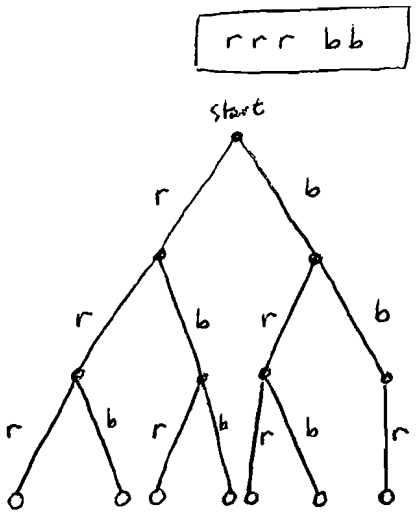
HW Section 2.R

Item	Ans	Reason
1	Sample space = $\{rrr, rrb, rbr, rbb, brr, brb, bbr\}$ $E = \{rrb, rbr, rbb, brr, brb, bbr\}$	 Sample space consists of the paths in the above tree diagram, from the "start" to a "leaf"
2	At most one blue ball: $\{rrr, rrb, rbr, brr\}$ Exactly one blue ball: $\{rrb, rbr, brr\}$	
3	1296	

stage	to do	# choices
1	select color room 1	6
2	" 2	6
3	" 3	6
4	" 4	6

#plans is $6 \cdot 6 \cdot 6 \cdot 6 = 1296$

Problem 4

Ans $w_1 = 1/5$

Reason

Abbreviate $w = w_5$

Outcome	1	2	3	4	5
wt	w	$\frac{1}{2}w$	$\frac{3}{2}w$	w	w

$$1 = w + \frac{w}{2} + \frac{3w}{2} + w + w$$
$$= 5w$$
$$w = 1/5$$

Problem 5

outcome	wt
1	1/12
2	1/12
3	1/4
4	1/4
5	1/12
6	1/4

Abbreviate $w = w_1$

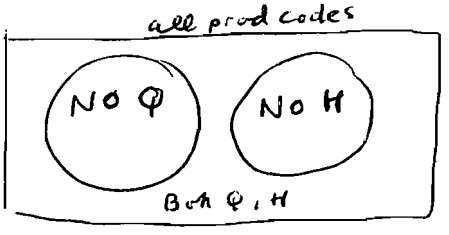
Outcome	1	2	3	4	5	6
wt	w	w	$3w$	$3w$	w	$3w$

$$1 = w + w + w + 3w + 3w + 3w$$
$$= 12w$$
$$w = 1/12$$

Problem 6

Ans 72

Only 5 letters available so each product code contains H or Q or both

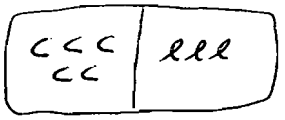


prod codes = $5 \cdot 4 \cdot 3 \cdot 2$

prod codes with no H = $4 \cdot 3 \cdot 2 \cdot 1$

prod codes with no Q = $4 \cdot 3 \cdot 2 \cdot 1$

prod codes with both H, Q = $5 \cdot 4 \cdot 3 \cdot 2 - 4 \cdot 3 \cdot 2 \cdot 1 - 4 \cdot 3 \cdot 2 \cdot 1 = 72$

Problem	Ans	Reason
7	$6 \cdot 4! \cdot 3! \cdot 2!$ $= 1728$	aaaa ccc hh A C H Find number of permutations of A, C, H this is $3 \cdot 2 \cdot 1 = 6$
8	30	

stage	to do	# choices
1	choose 2 Conservatives	$C(5,2) = 10$
2	choose 2 Liberals	$C(3,2) = 3$

$ans = 10 \cdot 3 = 30$

9	(a) 588	

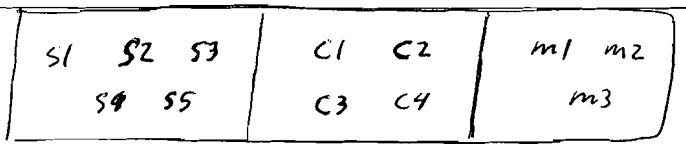
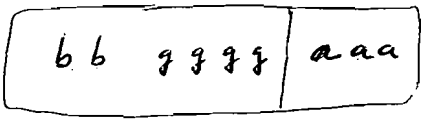
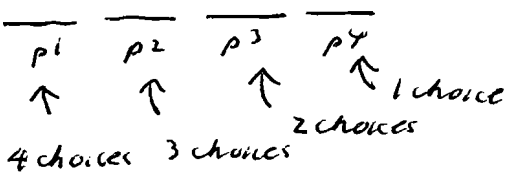
stage	to do	# choices
1	select cone	3
2	select lower scoop	14
3	select upper scoop	14

