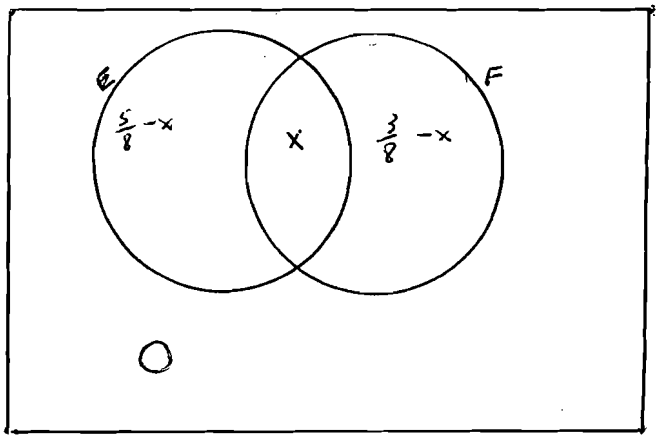
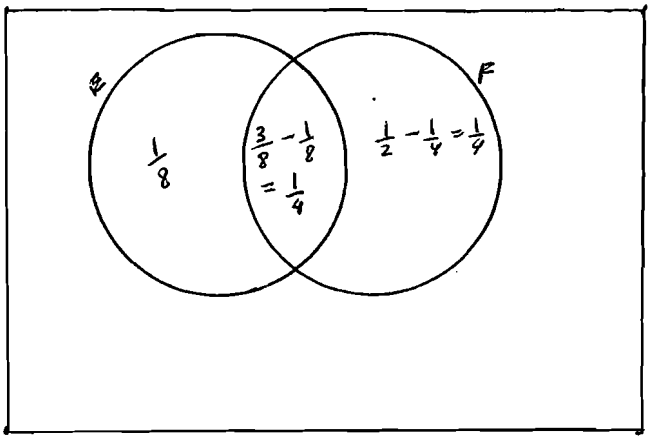
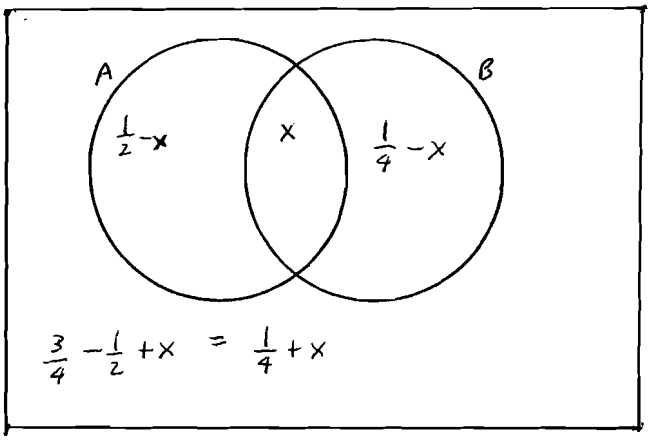
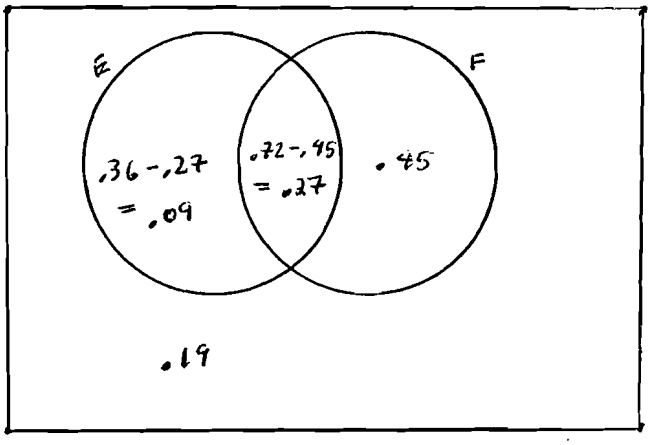
Section 3.2 HW Solutions

