2
Fair	Y	N



$$Pr[C1 \mid hh] = Pr[C1\ h\ h] / Pr[hh]$$

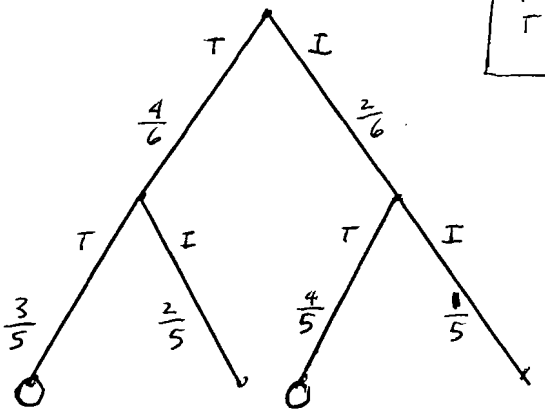
$$= \frac{\frac{1}{2}\frac{1}{2}\frac{1}{2}}{\frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{3}{5}\frac{3}{5}} = \frac{25}{61}$$

No. 3.4

Date 6

Problem	Ans	Reason -

12 $3/5$



T T	I
T T	I

Drawer

$$Pr\left[\text{first quarter is } T \mid \text{Last quarter is } T\right]$$

$$= \frac{\frac{4}{6}\frac{3}{5}}{\frac{4}{6}\frac{3}{5} + \frac{2}{6}\frac{4}{5}} = \frac{12}{12+8}$$

$$= \frac{12}{20} = 3/5$$

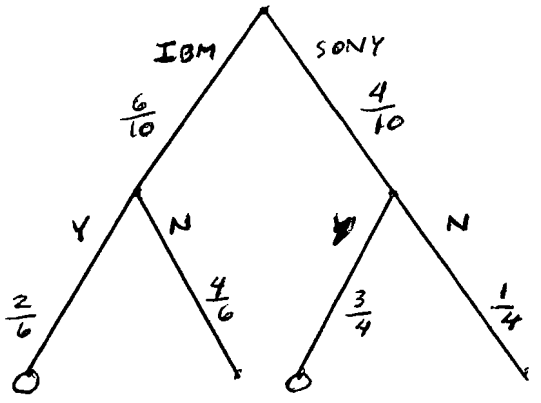
13 $3/5$

$$Pr\left[\text{first blue} \mid \text{and not blue}\right] = \frac{\frac{3}{6}\frac{1}{5} + \frac{3}{6}\frac{2}{5}}{\frac{2}{6}\frac{1}{5} + \frac{2}{6}\frac{1}{5} + \frac{1}{6}\frac{2}{5} + \frac{3}{6}\frac{2}{5} + \frac{3}{6}\frac{1}{5}}$$

$$= \frac{3+6}{2+2+2+6+3} = \frac{9}{15} = \frac{3\cdot 3}{3\cdot 5} = \frac{3}{5}$$

Problem	Ans	Reason

14 — 3/5

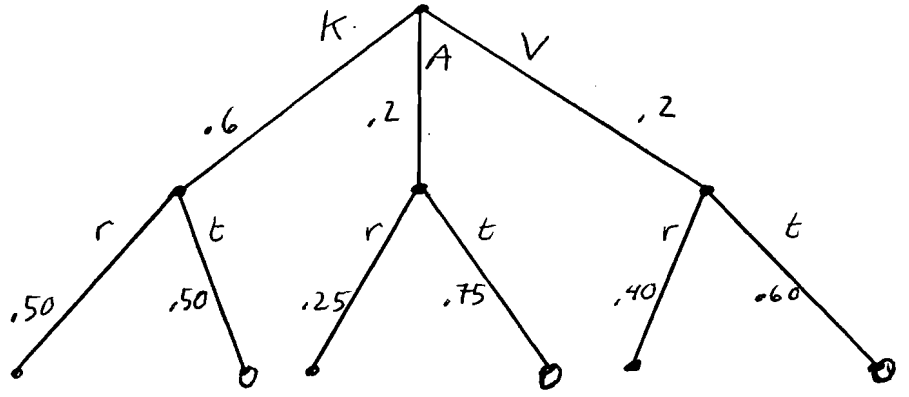


CO ?

