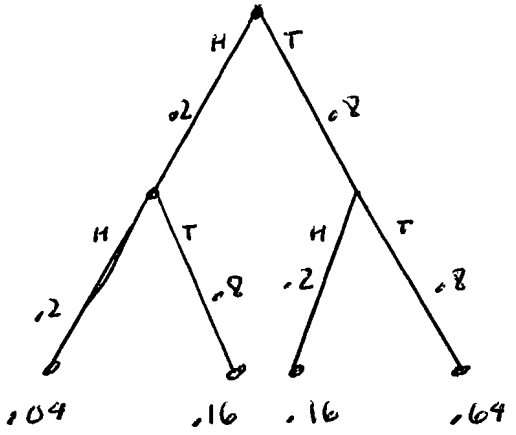
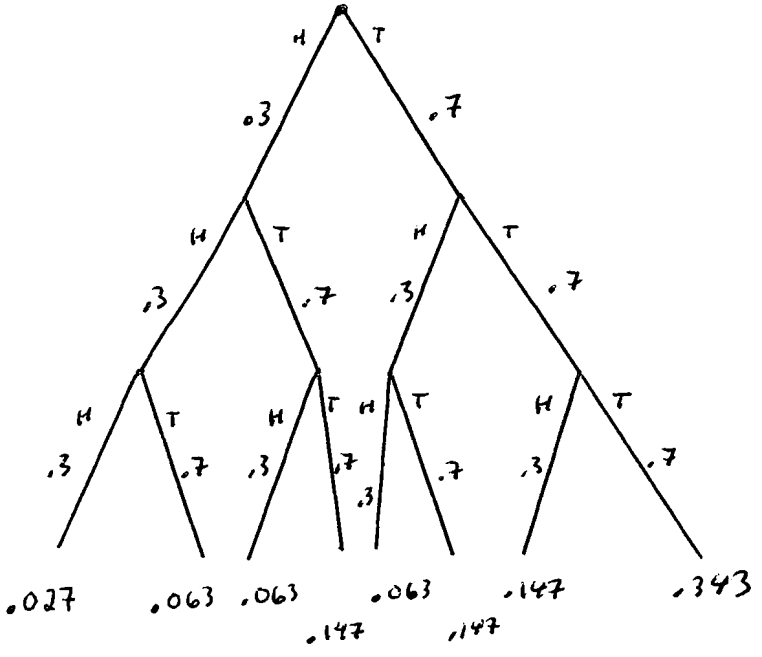
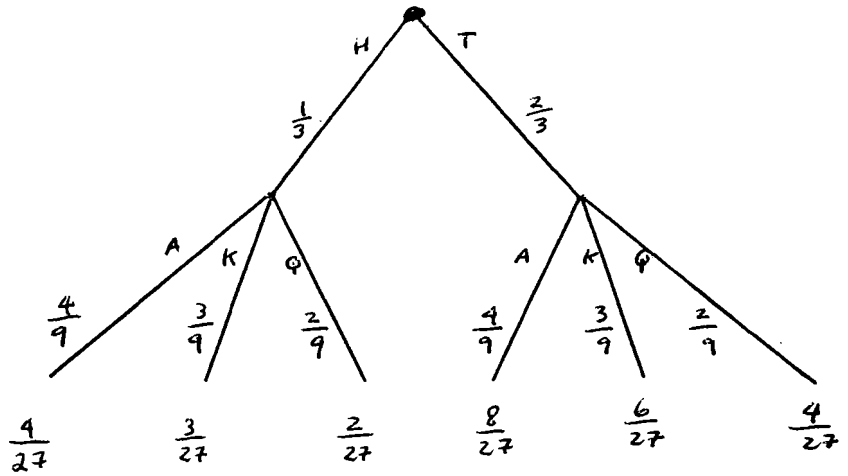
Section 3.3 HW Solutions

Problem	Ans	Reason
1		
2		

Problem	Ans	Reason

3 (a)

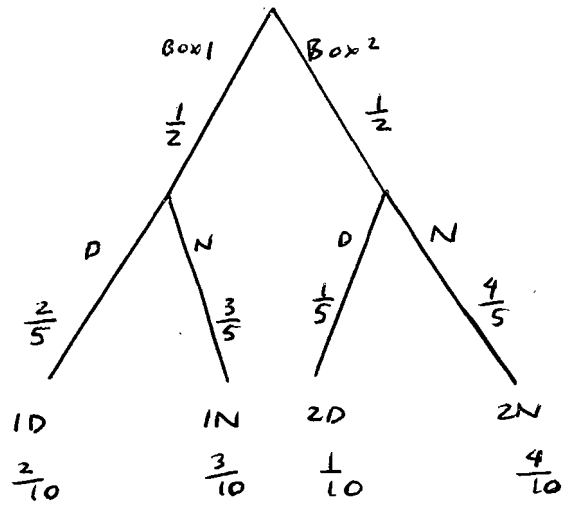


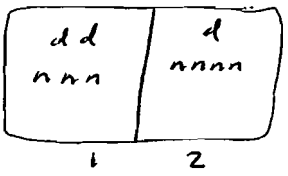
(b) 5/9 Event E =

$$\{ HA, HK, HQ, TK \}$$

$$Pr(E) = \frac{4}{27} + \frac{3}{27} + \frac{2}{27} + \frac{6}{27} = \frac{15}{27} = \frac{3 \cdot 5}{3 \cdot 9} = \frac{5}{9}$$