$3 \times 14 \times 14 = 588$

(b) 546

stage	to do	# choices
1	select cone	3
2	select lower scoop	14
3	select upper scoop	13

$3 \times 14 \times 13 = 546$

Problem	Ans	Reason
10	205	 choose 3, not all in same category total # ways to choose 3 = $C(12,3)$ $= \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 2 \cdot 11 \cdot 10 = 220$ # ways to choose 3 of type S = $C(5,3) = 10$ ... C = $C(4,3) = 4$ ... M = $C(3,3) = 1$ $220 - 10 - 4 - 1 = 205$
11	5/12	 choose 2, not both a $n(S) = C(9,2) = 36$ $n(E) = C(6,2) = 15$ $15/36 = \frac{3 \cdot 5}{3 \cdot 12} = \frac{5}{12}$
12	7/12	$1 - 5/12 = 7/12$
13	24	 $4 \cdot 3 \cdot 2 \cdot 1 = 24$

Problem 14 — Ans: 3360

Reason: $1, 2, 3, \cdots, 16$

choose 1st, 2nd, 3rd

stage	to do	# choices
1	select 1st prize winner	16
2	select 2nd prize winner	15
3	select 3rd prize winner	14

$16 \times 15 \times 14 = 3360$

Problem 15 — Ans: $\frac{1}{10,000} = .0001$

____ ____ ____ ____

↑ ↑ ↑ ↑

10 choices, 10 choices, 10 choices, 10 choices

sequences is $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$

Problem 16 — Ans: (a) 13/36

(b) $6/36 = 1/6$

	1	2	3	4	5	6
1	•	•	⊙	•	⊙	•
2	•	⊙	•	⊙	•	⊙
3	⊙	•	⊙	•	⊙	•
4	•	⊙	•	⊙	•	+
5	⊙	•	⊙	•	+	+
6	•	⊙	•	+	+	+

⊙: sum of 4, 6, 8

is 13

+: sum at least 10

is 6

No. 2, R

Date 6

Problem	Ans	Reason
17	7/18	

	1	2	3	4	5	6
1	'	'	'	'	'	⊙
2	'	'	'	'	'	⊙
3	'	'	'	'	'	⊙
4	'	'	'	'	⊙	⊙
5	'	'	'	⊙	⊙	⊙
6	⊙	⊙	⊙	⊙	⊙	⊙

$n(S) = 36 \qquad n(E) = 14$

$14/36 = 7/18$

18 | 1/10

n n	q q q

choose 2

$n(S) = C(5,2) = 10$

$n(E) = 1$

19 | 25/28

h h h h h	t t t

choose 2, not both t

$n(S) = C(8,2) = 28$

$n(E) = C(8,2) - C(3,2) = 25$

20 | 2^{20}

view as multistage experiment

$$\text{ans} = \underbrace{2 \times 2 \times 2 \cdots \times 2}_{10} \times \underbrace{4 \times 4 \times 4 \times 4 \times 4}_{5}$$

$$= 2^{10} 4^5 = 2^{20}$$

Problem	Ans	Reason
21	2510	Of the group of 12 students, any subset of at most 6 students could occur. $\# = C(12,0) + C(12,1) + C(12,2) + C(12,3) + C(12,4) + C(12,5) + C(12,6)$ $= 1 + 12 + 66 + 220 + 495 + 792 + 924$ $= 2510$
22	$\frac{28}{306}$	Find $n(S)$:

stage	to do	# choices
1	choose desk for tallest student	18
2	choose desk for shortest student	17

$n(S) = 18 \cdot 17 = 306$

Find $n(E)$:

stage	to do	# choices
1	choose tall desk for tallest student	4
2	choose short desk for shortest student	7