$$Pr[\text{sony} \mid \text{has CO}] = \frac{\frac{4}{10}\frac{3}{4}}{\frac{6}{10}\frac{2}{6} + \frac{4}{10}\frac{3}{4}} = \frac{3/10}{2/10 + 3/10}$$

$$= \frac{3}{2+3} = 3/5$$

15 — 30/57



$$Pr[K \mid t] = \frac{(.6)(.50)}{(.6)(.50) + (.2)(.75) + (.2)(.60)}$$

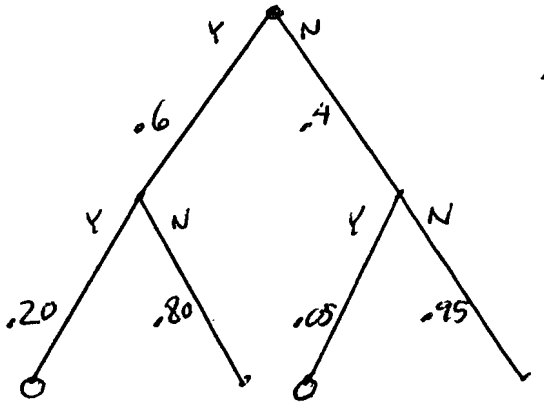
$$= \frac{30}{30 + 15 + 12} = \frac{30}{57}$$

No. 3.4

Date 8

Problem	Ans	Reason

16 $6/7$



4 years HS math?

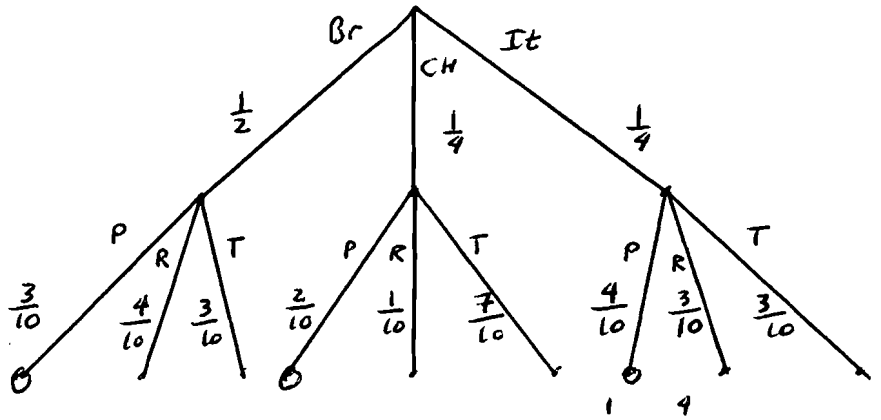
Major in Science?

$Pr[\text{4 years HS math} \mid \text{Science major}]$ $\quad .5 = \frac{1}{20}$

$$= \frac{\frac{6}{10}\,\frac{2}{10}}{\frac{6}{10}\,\frac{2}{10} + \frac{4}{10}\,\frac{1}{20}}$$

$$= \frac{6 \cdot 2}{6 \cdot 2 + 2 \cdot 1} = \frac{6}{6+1} = \frac{6}{7}$$

17 $\frac{1}{3}$

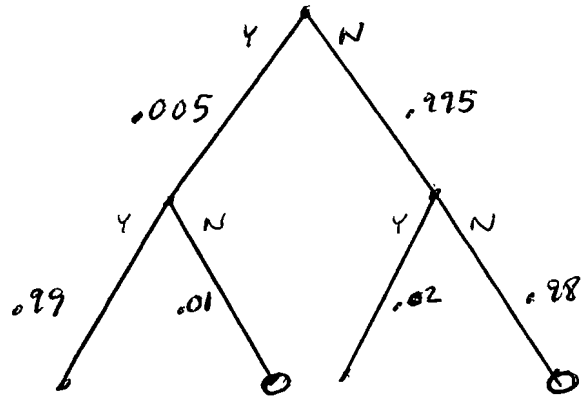


$$Pr[\text{It} \mid \text{P}] = \frac{\frac{1}{4}\,\frac{4}{10}}{\frac{1}{2}\,\frac{3}{10} + \frac{1}{4}\,\frac{2}{10} + \frac{1}{4}\,\frac{4}{10}}$$

$$= \frac{1}{\frac{3}{2} + \frac{1}{2} + 1} = \frac{1}{3}$$

Problem	Ans	Reason

18 $\frac{1}{19503}$



Infected?