4 3/10

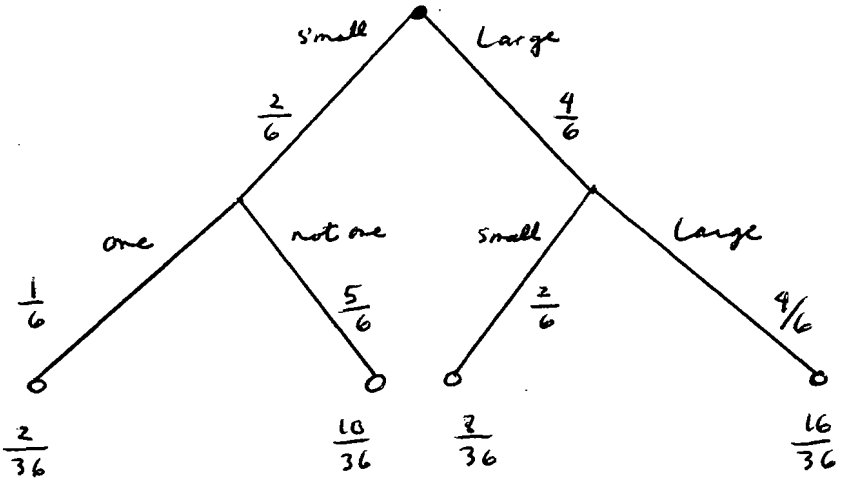




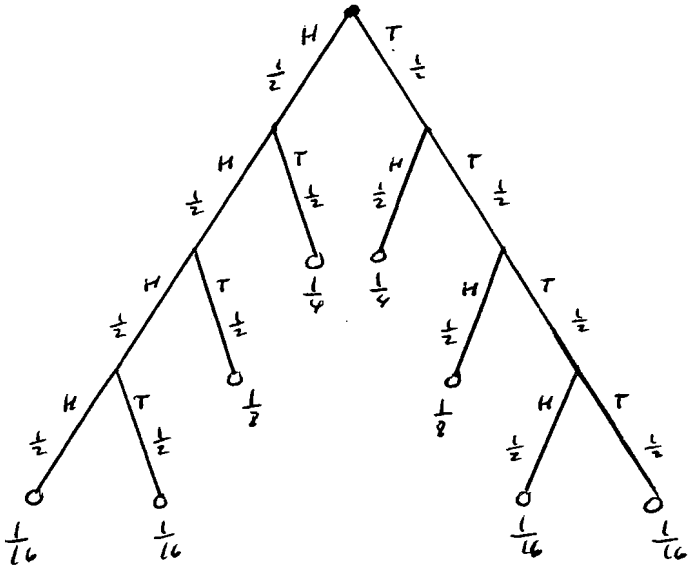
$$Pr\{\text{dime selected}\} = Pr\{1D, 2D\} = \frac{2}{10} + \frac{1}{10} = \frac{3}{10}$$

Problem	Ans	Reason

5

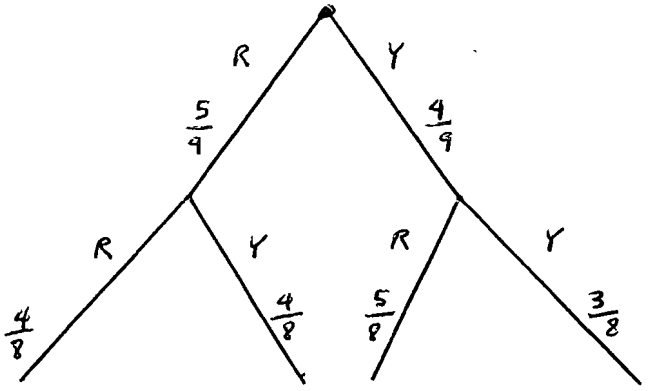


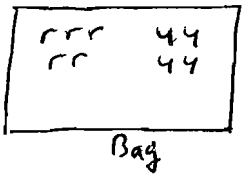
6 1/8



$E = \{ TTH \}$ $Pr(E) = \frac{1}{8}$

Problem	Ans	Reason
7	(a) 5/9	
	(b) 5/6	



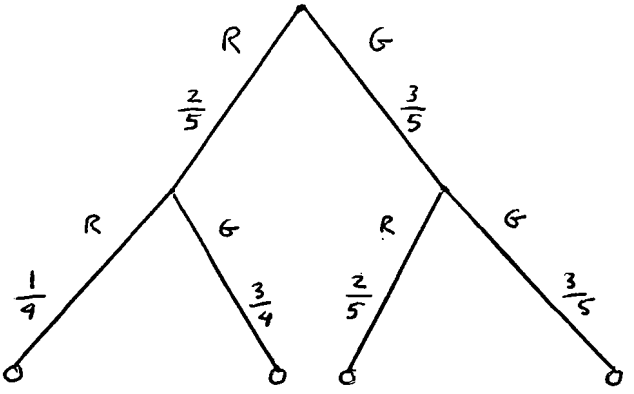


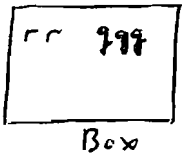
9 apples

$$Pr[\text{Second is red}] = \frac{5}{9}\frac{4}{8} + \frac{4}{9}\frac{5}{8} = \frac{40}{72} = \frac{8\cdot5}{8\cdot9} = \frac{5}{9}$$

$$Pr[\text{at least one red}] = \frac{5}{9}\frac{4}{8} + \frac{5}{9}\frac{4}{8} + \frac{4}{9}\frac{5}{8} = \frac{60}{72} = \frac{5}{6}$$

8	17/50	





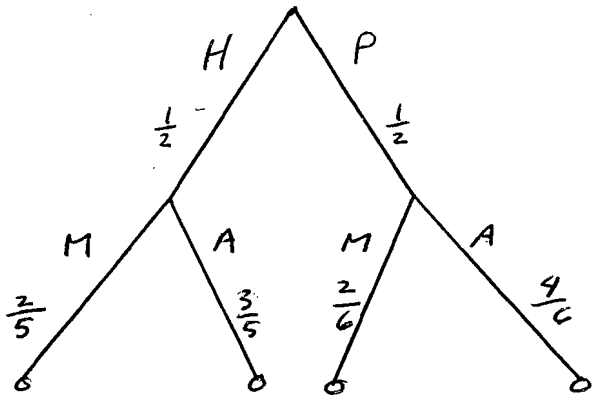
$$Pr[\text{Second ball Red}] = \frac{2}{5}\frac{1}{4} + \frac{3}{5}\frac{2}{5}$$