Problem	Ans	Reason
1	$Pr(A\|B) = 2/5$ $Pr(B\|A) = 1/2$	$Pr(A\|B) = \frac{Pr(A\cap B)}{Pr(B)} = \frac{.2}{.5} = \frac{2}{5}$ $Pr(B\|A) = \frac{Pr(A\cap B)}{Pr(A)} = \frac{.2}{.4} = \frac{1}{2}$
2	$Pr(E\|F) = \frac{32}{75}$ $Pr(F\|E) = \frac{2}{3}$	$Pr(E\|F) = \frac{Pr(E\cap F)}{Pr(F)} = \frac{.32}{.75} = \frac{32}{75}$ $Pr(F\|E) = \frac{Pr(E\cap F)}{Pr(E)} = \frac{.32}{.48} = \frac{32}{48} = \frac{2}{3}$
3	$Pr(E\|F) = 0$ $Pr(F\|E) = 0$	

$\frac{5}{8} - x + x + \frac{3}{8} - x = 1$

$x = 0$ $\quad Pr(E\cap F) = 0$

Problem	Ans	Reason
4	$Pr(E\|F) = \frac{1}{2}$ $Pr(F\|E) = \frac{2}{3}$	 $Pr(E\|F) = \frac{Pr(E\cap F)}{Pr(F)} = \frac{1/4}{1/2} = \frac{1}{2}$ $Pr(F\|E) = \frac{Pr(E\cap F)}{Pr(E)} = \frac{1/4}{3/8} = \frac{2}{3}$
5	$Pr(B\|A) = 3/8$ $Pr(B\|A') = 1/8$	 $3/4 = Pr(A\|B) = Pr(A\cap B)/Pr(B)$ $= \frac{x}{1/4} = 4x$ $x = 3/16$ $Pr(B\|A) = Pr(A\cap B)/Pr(A) = x/\frac{1}{2} = 2x = \frac{3}{8}$ $Pr(B\|A') = Pr(A'\cap B)/Pr(A') = \frac{\frac{1}{4}-x}{1/2} = \frac{1}{8}$

Problem	Ans	Reason
6	$Pr(E\|F) = 3/8$	
	$Pr(F\|E) = 3/4$	

$$Pr(E|F) = \frac{Pr(E\cap F)}{Pr(F)} = \frac{.27}{.72} = \frac{27}{72} = \frac{9\cdot 3}{9\cdot 8} = \frac{3}{8}$$

$$Pr(F|E) = \frac{Pr(E\cap F)}{Pr(E)} = \frac{.27}{.36} = \frac{27}{36} = \frac{9\cdot 3}{9\cdot 4} = \frac{3}{4}$$