$n(E) = 4 \cdot 7 = 28$

Problem	Ans	Reason
23	1/5	

	1	2	3	4	5
1	,	,	,	,	①
2	,	,	,	①	,
3	,	,	①	,	,
4	,	①	,	,	,

0: sum is 6

$n(S) = 4 \cdot 5 = 20$

$n(E) = 4$

$4/20 = 1/5$

24 (a) $\frac{4}{91}$

(b) $\frac{53}{65}$

a a	m m n
a a	m m n
a a	m m n

choose 3

$n(S) = C(15,3)$

(a) $n(E) = C(6,3)$

(b) $n(E) = C(15,3) - C(9,3)$

25 (a) 2024
(b) 20
(c) 816
(d) 1734

# diet	# choices
0	$C(6,3) = 20$
1	$C(18,1)\,C(6,2) = 270$
2	$C(18,2)\,C(6,1) = 918$
3	$C(18,3) = 816$

$C(24,3) = 2024$

No. 2.R

Date 9

Problem	Ans	Reason
26	(a) $\frac{20}{2024}$	
	(b) $\frac{816}{2024}$	
	(c) $\frac{1734}{2024}$	
	(d) $\frac{1188}{2024}$	$\frac{1188}{2024} = 1 - \frac{20}{2024} - \frac{816}{2024}$
27	(a) $\frac{48}{C(52,5)}$	
	(b) $\frac{13048}{C(52,5)}$	
	(c) $\frac{40}{C(52,5)}$	

Problem 28

Ans

(a) $\dfrac{13 \cdot 4 \cdot C(12,2) \cdot 4 \cdot 4}{C(52,5)}$

Reason

stage	to do	# choices
1	3 of kind: pick the kind	13
2	3 of kind: pick suits	$C(4,3)$
3	pick kinds for remaining cards	$C(12,2)$
4	pick suits for rem cards	$4 \cdot 4$

$n(E) = 13 \cdot 4 \cdot C(12,2) \cdot 4 \cdot 4$

$n(S) = C(52,5)$

(b)

$\dfrac{C(13,2)\,C(4,2)^2\,44}{C(52,5)}$

stage	to do	# choices
1	2 pairs: pick the kinds	$C(13,2)$
2	2 pairs: pick the suits	$C(4,2)^2$
3	Last card	44

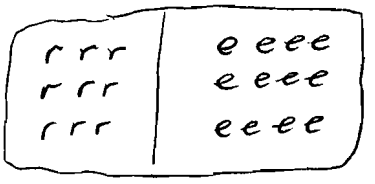
(c)

$\dfrac{13 \cdot C(4,2) \cdot C(12,3)\,4^3}{C(52,5)}$

stage	to do	# choices
1	pair: pick kind	13
2	pair: pick suits	$C(4,2)$
3	pick kinds of rem cards	$C(12,3)$
4	rem cards: pick suits	4^3

Problem	Ans	Reason

29 80262



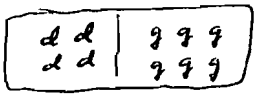
choose 4, 5, 6 not all r

situation	number
Course selections with 4 courses	$C(21,4) - C(9,4)$
Course selections with 5 courses	$C(21,5) - C(9,5)$
Course selections with 6 courses	$C(21,6) - C(9,6)$

Total number of course selections is

$$C(21,4) + C(21,5) + C(21,6) - C(9,4) - C(9,5) - C(9,6) = 80262$$

30 $2/3$



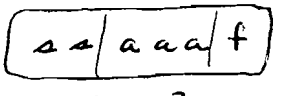
choose 2

$n(S) = C(10,2) = 45$

$n(E) = C(10,2) - C(6,2) = 30$

$$\frac{30}{45} = \frac{2}{3}$$

31 (a) $\frac{1}{5}$



choose 2

$$\frac{C(3,2)}{C(6,2)} = \frac{1}{5}$$

(b) $\frac{4}{5}$

$$1 - \frac{C(3,2)}{C(6,2)} = \frac{4}{5}$$

(c) $\frac{4}{15}$

$$\frac{C(2,2) + C(3,2)}{C(6,2)} = \frac{4}{15}$$

Problem	Ans	Reason

32 — Ans: 3744

stage	to do	# choices
1	2-kind, pick kind	13
2	2-kind, pick suits	$C(4,2) = 6$
3	3-kind, pick kind	12
4	3-kind, pick suits	$C(4,3) = 4$