$$= \frac{2}{5}\left(\frac{1}{4} + \frac{3}{5}\right) = \frac{2}{5}\cdot\frac{17}{20}$$

$$= \frac{17}{5\cdot10} = \frac{17}{50}$$

Problem	Ans	Reason

9 $\frac{19}{30}$

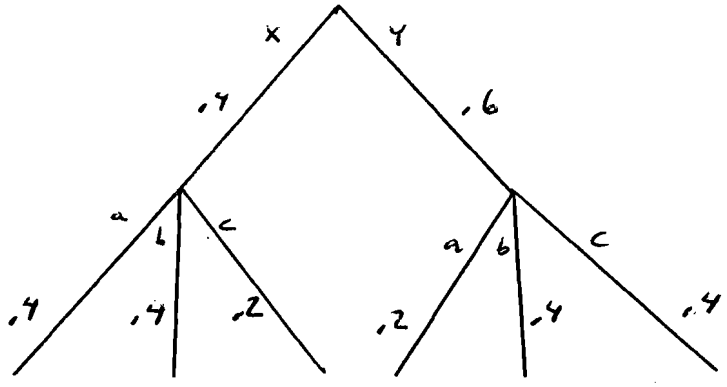


$Pr\left[\text{selected course is in afternoon}\right] = \frac{1}{2}\frac{3}{5} + \frac{1}{2}\frac{4}{6}$

$= \frac{1}{2}\left(\frac{3}{5} + \frac{4}{6}\right)$

$= \frac{1}{2}\frac{19}{15} = \frac{19}{30}$

10 .28

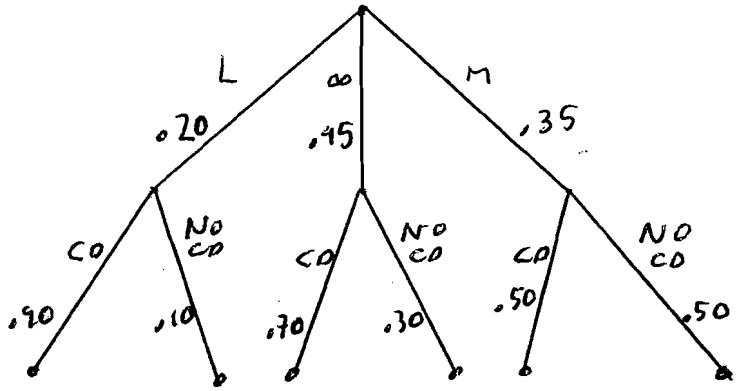


$Pr\left[\text{outcome of stage 2 is a}\right] = (.4)(.4) + (.6)(.2)$

$= .16 + .12$

$= .28$

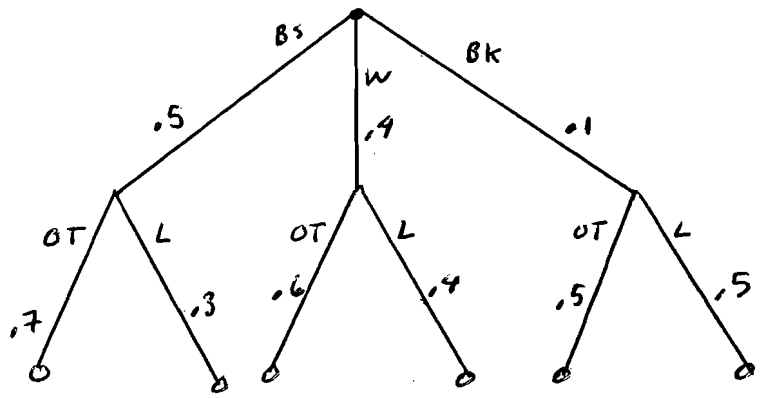
Problem	Ans	Reason
11	(a) .67	
	(b) .845	



$$Pr[\text{car has CO}] = (.20)(.90) + (.45)(.70) + (.35)(.50)$$
$$= .18 + .315 + .175$$
$$= .67$$

$$Pr[\text{M or has CO}] = (.20)(.90) + (.45)(.70) + .35$$
$$= .180 + .315 + .350$$
$$= .845$$

12	.36	



$$Pr[\text{Joe Late}] = (.5)(.3) + (.4)(.4) + (.1)(.5)$$
$$= .15 + .16 + .05$$
$$= .36$$

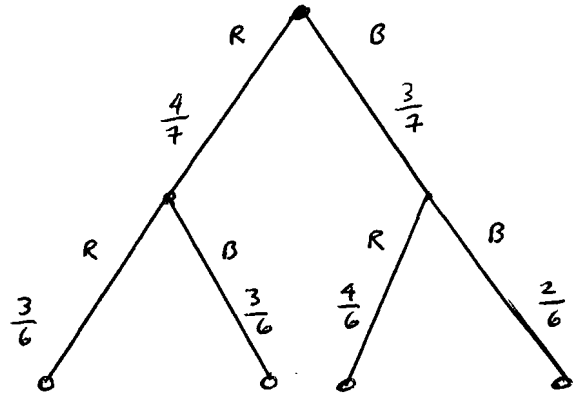
Problem	Ans	Reason

13 (a) $4/7$

(b) $3/5$

rr	bb
rr	b

7 balls in box



$$Pr[\text{exactly one blue}] = \frac{4}{7}\frac{3}{6} + \frac{3}{7}\frac{4}{6} = \frac{4}{7}$$