7 $Pr(E|F) = .32$

$Pr(E'|F) = .68$

E, F are indep so $Pr(E\cap F) = Pr(E)\,Pr(F)$

$$Pr(E|F) = \frac{Pr(E\cap F)}{Pr(F)} = \frac{Pr(E)\,Pr(F)}{Pr(F)} = Pr(E)$$

$$Pr(E'|F) = \frac{Pr(E'\cap F)}{Pr(F)} = \frac{Pr(F) - Pr(E\cap F)}{Pr(F)}$$

$$= 1 - Pr(E|F)$$

$$= .68$$

Problem	Ans	Reason

8 $Pr(E'|F) = .6$

$Pr(E|F') = .4$

$$Pr(E'|F) = \frac{Pr(E' \cap F)}{Pr(F)} = \frac{Pr(F) - Pr(E \cap F)}{Pr(F)}$$

$$= \frac{Pr(F) - Pr(E)\,Pr(F)}{Pr(F)} = 1 - Pr(E)$$

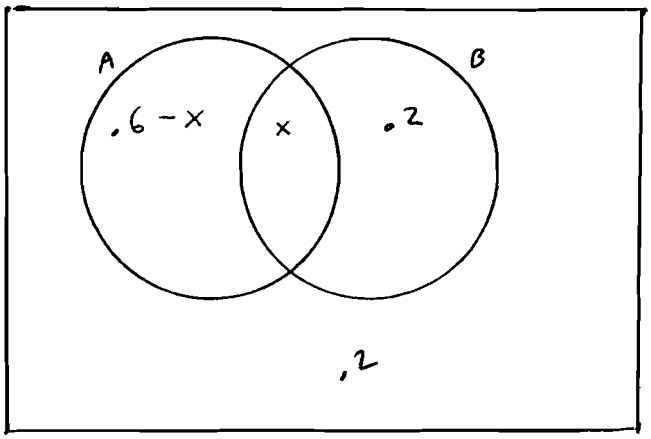
$$= .6$$

$$Pr(E|F') = \frac{Pr(E \cap F')}{Pr(F')} = \frac{Pr(E) - Pr(E \cap F)}{1 - Pr(F)}$$

$$= \frac{Pr(E) - Pr(E)\,Pr(F)}{1 - Pr(F)} = Pr(E) = .4$$

9 (a) $Pr(B) = .5$

(b) $Pr(B) = .2$



A, B indep implies $x = Pr(A)\,Pr(B)$

$= (.6)(x + .2)$

$= .6x + .12$

so $.4x = .12$

$4x = 1.2$

$x = .3$

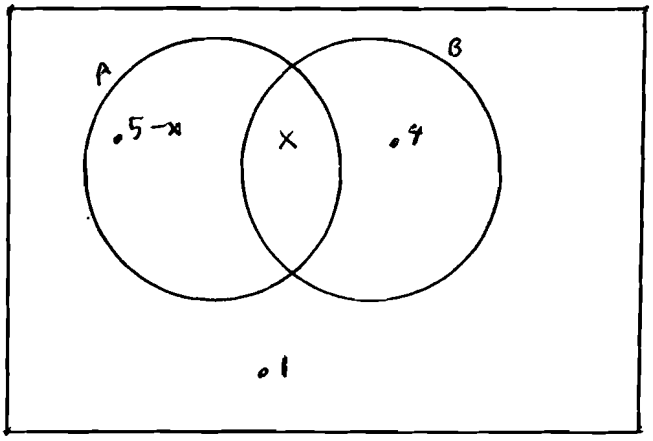
$Pr(B) = x + .2 = .3 + .2 = .5$

A, B disjoint implies $x = 0$ so $Pr(B) = .2$

Problem	Ans	Reason

10 (a) $Pr(B) = .8$

(b) $Pr(B) = .4$



A, B indep implies

$x = Pr(A \cap B) = Pr(A)Pr(B)$

$= (.5)(x + .4)$

$= .5x + .2$

so $.5x = .2$ $\quad 5x = 2$ $\quad x = 2/5 = .4$

$Pr(B) = x + .4 = .4 + .4 = .8$

A, B disjoint implies $x = 0$ so $Pr(B) = .4$

11 A, B NOT indep

$$\frac{1}{2} = Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{\frac{1}{5}}{Pr(B)}$$

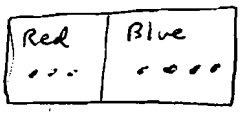
so $Pr(B) = \frac{2}{5}$

$$\frac{1}{3} = Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)} = \frac{\frac{1}{5}}{Pr(A)}$$

so $Pr(A) = \frac{3}{5}$

$$Pr(A \cap B) \overset{?}{=} Pr(A)\,Pr(B)$$

$$\frac{1}{5} \qquad \frac{3}{5} \cdot \frac{2}{5} = \frac{6}{25} \neq \frac{1}{5}$$

Problem	Ans	Reason
12	4/5	 Box

Sample space S: 2-element subsets

$n(S) = C(7,2) = 7 \cdot 6/2 = 21$

elements in S equally likely

event A: exactly one red ball

event B: at least one red ball

$A \subset B$

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A)}{Pr(B)} = \frac{n(A)}{n(B)}$$