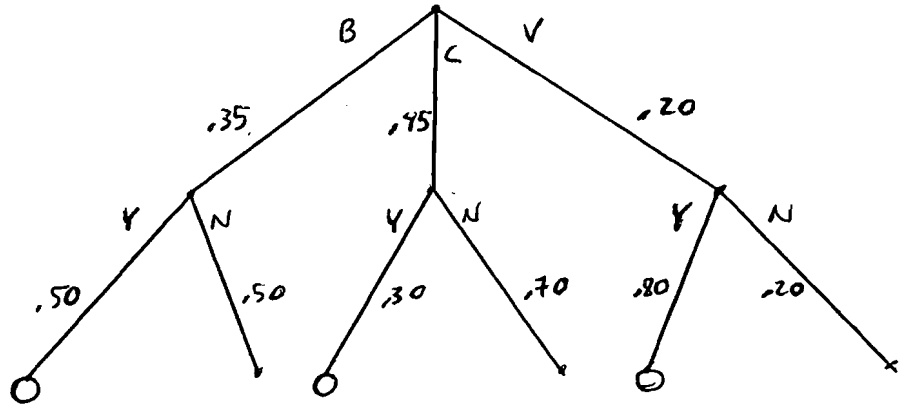
test pos?

$Pr[\text{infected} \mid \text{test neg}]$

$= \frac{(.005)(.01)}{(.005)(.01) + (.995)(.98)} = \frac{5}{97515}$

$= \frac{1}{19503}$

19 $\frac{67}{94}$



2 airbags?

$Pr[\text{B or V} \mid \text{2 airbags}]$

$= \frac{(.35)(.50) + (.20)(.80)}{(.35)(.50) + (.45)(.30) + (.20)(.80)}$

$= \frac{67}{94}$

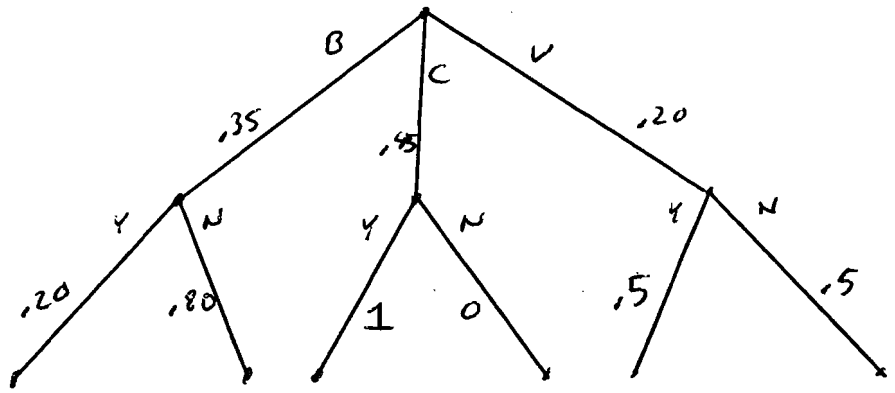
$.47 = (.35)(.50) + (.45)(.30) + (.20)(.80)$

No. 3.4

Date 10

Problem	Ans	Reason

20 $\frac{45}{62}$ $\frac{30}{47}$



white?

$Pr[C \mid \text{white and 2 airbags}]$

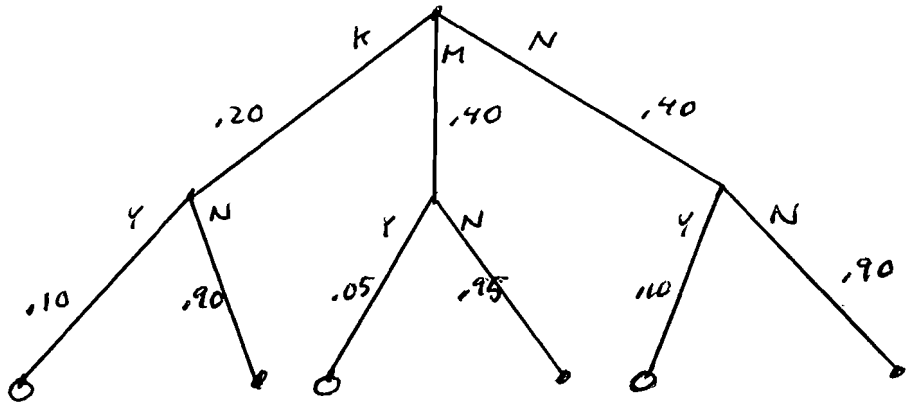
$= \dfrac{Pr[C \text{ and 2 airbags}]}{Pr[\text{white}]\, Pr[\text{2 airbags}]}$

$Pr[\text{white}] = (.35)(.20) + (.45)(1) + (.20)(.5)$

$= .62$

$\text{Ans} = \dfrac{(.45)(.30)(1)}{(.62)(.47)} = \dfrac{45 \cdot 30}{62 \cdot 47}$

21 $\frac{1}{2}$



Lose Signal?