$13 \times 6 \times 12 \times 4 = 3744$

33 — Ans: $\frac{1}{24}$

$4! = 4 \cdot 3 \cdot 2 \cdot 1$
$= 24$

34 — (a) Ans: $1/6$

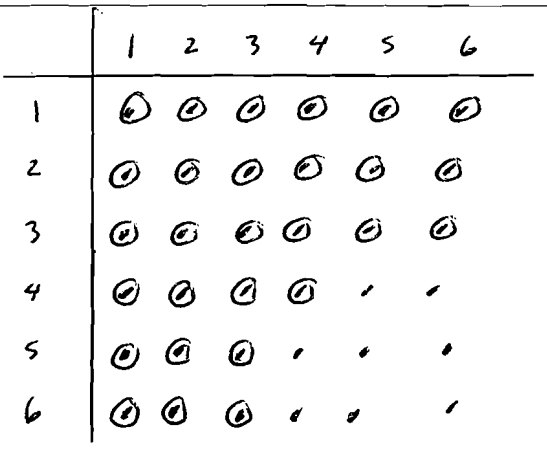
	1	2	3	4	5	6
1	⊙	·	·	·	·	·
2	·	⊙	·	·	·	·
3	·	·	⊙	·	·	·
4	·	·	·	⊙	·	·
5	·	·	·	·	⊙	·
6	·	·	·	·	·	⊙

$\frac{6}{36} = \frac{1}{6}$

(b) Ans: $\frac{1}{2}$

	1	2	3	4	5	6
1	⊙	·	⊙	·	⊙	·
2	·	⊙	·	⊙	·	⊙
3	⊙	·	⊙	·	⊙	·
4	·	⊙	·	⊙	·	⊙
5	⊙	·	⊙	·	⊙	·
6	·	⊙	·	⊙	·	⊙

$\frac{18}{36} = \frac{1}{2}$

Problem	Ans	Reason
34	(c) 7/9	 $\frac{28}{36} = \frac{7}{9}$
35	(a) 180	

stage	to do	# choices
1	pick female	3
2	pick child	2
3	pick 1st male	6
4	pick 2nd male	5

$3 \cdot 2 \cdot 6 \cdot 5 = 180$

(b) 90

Pick Sam but not Heidi

stage	to do	# choices
1	pick female	3
2	pick child	1
3	pick Sams part	2
4	pick other male	5

$3 \cdot 1 \cdot 2 \cdot 5 = 30$

Problem	Ans	Reason

35, cont

Pick Heidi and not Sam

Stage	To do	# choices
1	Pick female	3
2	Pick child	1
3	Pick 1st male	5
4	Pick 2nd male	4

$3 \cdot 1 \cdot 5 \cdot 4 = 60$

$30 + 60 = 90$

(c) $1/6$

Pick both Sam. Heidi

Stage	To do	# choices
1	Pick female	3
2	Pick child	1
3	Pick Sam's part	2
4	Pick other male	5,

$3 \cdot 1 \cdot 2 \cdot 5 = 30$ $\quad 30/180 = 1/6$

36 **0** defects

defects	good widgets
5	45

choose 3

# defects	n(E)
0	$C(45,3)$
1	$C(45,2) \cdot 5$
2	$C(45,1)\,C(5,2)$
3	$C(5,3)$

$C(45,3) > C(45,2) \cdot 5 > C(45,1) \cdot C(5,2) > C(5,3)$

14190 $\quad$ 4950 $\quad$ 450 $\quad$ 10

Problem	Ans	Reason
37	(skip)	
38	(a) $\frac{3n(50-n)(49-n)}{50 \cdot 49 \cdot 48}$	

defective items	good items
n	$50-n$

choose 3

$$n(S) = C(50,3)$$
$$n(E) = n\,C(50-n,2)$$

$$\frac{n(E)}{n(S)} = \frac{n(50-n)(49-n)}{50 \cdot 49 \cdot 48}\,3$$

(b) $n = 16, 17$

Maximize

$$\frac{3n(50-n)(49-n)}{50 \cdot 49 \cdot 48}$$

Maximize

$$n(50-n)(49-n) \quad *$$

at $n = 16, 17$ $*$ is

$16 \cdot 34 \cdot 33$

For $1 \le n \le 49$ we have

$$n(50-n)(49-n) - 16 \cdot 34 \cdot 33$$
$$= (n-16)(n-17)(n-66)$$
$$\le 0 \quad \text{with equality at } n = 16, 17.$$