$n(A) = 3 \cdot 4 = 12$

$n(B) = n(A) + 3 = 15$

$Pr(A|B) = 12/15 = 4/5$

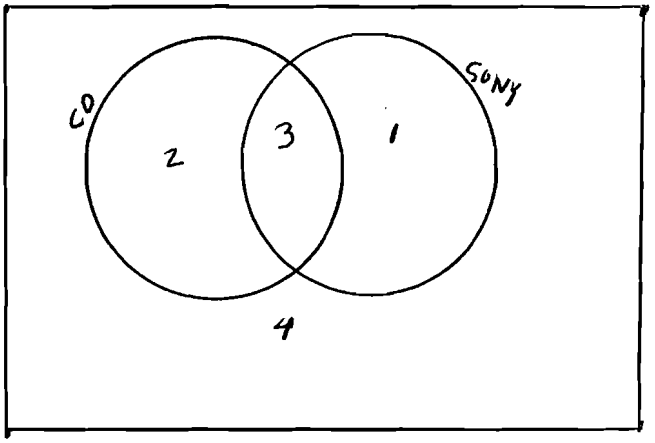
13 Not indep

$Pr(F) = 1/8$

$Pr(B) = 4/8 = 1/2$

$Pr(F \cap B) = Pr(F) = \frac{1}{8}$

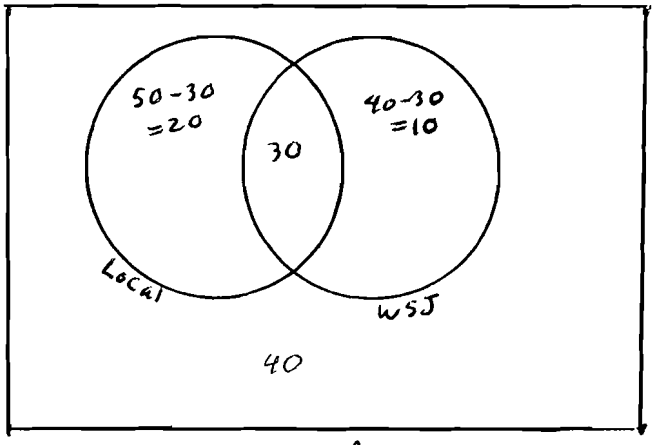
$Pr(F \cap B) \neq Pr(F)\, Pr(B)$

Problem	Ans	Reason
14	3/5	

View IBM = (SONY)'

$$Pr(Sony \mid CD) = \frac{Pr(Sony \cap CD)}{Pr(CD)}$$

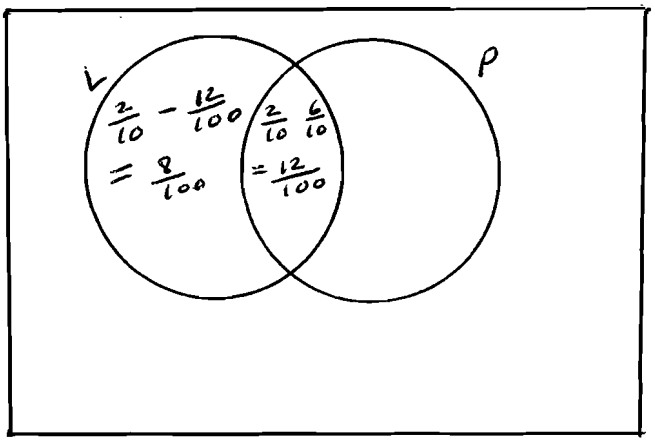
$$= 3/5$$

15	2/3	

L : reads Local paper
E : reads exactly one paper

$$Pr(L \mid E) = \frac{Pr(L \cap E)}{Pr(E)} = \frac{n(L \cap E)}{n(E)}$$

$$= \frac{20}{20+10} = \frac{2}{3}$$

Problem	Ans	Reason
16	$\frac{8}{100}$	
17	(a) $2/5$	
	(b) $1/5$	
	(c) $\frac{2}{11}$	