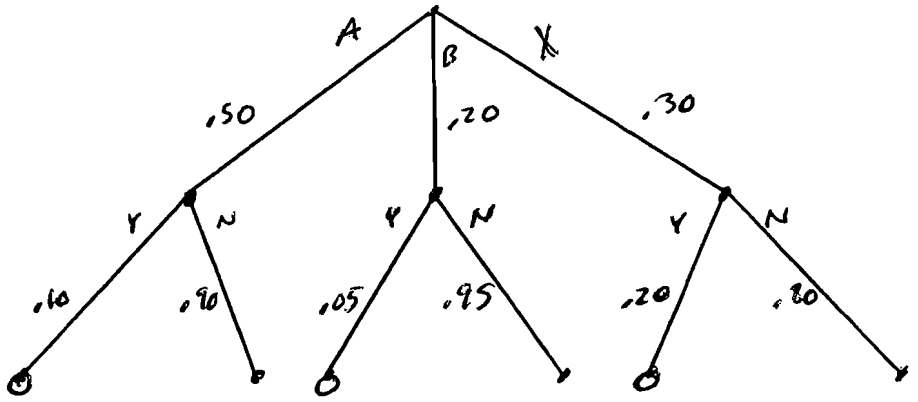
$Pr[N \mid \text{lose signal}]$

$= \dfrac{(.40)(.10)}{(.20)(.10) + (.40)(.05) + (.40)(.10)} = \dfrac{4}{2+2+4}$

$= \frac{4}{8} = \frac{1}{2}$

Problem	Ans	Reason

22 $\frac{1}{2}$



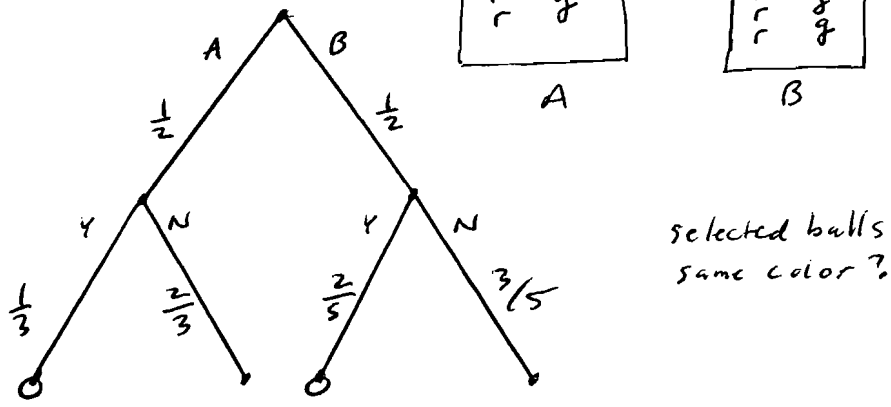
Brand

Defective?

$$Pr[\text{Brand X} \mid \text{defective}] = \frac{(.30)(.20)}{(.50)(.10) + (.20)(.05) + (.30)(.20)} = \frac{6}{5+1+6}$$

$$= \frac{6}{12} = \frac{1}{2}$$

23 $6/11$



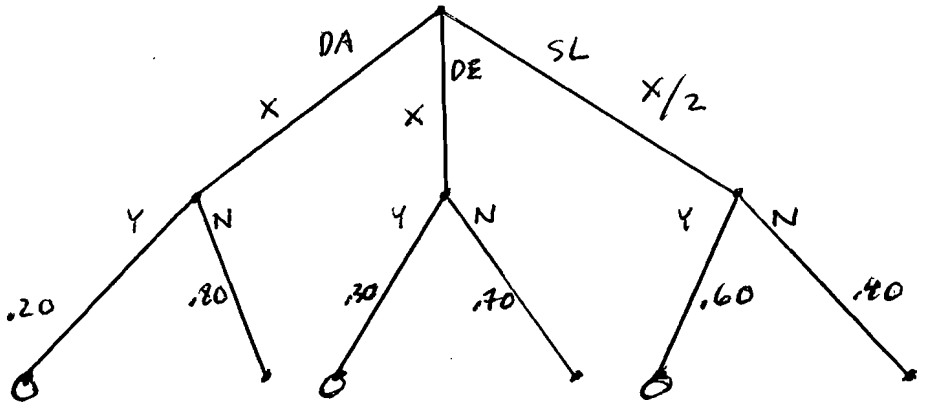
selected balls same color?

