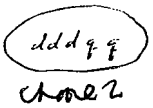
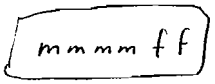
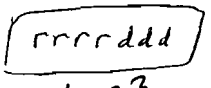
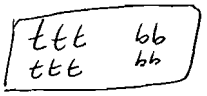
No. 2.4

Date /

Problem	Ans	Reason
		In what follows S denotes the Sample Space and E denotes the event
1	3/10	 chose 2 $n(S) = C(5,2) = 10$ $n(E) = C(3,2) = 3$
2	1/15	 chose 2 $n(S) = C(6,2) = 15$ $n(E) = C(2,2) = 1$
3	$C(13,3)/C(52,3) = \frac{11}{850}$	$n(S) = C(52,3)$ $n(E) = C(13,3)$
4	$1/C(52,4)$	$n(S) = C(52,4)$ $n(E) = C(4,4) = 1$
5	4/35	 chose 3 $n(S) = C(7,3) = 35$ $n(E) = C(4,3) = 4$
6	7/15	 chose 2 $n(S) = C(10,2) = 45$ $n(E) = C(6,2) + C(4,2)$ $= 15 + 6 = 21$ $21/45 = \frac{3 \cdot 7}{9 \cdot 5} = \frac{7}{3 \cdot 5} = \frac{7}{15}$
7	1/216	$n(S) = 6 \times 6 \times 6 = 216$

Problem	Ans	Reason
8	5/72	$E = \{115, 124, 133, 142, 151, 214, 223, 232, 241, 313, 322, 331, 412, 421, 511\}$
		$n(E) = 15$ $\quad \frac{15}{216} = \frac{3 \cdot 5}{3 \cdot 2 \cdot 6 \cdot 6} = 5/72$
	1/2	For each role x and $7-x$ are equally likely
		$x+y+z$ even $\Leftrightarrow$ $7-x+7-y+7-z$ odd
		$n(E) = n(S)/2$
9	3/10	hwppp chpt 3 $\quad n(S) = C(5,3) = 10$
		$n(E) = C(3,1) = 3$
		$n(E) = C(2,1)C(3,2)$
		$= 2 \times 3 = 6$
	3/5	$6/10 = 3/5$
10	7/8	$n(S) = 2^4 = 16$
		$n(E) = 16 - 1 - 1 = 14$
		$14/16 = 7/8$

Problem	Ans	Reason
11	1/28	ff tt tttt $n(S) = C(8,2) = 28$ $n(E) = C(2,2) = 1$
12	2/5	View 1: $n(S) = 5 \cdot 4 \cdot 3$ $n(E) = 2 \cdot 4 \cdot 3$ View 2: $n(S) = 5$ $n(E) = 2$ Focus on first digit
13	125/126	ooooo 44444. $n(S) = C(10,5) = 252$ $n(E) = n(S) - 1 - 1 = 250$ $\frac{250}{252} = \frac{125}{126}$
14	50/63	E : 2 yellow, 3 orange or 3 yellow, 2 orange $n(E) = C(5,2)C(5,3) + C(5,3)C(5,2)$ $= 10 \cdot 10 + 10 \cdot 10$ $= 200$ $\frac{200}{252} = \frac{100}{126} = \frac{50}{63}$
15	10/21	bbbbb wwwww $n(S) = C(10,9) = 210$ $n(E) = C(5,2)C(5,2)$ $= 10 \cdot 10 = 100$ $\frac{100}{210} = \frac{10}{21}$

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Problem	Ans	Reason
16	1/42	$S = \{(a,b,c,d) : a,b,c,d \text{ mutually distinct}, 1 \le a,b,c,d \le 7\}$

$n(S) = 7 \cdot 6 \cdot 5 \cdot 4 = 840$

$E = \{(1,b,3,d) : b \neq d,\ b \neq 1,\ b \neq 3,\ d \neq 1,\ d \neq 3\}$

$n(E) = 5 \cdot 4 = 20$

$\frac{20}{840} = \frac{2}{84} = \frac{1}{42}$

7	1/7	De, Da, f, f, f, f, f	$n(S) = C(7,3) = 35$ $n(E) = C(5,1) = 5$ $5/35 = 1/7$
18	2/9	t t t t m m m m m m m s s s s s s s	focus on tallest student $n(S) = 4 + 7 + 7 = 18$ $n(E) = 4$ $4/18 = 2/9$

Problem	Ans	Reason

19 $5/6$

PPPP
CCC
SS

$$n(S) = C(9,5) = 126$$
$$n(E) = C(4,2)C(5,3) + C(4,3)C(5,2) + C(4,4)C(5,1)$$
$$= 6 \cdot 10 + 4 \cdot 10 + 1 \cdot 5$$
$$= 105$$
$$105/126 = 5/6$$

20 $1/210$

1 2 3 4 5 6 7

$$n(S) = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$$
$$n(E) = 4 \cdot 3$$
$$n(E)/n(S) = 1/7 \cdot 6 \cdot 5 = 1/210$$

21 (a) $3/10$

s | ffff
mm | fff

|||

ttr

$$n(S) = C(10,3)$$
$$n(E) = C(9,2)$$
$$\frac{n(E)}{n(S)} = \frac{9 \cdot 8}{2 \cdot 1} \cdot \frac{3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8} = 3/10$$

(b) $1/10$

$$n(S) = 10$$
$$n(E) = 1$$

22 (a) $7/10$

$$n(S) = C(10,3) = 120$$
$$n(E) = C(10,3) - C(3,3) - C(7,3)$$
$$= 120 - 1 - 35$$
$$= 84$$
$$\frac{84}{120} = \frac{2 \cdot 2 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5} = 7/10$$