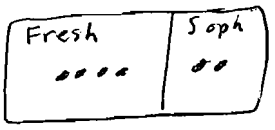
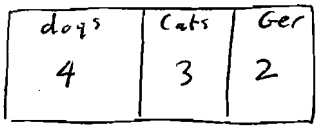
	1	2	3	4	5	6
1	.	.	.	⊙	+	.
2	.	.	.	⊕	.	.
3	.	.	+	⊙	.	.
4	⊙	⊕	⊙	.	⊙	⊙
5	+	.	.	⊙	.	.
6	.	.	.	⊙	.	.

$$Pr(O|+) = \frac{Pr(O \cap +)}{Pr(+)} = \frac{n(O \cap +)}{n(+)} = \frac{2}{5}$$

$$Pr(+|O) = \frac{Pr(O \cap +)}{Pr(O)} = \frac{n(O \cap +)}{n(O)} = \frac{2}{10} = \frac{1}{5}$$

$$O' = O \cup \{(4,4)\} \qquad n(O') = 11$$

$$Pr(+|O') = \frac{Pr(+ \cap O')}{Pr(O')} = \frac{n(+ \cap O')}{n(O')} = \frac{2}{11}$$

Problem	Ans	Reason
18	3/7	 choose 2 Sample space S: 2-element subsets elements of S equally likely Event A: both Fresh Event B: at least one is Fresh $A \subset B$ $Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A)}{Pr(B)} = \frac{n(A)}{n(B)}$ $n(A) = C(4,2) = \frac{4 \cdot 3}{2} = 6$ $n(B) = C(6,2) - 1 = 15 - 1 = 14$ $6/14 = 3/7$
19	$6/10 = 3/5$	 choose 2 Sample space S: 2-element subsets Event A: both are dogs Event B: both same type of animal. $A \subset B$ Each element of S equally likely $Pr(A \mid B) = \frac{Pr(A)}{Pr(B)} = \frac{n(A)}{n(B)}$ $n(A) = C(4,2) = 4 \cdot 3/2 = 6$ $n(B) = C(4,2) + C(3,2) + C(2,2) = 6 + 3 + 1 = 10$

No. 3.2

Date 10

Problem	Ans	Reason
20	(a) 1/18	
	(b) 1/9	

Beef	Ham	Tur
3	2	2

choose 2

Sample space S: 2-element subsets

Event A: at least one not beef

Event B: both turkey $B \subset A$

Event C: both same type: turkey or Ham. $C \subset A$

elements in S equally likely

$$Pr(B|A) = \frac{Pr(B \cap A)}{Pr(A)} = \frac{n(B \cap A)}{n(A)} = \frac{n(B)}{n(A)}$$

$n(A) = C(7,2) - C(3,2) = 21 - 3 = 18$

$n(B) = 1$

$$Pr(C|A) = \frac{Pr(A \cap C)}{Pr(A)} = \frac{Pr(C)}{Pr(A)} = \frac{n(C)}{n(A)}$$

$n(C) = 1 + 1 = 2$ $\quad 2/18 = 1/9$

21 2/3

Chev	Ford
3	4

choose 2

Sample space S: 2-element subsets

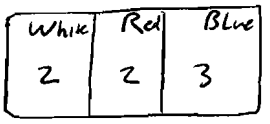
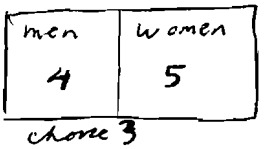
Event A: same make

Event B: both Fords $B \subset A$

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)} = \frac{Pr(B)}{Pr(A)} = \frac{n(B)}{n(A)}$$