$$\frac{C(3,2) + C(2,2)}{C(5,2)} = \frac{3+1}{10} = \frac{2}{5}$$

$$Pr[B \mid \text{selected balls same color}] = \frac{\frac{1}{2}\frac{2}{5}}{\frac{1}{2}\frac{1}{3} + \frac{1}{2}\frac{2}{5}} = \frac{6}{11}$$

Problem	Ans	Reason

24 $3/8$



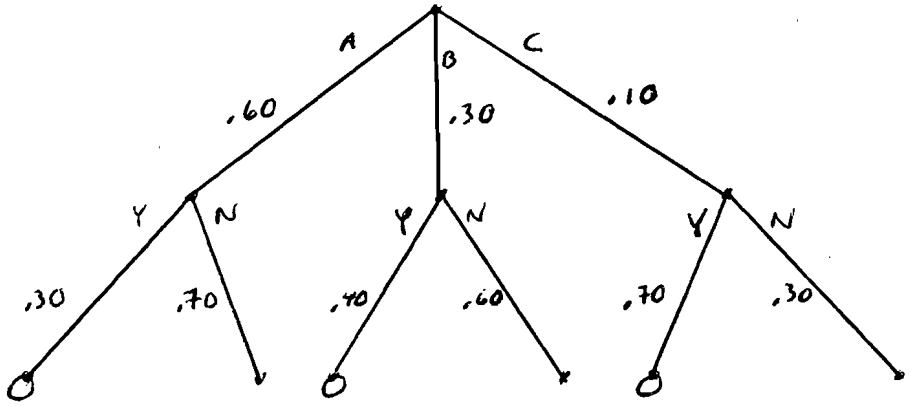
$x + x + x/2 = 1 \qquad \frac{5}{2}x = 1 \qquad x = 2/5$

$Pr[DE \mid late]$

$$= \frac{x(.30)}{x(.20) + x(.30) + \frac{x}{2}(.60)}$$

$$= \frac{3}{2 + 3 + 3} = \frac{3}{8}$$

25 $18/37$



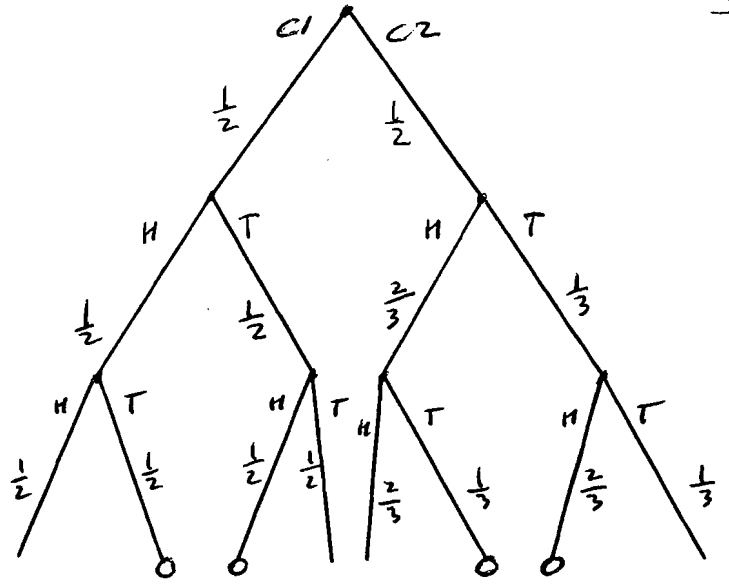
$$Pr[A \mid \text{germinate}] = \frac{(.60)(.30)}{(.60)(.30) + (.30)(.40) + (.10)(.70)}$$