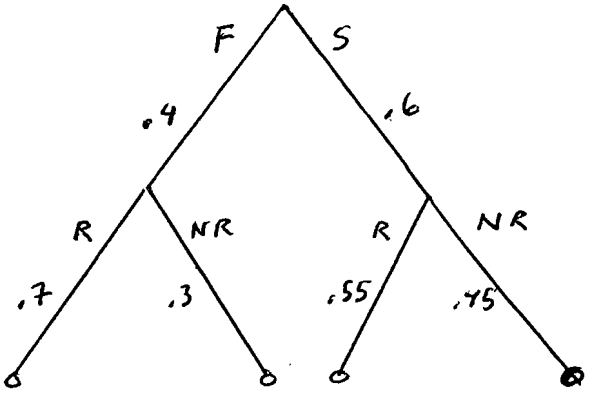
$$Pr[\text{2nd ball blue} \mid \text{at least one blue}] = \frac{Pr[\text{2nd ball blue}]}{Pr[\text{at least one blue}]}$$

$$Pr[\text{2nd ball blue}] = \frac{4}{7}\frac{3}{6} + \frac{3}{7}\frac{2}{6} = \frac{18}{42}$$

$$Pr[\text{at least one blue}] = \frac{4}{7}\frac{3}{6} + \frac{3}{7}\frac{2}{6} + \frac{3}{7}\frac{4}{6} = \frac{30}{42}$$

$$\text{Ans} = \frac{18}{42}\frac{42}{30} = \frac{18}{30} = \frac{6\cdot 3}{6\cdot 5} = \frac{3}{5}$$

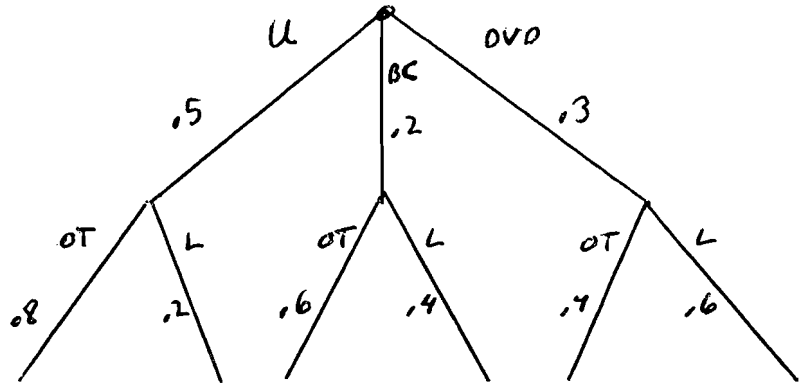
14 .39



$$Pr[\text{student is NR}] = (.4)(.3) + (.6)(.45)$$
$$= .12 + .27$$
$$= .39$$

Problem	Ans	Reason

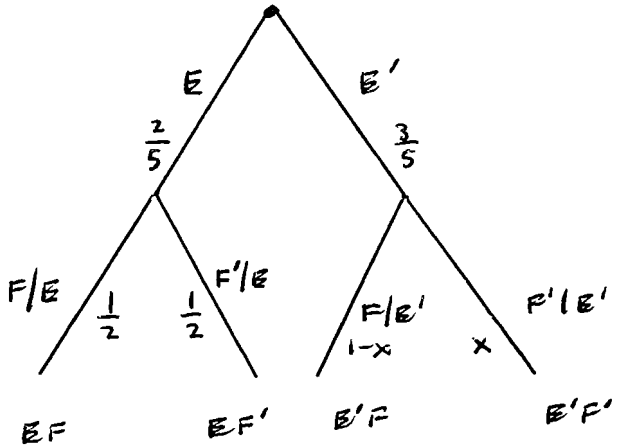
15 .64



$$Pr[\text{on time}] = (.5)(.8) + (.2)(.6) + (.3)(.4)$$
$$= .40 + .12 + .12$$
$$= .64$$

16 $Pr[E] = 3/5$

$Pr[E \cap F] = \frac{1}{5}$

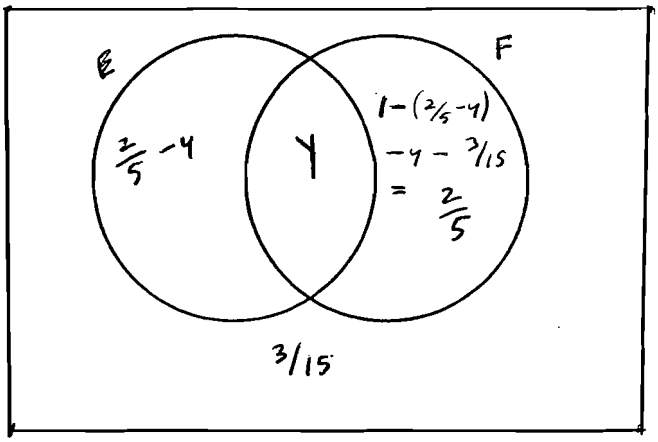


$$\frac{3}{15} = \frac{3}{5} \cdot x \qquad x = \frac{1}{3}$$

$$Pr[F] = Pr[\{EF, E'F\}]$$
$$= \frac{2}{5}\frac{1}{2} + \frac{3}{5}\frac{2}{3} = \frac{3}{5}$$

Problem	Ans	Reason

16, cont



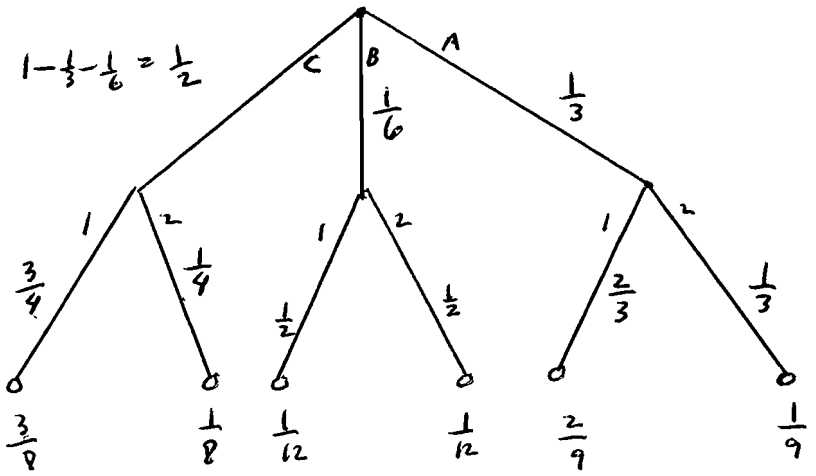
$$\frac{1}{2} = Pr(F|E) = \frac{Pr(E \cap F)}{Pr(E)}$$