$n(B) = C(4,2) = 6$

$n(A) = C(4,2) + C(3,2) = 6 + 3 = 9$ $\quad \frac{6}{9} = \frac{2}{3}$

Problem	Ans	Reason
22	1/6	 Sample space S: 2-element subsets Event A: neither is white Event B: neither is blue Elements in S equally likely $Pr(A\|B) = \frac{Pr(A\cap B)}{Pr(B)} = \frac{n(A\cap B)}{n(B)}$ $n(A\cap B) = 1$ "both red" $n(B) = C(4,2) = 6$
23	2/7	 Sample space S: 3-element subsets Event A: all are male Event B: all same sex $A \subset B$ elements in S equally likely $Pr(A\|B) = \frac{Pr(A\cap B)}{Pr(B)} = \frac{Pr(A)}{Pr(B)} = \frac{n(A)}{n(B)}$ $n(A) = C(4,3) = 4$ $n(B) = C(4,3) + C(5,3) = 4 + 10 = 14$ $4/14 = \frac{2\cdot 2}{2\cdot 7} = 2/7$

Problem	Ans	Reason

24 **3/8**

women	men
6	4

choose 3

Sample Space S: 3-element subsets

Event A: 2 men and 1 woman

Event B: 2 men, 1 woman or 1 man, 2 women

$A \subset B$

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A)}{Pr(B)} = \frac{n(A)}{n(B)}$$

$n(A) = C(4,2) \cdot C(6,1) = 6 \cdot 6 = 36$

$n(B) = 36 + C(6,2) \cdot C(4,1)$

$= 36 + 60 = 96$

$\frac{36}{96} = \frac{6 \cdot 6}{6 \cdot 16} = \frac{6}{16} = \frac{3}{8}$

25 **2/11**

F	S	J
5	3	2

chose 3

sample space S: 3-element subsets

event A: 2F and 1J

event B: 1F or 2F or 3F. $A \subset B$

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A)}{Pr(B)} = \frac{n(A)}{n(B)}$$

$n(A) = C(5,2)C(2,1) = 10 \cdot 2 = 20$

$n(B) = C(10,3) - C(5,3)$

$= 120 - 10 = 110$

$\frac{20}{110} = \frac{2}{11}$

Problem	Ans	Reason

26 $\frac{35}{131}$

Soph	Jun	Sen
8	7	5

20 people

choose 2

sample set S: 2-element subsets

Event A: 1J and 1S

Event B: not both in same class $A \subset B$

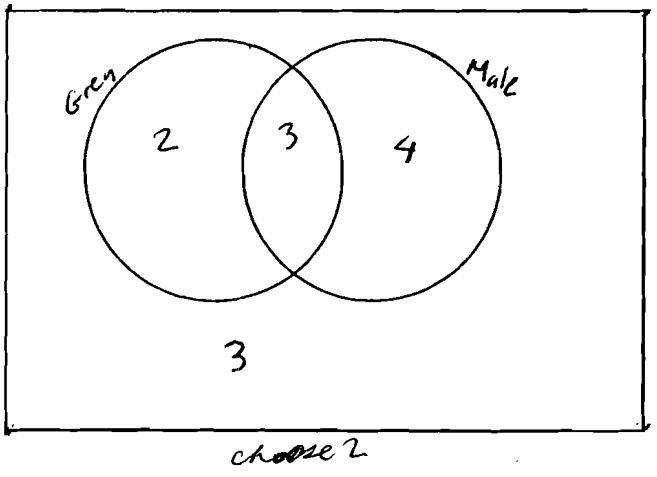
$$Pr(A|B) = \frac{n(A)}{n(B)}$$

$$n(A) = 7 \cdot 5 = 35$$

$$n(B) = C(20,2) - C(8,2) - C(7,2) - C(5,2)$$
$$= 190 - 28 - 21 - 10$$
$$= 190 - 59 = 131$$

27 (a) $1/10$

(b) $3/5$



sample space S: 2-element subsets (of set of 12 mice)