$$= 18/37$$

Problem	Ans	Reason

26 $8/17$

Coin	1	2
Fair	Y	N



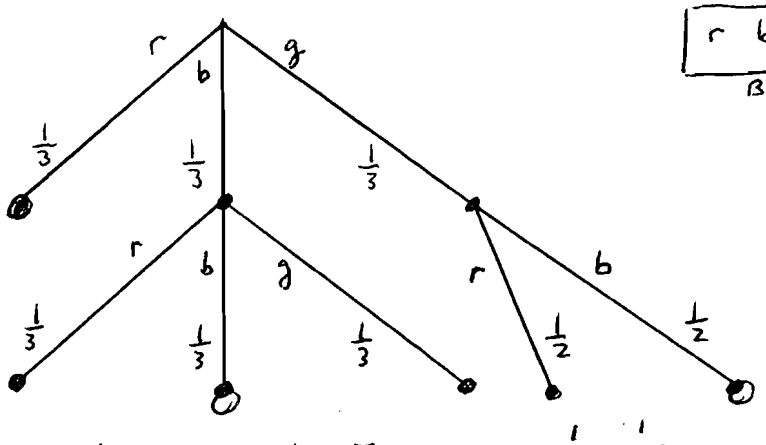
$$Pr[\text{unfair coin} \mid HT \text{ or } TH] = \frac{\frac{1}{2}\frac{2}{3}\frac{1}{3} + \frac{1}{2}\frac{1}{3}\frac{2}{3}}{\frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{2}{3}\frac{1}{3} + \frac{1}{2}\frac{1}{3}\frac{2}{3}}$$

$$= \frac{4/9}{1/2 + 4/9} = \frac{4/9}{17/18} = \frac{4}{9}\cdot\frac{2\cdot 9}{17} = \frac{8}{17}$$

27 $2/5$

r	b	g

Box — 3 balls



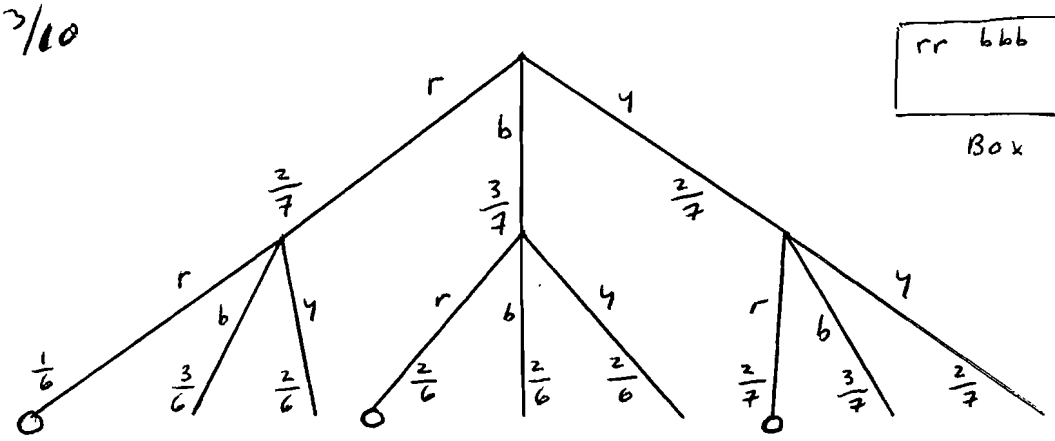
$$Pr[\text{1st ball blue} \mid \text{2nd ball blue}] = \frac{\frac{1}{3}\frac{1}{3}}{\frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{2}} = 2/5$$

No. 3.4

Date 14

Problem	Ans	Reason

28 $3/10$

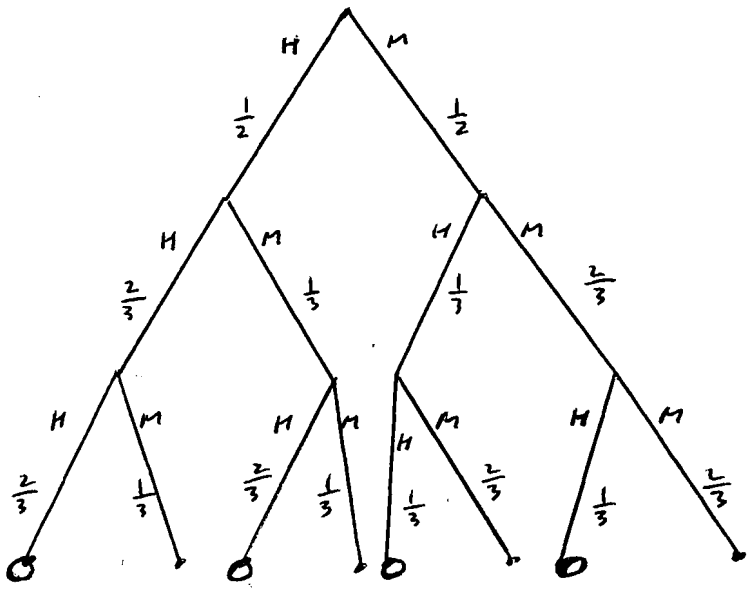


$$Pr[\text{1st ball yellow} \mid \text{2nd ball red}] = \frac{\frac{2}{7}\frac{2}{7}}{\frac{2}{7}\frac{1}{6} + \frac{3}{7}\frac{2}{6} + \frac{2}{7}\frac{2}{7}} = \frac{\frac{2}{7}}{\frac{1}{6} + \frac{3}{6} + \frac{2}{7}}$$