$$= \frac{y}{2/5} = \frac{5}{2}y$$

$y = \frac{1}{5}$ so $Pr(E \cap F) = \frac{1}{5}$

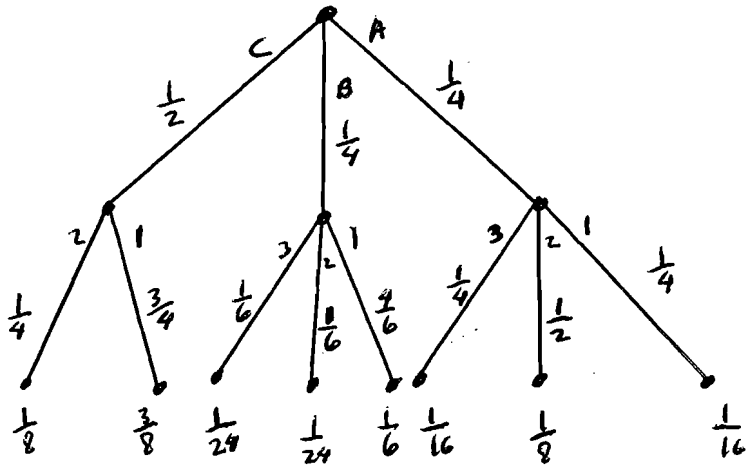
$Pr(F) = y + \frac{2}{5} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

17



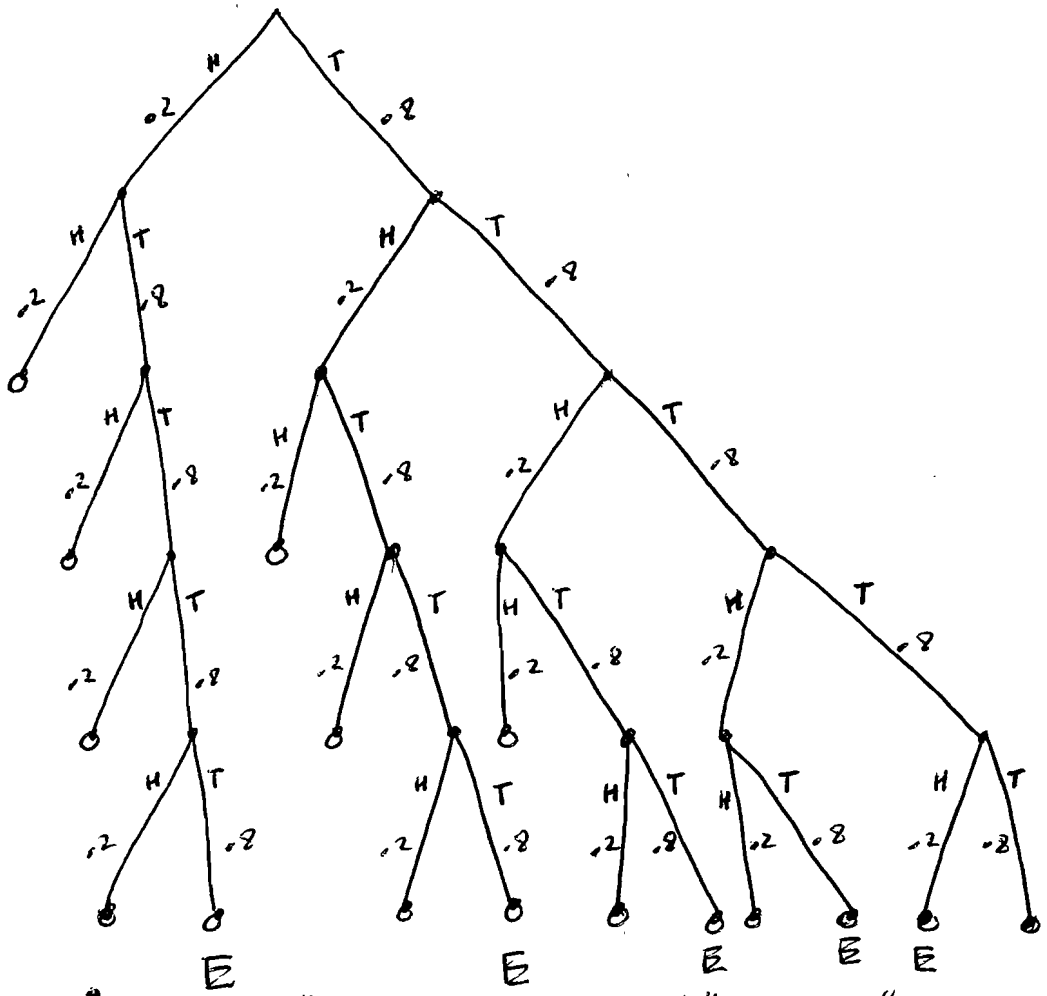
Problem	Ans	Reason

18



$\frac{1}{8} + \frac{3}{8} = \frac{1}{2}$ $\quad 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