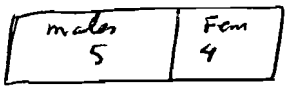
event A: both Female

event B: both Grey

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{n(A \cap B)}{n(B)} = \frac{C(2,2)}{C(5,2)} = \frac{1}{10}$$

event C: 1 male and Female

$$Pr(C|B) = \frac{Pr(C \cap B)}{Pr(B)} = \frac{n(C \cap B)}{n(B)} = \frac{2 \cdot 3}{10} = \frac{3}{5}$$

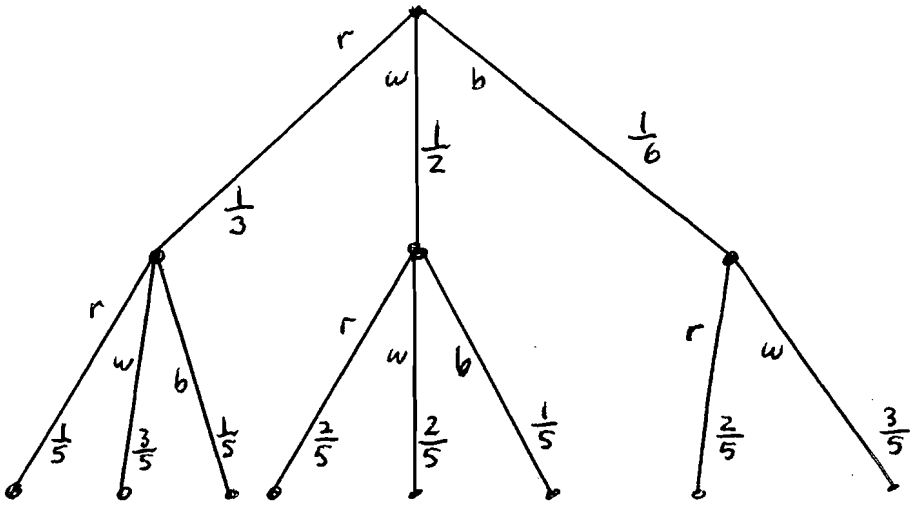
Problem	Ans	Reason
28	(a) 6/7	Event D: 1M or 2M Event E: Both white
	(b) 17/35	$Pr(D/E) = \frac{Pr(D \cap E)}{Pr(E)} = \frac{n(D \cap E)}{n(E)}$ $n(D \cap E) = C(7,2) - C(3,2) = 21 - 3 = 18$ $n(E) = C(7,2) = 21$ $18/21 = \frac{6 \cdot 3}{7 \cdot 3} = 6/7$ Event F: 1 Grey and 1 white $Pr(C/F) = \frac{n(C \cap F)}{n(F)}$ $n(C \cap F) = 3 \cdot 3 + 2 \cdot 4 = 9 + 8 = 17$ $n(F) = 5 \cdot 7 = 35$
29	3/5	 choose 3 males, 2 females Sample space is 5-element subset consisting of 3 males and 2 females Event A: Alex selected Event B: exactly one of Alex, Zelda selected $Pr(A/B) = \frac{n(A \cap B)}{n(B)}$ $A \cap B$: Alex selected and Zelda not $n(A \cap B) = C(4,2)\,C(3,2) = 6 \cdot 3 = 18$ $n(B) = 18 + 12 = 30$ $18/30 = \frac{3 \cdot 6}{3 \cdot 10} = \frac{6}{10} = \frac{3}{5}$