$$= \frac{\frac{2}{7}}{\frac{2}{3} + \frac{2}{7}} = \frac{\frac{1}{7}}{\frac{1}{3} + \frac{1}{7}} = \frac{\frac{1}{7}}{\frac{10}{3 \cdot 7}} = \frac{1}{7}\frac{3 \cdot 7}{10} = \frac{3}{10}$$

29 (a) $2/3$

(b) $4/9$



$$Pr[\text{Hits 1st shot} \mid \text{Hits 3rd shot}] = \frac{\frac{1}{2}\frac{2}{3}\frac{2}{3} + \frac{1}{2}\frac{1}{3}\frac{2}{3}}{\frac{1}{2}\frac{2}{3}\frac{2}{3} + \frac{1}{2}\frac{2}{3}\frac{1}{3} + \frac{1}{2}\frac{1}{3}\frac{1}{3} + \frac{1}{2}\frac{2}{3}\frac{1}{3}} = 2/3$$

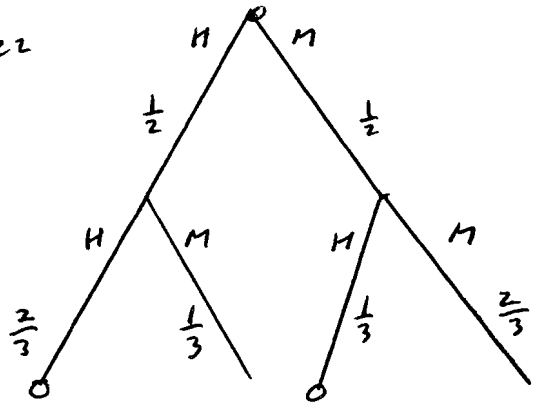
$$Pr[\text{Hits all shots} \mid \text{Hits 3rd shot}] = \frac{\frac{1}{2}\frac{2}{3}\frac{2}{3}}{\frac{1}{2}\frac{2}{3}\frac{2}{3} + \frac{1}{2}\frac{2}{3}\frac{1}{3} + \frac{1}{2}\frac{1}{3}\frac{1}{3} + \frac{1}{2}\frac{2}{3}\frac{1}{3}} = 4/9$$

Problem	Ans	Reason

30

(a) $\frac{1}{2}$ for all $n \geq 2$

(b) $\frac{2}{3}$ for all $n \geq 2$



1st shot

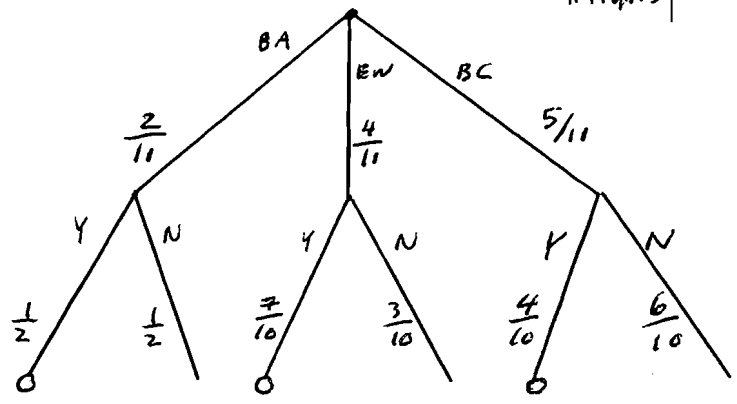
nth shot $n \geq 2$

$$\Pr[\text{hit 1st shot} \mid \text{hit nth shot}] = \frac{\frac{1}{2}\frac{2}{3}}{\frac{1}{2}\frac{2}{3} + \frac{1}{2}\frac{1}{3}}$$