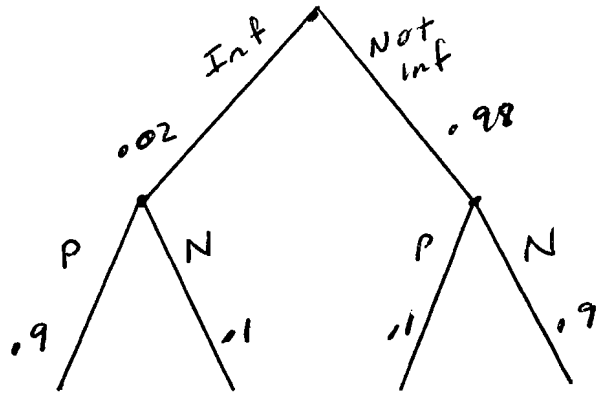
19 .4096



$Pr(E) = (.2)(.8)^4 + (.2)(.8)^4 + (.2)(.8)^4 + .2(.8)^4 + .2(.8)^4 = .4096$

Problem	Ans	Reason

20 .116



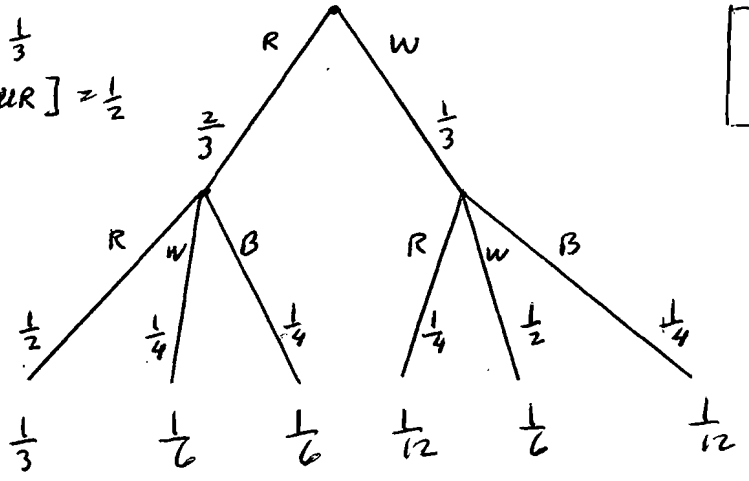
Inf ?

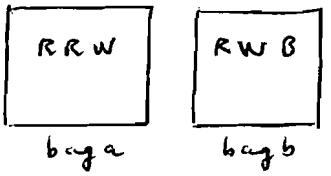
Test

$$Pr[\text{test pos}] = (.02)(.9) + (.98)(.1)$$
$$= .018 + .098$$
$$= .116$$

21 $Pr[RR] = \frac{1}{3}$

$Pr[RR \mid \text{1st ball R}] = \frac{1}{2}$



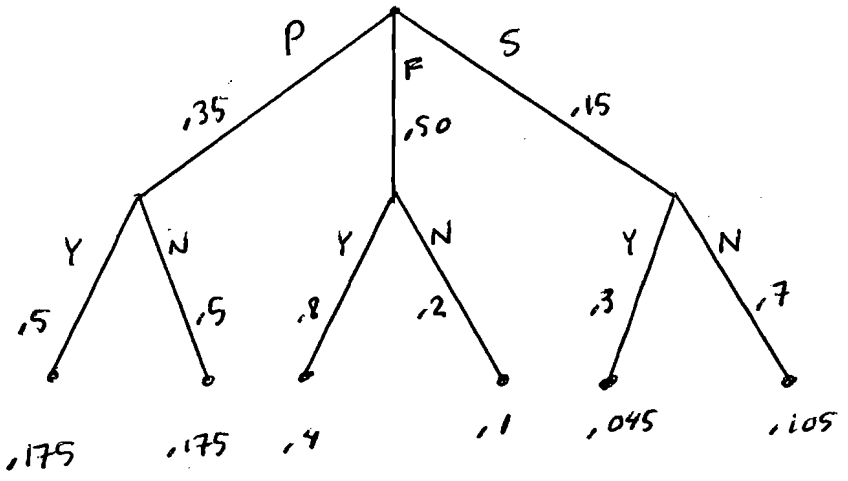


$$Pr[\text{2 red balls}] = \frac{1}{3}$$

$$Pr[\text{2 red} \mid \text{First ball red}] = \frac{1}{2}$$

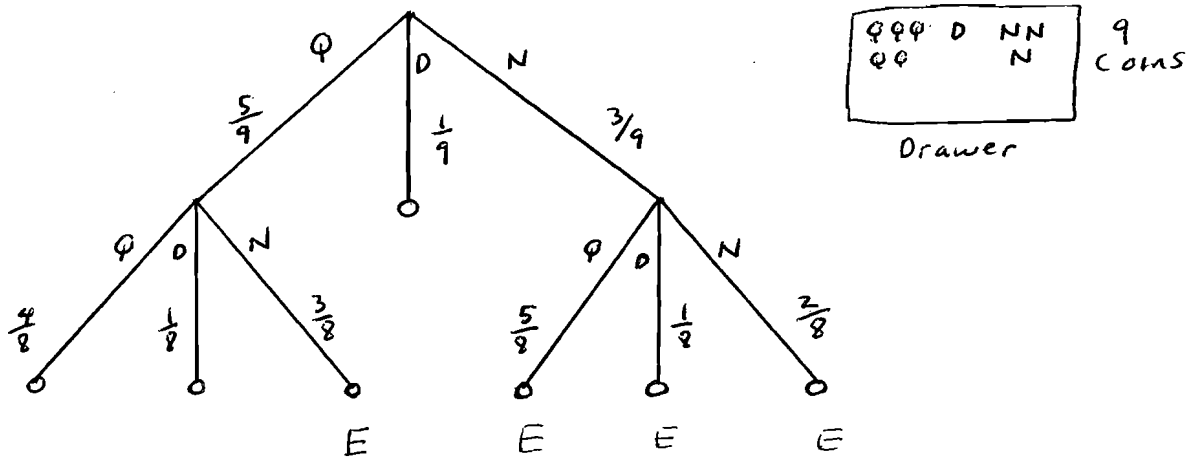
Problem

22 .62



$$Pr[\text{satisfied}] = .175 + .4 + .045$$
$$= .62$$

23 $\frac{13}{24}$



$$Pr(E) = \frac{5}{9}\frac{3}{8} + \frac{3}{9}\frac{5}{8} + \frac{3}{9}\frac{1}{8} + \frac{3}{9}\frac{2}{8}$$