Problem	Ans	Reason

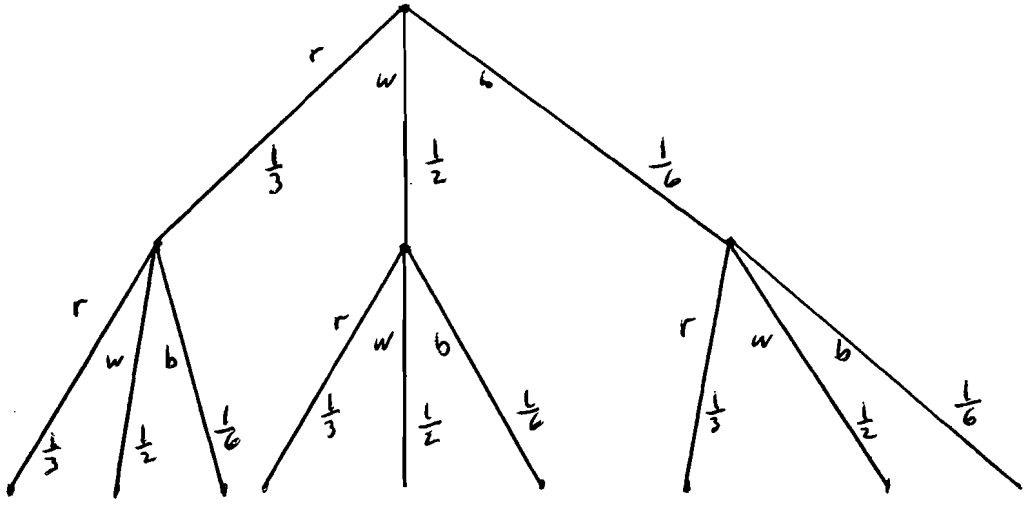
30

Urn

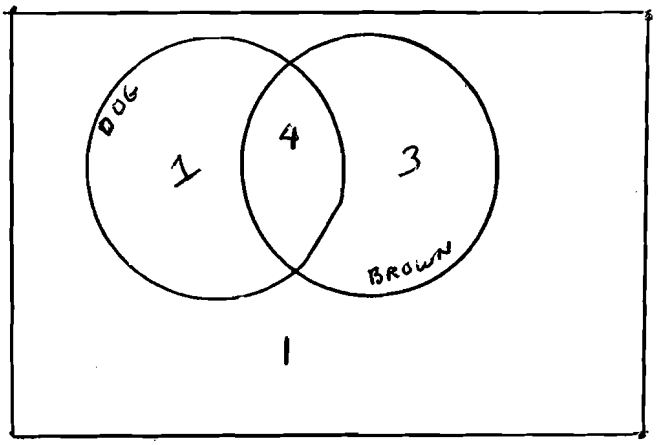
rr	wwww	b



31



Problem	Ans	Reason
32	11/15	



choose 2

View $(DOG)' = CAT$

$(Brown)' = GREY$

Choose 2-element subset

Sample Space S: 2-element subsets of set of 9 animals

Each element equally likely

Event A: at least one is a cat

Event B: at least one is grey

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{n(A \cap B)}{n(B)}$$

$$n(B) = C(9,2) - C(7,2) = 15$$

$A \cap B$: {grey cat + arb} or {grey dog, brown cat}

$$n(A \cap B) = 1 \cdot 8 + 1 \cdot 3 = 11$$

No. 3.2

Date 17

Problem	Ans	Reason
33	1/5	1 2 3 4 5 (seats) / A B C D E (people)

Sample space S: permutations of 5 people

Event X: A to left of B

Event Y: A has position __A__

$$Pr(X|X) = \frac{Pr(X \cap Y)}{Pr(X)} = \frac{n(X \cap Y)}{n(X)}$$

$n(S) = 5! = 120$

$n(X) = n(S)/2 = 60$

$n(X \cap Y) = 2 \cdot 3 \cdot 2 \cdot 1 = 12$

↑ choose remaining blanks

choose Loc of B

__ __ A __ B __

$12/60 = 1/5$

3/10

Event Z: A has position __A___

$Pr(Z|X) = n(X \cap Z)/n(X)$

$n(X \cap Z) = 3 \cdot 3 \cdot 2 \cdot 1 = 18$

$18/60 = 3/10$

34 Vague, skip

35