$$= \frac{2}{2+1} = \frac{2}{3}$$

31

airline	BA	EW	BC
#flights	2	4	5



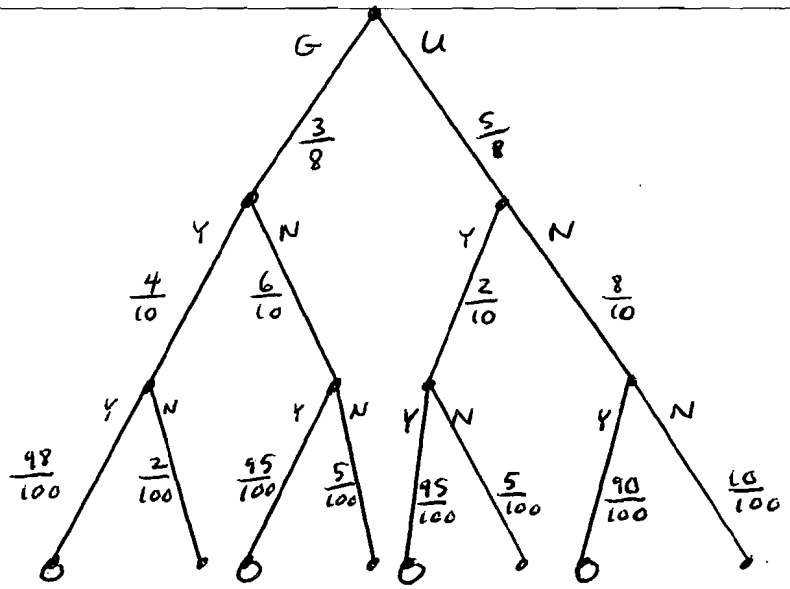
On Time ?

Airline	BA	EW	BC
Prob.	$\frac{10}{58}$	$\frac{28}{58}$	$\frac{20}{58}$

No. 3.4

Date 16

Problem	Ans	Reason
32	$\frac{2886}{7436}$	



Technical field?

Employed?

$$\Pr[\text{Grad deg} \mid \text{employed}] = \frac{\frac{3}{8}\frac{4}{10}\frac{98}{100} + \frac{3}{8}\frac{6}{10}\frac{95}{100}}{\frac{3}{8}\frac{4}{10}\frac{98}{100} + \frac{3}{8}\frac{6}{10}\frac{95}{100} + \frac{5}{8}\frac{2}{10}\frac{95}{100} + \frac{5}{8}\frac{8}{10}\frac{90}{100}}$$

$$= \frac{12\cdot 98 + 18\cdot 95}{12\cdot 98 + 18\cdot 95 + 10\cdot 95 + 40\cdot 90}$$

$$= \frac{1176 + 1710}{1176 + 1710 + 950 + 3600} = \frac{2886}{7436}$$

33 Undergrad degree more likely $\frac{2886}{7436} < \frac{1}{2}$

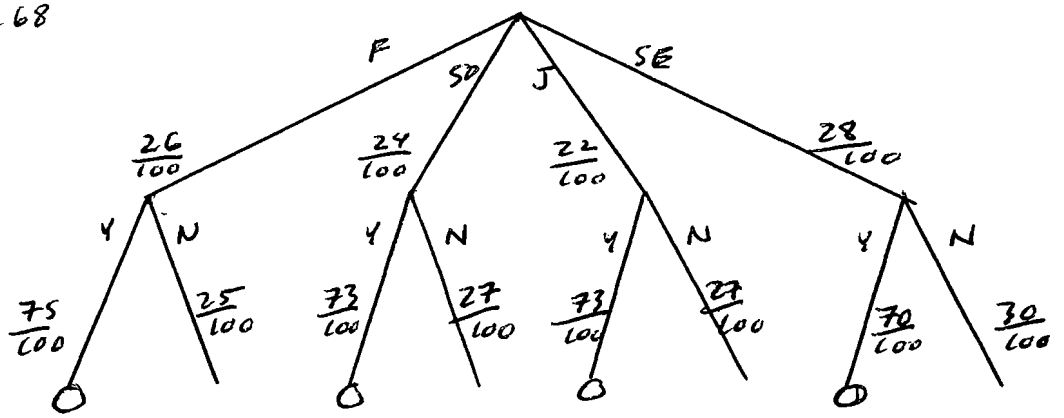
No. 3.4

Date 17

Problem	Ans	Reason

34

$\frac{1950}{7268}$



$$Pr[F \mid \text{state res}] = \frac{\frac{26}{100}\frac{75}{100}}{\frac{26}{100}\frac{75}{100} + \frac{24}{100}\frac{73}{100} + \frac{22}{100}\frac{73}{100} + \frac{28}{100}\frac{70}{100}}$$

$$= \frac{26.75}{26.75 + 24.73 + 22.73 + 28.70}$$

$$= \frac{1950}{1950 + 1752 + 1606 + 1960}$$

$$= \frac{1950}{7268}$$