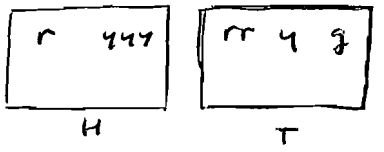
$$= \frac{5}{9}\frac{3}{8} + \frac{3}{9} = \frac{1}{3}\left(1 + \frac{5}{8}\right)$$

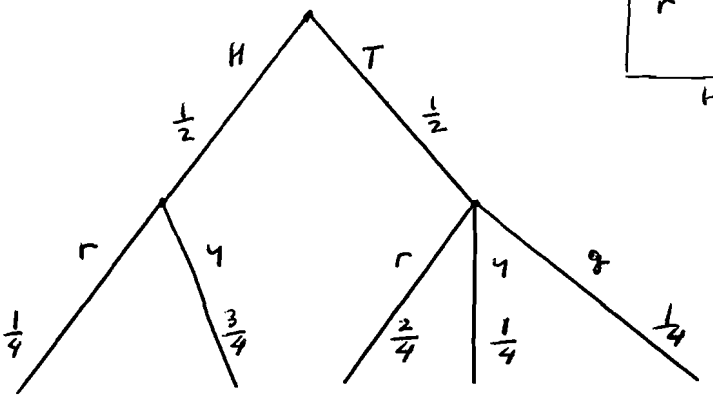
$$= \frac{1}{3}\frac{13}{8} = \frac{13}{24}$$

Problem	Ans	Reason

24 (a)

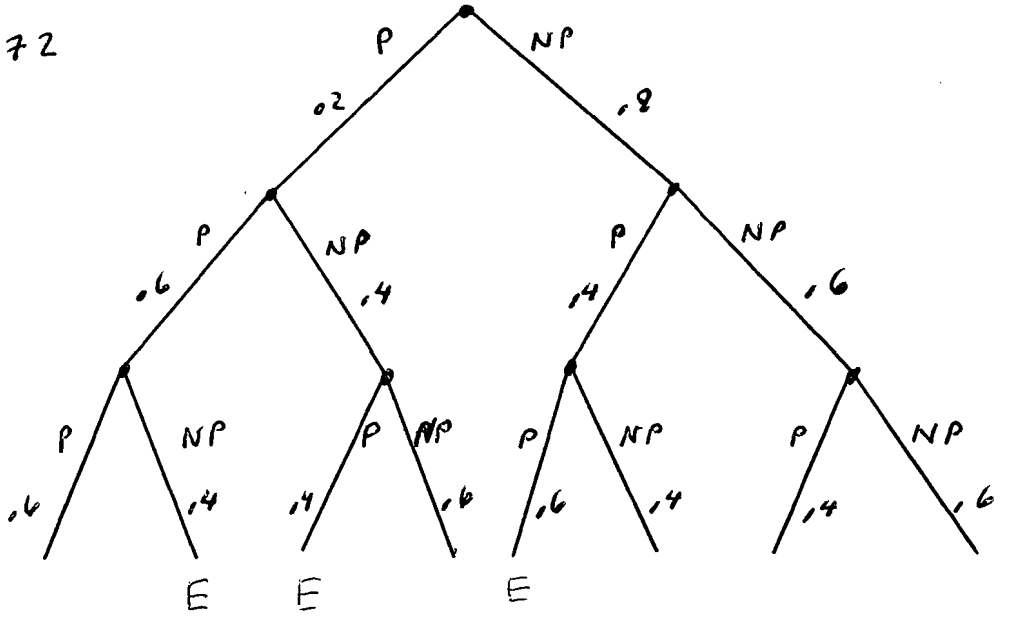
(b) 3/8





$$Pr[\text{r ball drawn}] = \frac{1}{2}\frac{1}{4} + \frac{1}{2}\frac{2}{4} = \frac{3}{8}$$

25 .272



$$Pr(E) = (.2)(.6)(.4) + (.2)(.4)(.4) + (.8)(.4)(.6)$$

$$= .272$$

Problem	Ans	Reason

26 .2496

Event E = {exactly two consecutive heads}

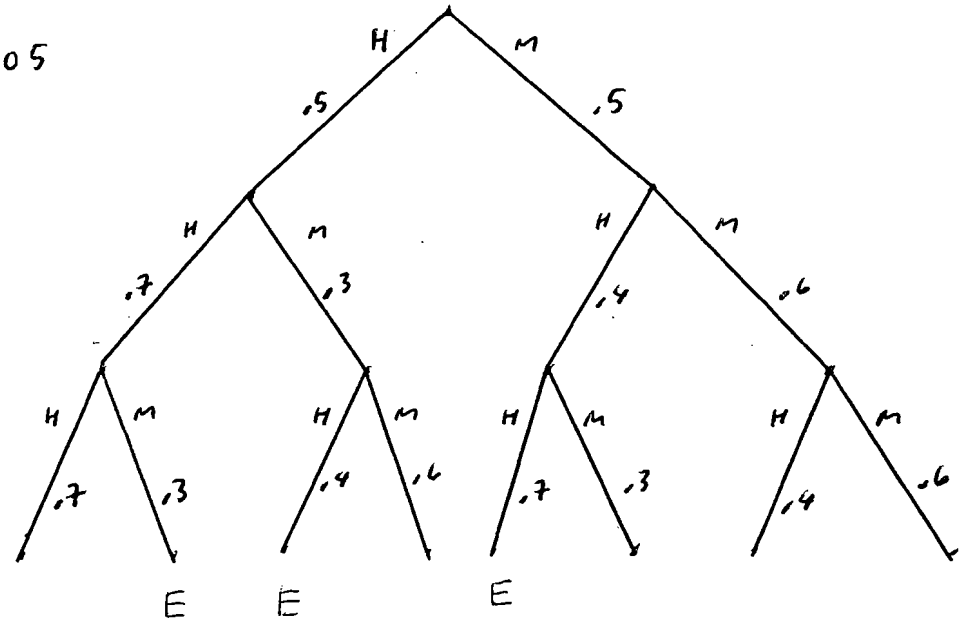
$$E = \{ HHTH,\ HHTT,\ THHT,\ TTHH,\ HTHH \}$$

outcome	HHTH	HHTT	THHT	TTHH	HTHH
wt	$(.4)^3(.6)$	$(.4)^2(.6)^2$	$(.4)^2(.6)^2$	$(.4)^2(.6)^2$	$(.4)^3(.6)$

$Pr(E)$ = sum of wts

= .2496

27 .305

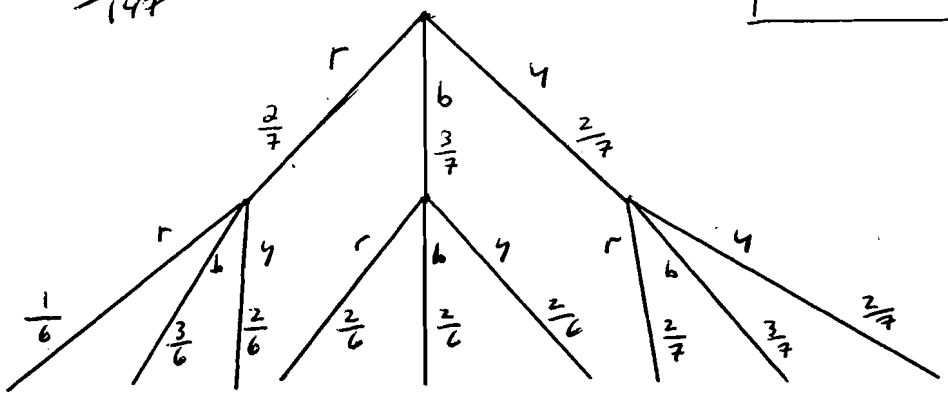


$$E = \{ HHM,\ HMH,\ MHH \}$$

$$Pr(E) = (.5)(.7)(.3) + (.5)(.3)(.4) + (.5)(.4)(.7)$$

$$= .305$$

Problem	Ans	Reason
28	(a) $\frac{47}{147}$	rr bbb yy — 7 balls in box
	(b) $\frac{40}{147}$	



$$Pr[\text{2nd ball yellow}] = \frac{2}{7}\frac{2}{6} + \frac{3}{7}\frac{2}{6} + \frac{2}{7}\frac{2}{7}$$
$$= \frac{2}{7}\left(\frac{2}{6} + \frac{3}{6} + \frac{2}{7}\right)$$
$$= \frac{2}{7}\left(\frac{5}{6} + \frac{2}{7}\right) = \frac{2}{7}\frac{47}{42}$$
$$= \frac{47}{147}$$

$$Pr[\text{2nd ball red}] = \frac{2}{7}\frac{1}{6} + \frac{3}{7}\frac{2}{6} + \frac{2}{7}\frac{2}{7}$$
$$= \frac{2}{7}\left(\frac{1}{6} + \frac{3}{6} + \frac{2}{7}\right)$$
$$= \frac{2}{7}\left(\frac{4}{6} + \frac{2}{7}\right) = \frac{2}{7}\frac{40}{42} = \frac{40}{147}$$

29 $\frac{27}{98}$

$$E = \left\{ \begin{matrix} 15 & 25 & 35 & 45 & 65 & 75 \\ 51 & 52 & 53 & 54 & 56 & 57 \end{matrix} \right\}$$

outcome	15	25	35	45	65	75	51	52	53	54
wt	$\frac{1}{7}\frac{1}{6}$	$\frac{1}{7}\frac{1}{7}$	$\frac{1}{7}\frac{1}{6}$	$\frac{1}{7}\frac{1}{7}$	$\frac{1}{7}\frac{1}{7}$	$\frac{1}{7}\frac{1}{6}$	$\frac{1}{7}\frac{1}{6}$	$\frac{1}{7}\frac{1}{6}$	$\frac{1}{7}\frac{1}{6}$	$\frac{1}{7}\frac{1}{6}$

56	57
$\frac{1}{7}\frac{1}{6}$	$\frac{1}{7}\frac{1}{6}$

$Pr(E)$ = sum of wts = $\frac{27}{98}$

Problem	Ans	Reason

30

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

$$Pr(E|F') = \frac{Pr(E \cap F')}{Pr(F')}$$

so

$$Pr(E|F)\,Pr(F) + Pr(E|F')\,Pr(F')$$

$$= Pr(E \cap F) + Pr(E \cap F')$$

$E \cap F$, $E \cap F'$ are disjoint events so

$$Pr(E \cap F) + Pr(E \cap F') = Pr\left((E \cap F) \cup (E \cap F')\right)$$

$$= Pr\left(E \cap (F \cup F')\right)$$

$$= Pr(E) \quad \checkmark$$

31

total number of hands: $P(36,5) = 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32$

Number of hands with dist ranks:

$36 \cdot 32 \cdot 28 \cdot 24 \cdot 20$ *

Number of hands with ranks in increasing order:

[In *, each of 5! possible orderings equally likely]

$$\frac{36 \cdot 32 \cdot 28 \cdot 24 \cdot 20}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

Ans = $\dfrac{36 \cdot 32 \cdot 28 \cdot 24 \cdot 20}{36 \cdot 35 \cdot 34 \cdot 33 \cdot 32} \quad \dfrac{1}{5!}$

Prob'lem	Ans	Reason
32		Sample space S consists of permutations involving 5 cards from deck of 100 Each equally likely $n(S) = P(100, 5)$ Event E: ranks are consecutive after reordering events in E

ranks	#
1 2 3 4 5	$5! \cdot 10^5$
2 3 4 5 6	$5! \cdot 10^5$
3 4 5 6 7	$5! \cdot 10^5$
4 5 6 7 8	$5! \cdot 10^5$
5 6 7 8 9	$5! \cdot 10^5$
6 7 8 9 10	$5! \cdot 10^5$

$$n(E) = 6! \cdot 10^5$$

$$Pr(E) = \frac{6! \cdot 10^5}{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}$$