(b) $\frac{83}{120}$

$$n(S) = 120$$
$$n(E) = 120 - C(9,2) - 1 = 83$$

↑ ↑ ... from

Problem	Ans		Reason
23	5/8	ddd rr dd rr	rrr : $C(4,3) = 4$ drr : $C(5,1)C(4,2) = 30$ ddr : $C(5,2)C(4,1) = 40$ ddd : $C(5,3) = 10$

$n(S) = 30 + 40 + 10 = 80$

$n(E) = 40 + 10 = 50$

$50/80 = 5/8$

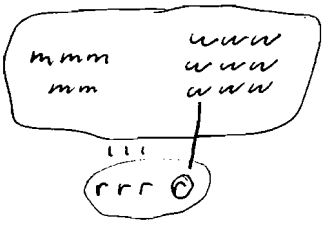
24 $\frac{40}{143}$

$n(S) = C(13,3) \cdot 9$

$n(E) = C(5,2)C(8,1) \cdot 9$

$$\frac{n(E)}{n(S)} = \frac{5 \cdot 4}{2} \cdot \frac{8}{1} \cdot \frac{3 \cdot 2 \cdot 1}{13 \cdot 12 \cdot 11}$$

$$= \frac{5 \cdot 8}{13 \cdot 11} = \frac{40}{143}$$



25 $\frac{7}{36}$

$n(S) = 6 \times 6 = 36$

$n(E) = 5 + 2 = 7$

	1	2	3	4	5	6
1						
2				P		▲
3					▲	
4		P		▲		
5			▲			
6		▲				

Problem	Ans	Reason
26	$\frac{2080}{11440} = \frac{208}{1144}$	rrr, rrw, www, wwb, bbb

$$n(S) = C(16,9) = 11440$$

$$n(E) = C(10,9) + C(6,1)C(10,8) + C(6,2)C(10,7)$$
$$= 10 + 270 + 1800$$
$$= 2080$$

27 $\frac{1}{4}$

$$S = \{w,L\} \times \{w,L\} \times \{w,L\} \times \{w,L\} \times \{w,L\}$$
$$n(S) = 2^5 = 32$$

$E = \{$ wwwww, wwwwL, Lwwwww, wwwLL, LwwwL, LLwww, wwwLw, wLwww $\}$

$n(E) = 8$ $\quad \frac{8}{32} = \frac{1}{4}$

28 $5/14$

$$n(S) = C(8,2) = 28$$
$$n(E) = C(5,2) = 10$$

PPPP bbb f — choose 2

$$10/28 = \frac{5}{14}$$

29 (a) $\frac{10}{21}$ (b) $1/3$

NNN ttt SS

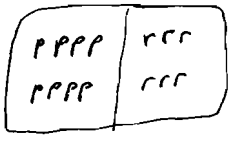
(a) $n(S) = C(9,3) = 84$

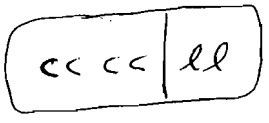
$n(E) = C(4,1)C(5,2)$
$= 4 \cdot 10 = 40$

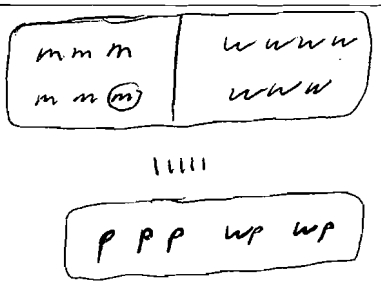
(b) $n(E) = C(8,2) = 28$

$28/84 = 1/3$

Problem	Ans	Reason
30	(a) $1/28$	$n(S) = 84$ $n(E) = C(3,1)C(2,2) = 3$ $3/84 = 1/28$
	(b) $\frac{7}{12}$	$n(S) = 84$ $n(E) = n(S) - C(7,3)$ $= 84 - 35$ $= 49$ $49/84 = \frac{7 \cdot 7}{7 \cdot 12} = \frac{7}{12}$
31	$\frac{3248}{3432} = \frac{406}{429}$	$n(S) = C(14,7) = 3432$ $n(E) = C(8,2)C(6,5) + C(8,3)C(6,4) + C(8,4)C(6,3) + C(8,5)C(6,2)$ $= 7 \cdot 8 \cdot 2 \cdot 29$ $= 3248$
32	$4/5$	$n(S) = C(6,3) = 20$ $n(E) = 20 - C(4,3)$ $= 20 - 4 = 16$ $16/20 = 4/5$





Problem	Ans	Reason
33	(a) 41,580	
		# ways = $7 \cdot 6 \cdot 11 \cdot 10 \cdot 9 = 41,580$
	(b) 3/11	# ways that involve Shawn getting picked $= 7 \cdot 6 \cdot 10 \cdot 9 \cdot 3$
		$\frac{7 \cdot 6 \cdot 10 \cdot 9 \cdot 3}{7 \cdot 6 \cdot 11 \cdot 10 \cdot 9} = \frac{3}{11}$
34	$\frac{1}{720}$	$n(S) = 6! = 720$
35	$n \geq 7$	$n(S) = C(n+3, 3)$
		$n(E) = C(3,3) = 1$
		$Pr(E) = \frac{1}{C(n+3,3)} = \frac{3 \cdot 2 \cdot 1}{(n+3)(n+2)(n+1)}$
		when is $\frac{3 \cdot 2 \cdot 1}{(n+3)(n+2)(n+1)} < \frac{1}{100}$?
36	$n = 1, 2$	$n(S) = C(n+3, 3)$ $\quad n(E) = 3n$
		when is $\frac{3n \cdot 3 \cdot 2 \cdot 1}{(n+3)(n+2)(n+1)} > \frac{1}{2}$?
	$n \geq 11$	when is $\frac{3n \cdot 3 \cdot 2 \cdot 1}{(n+3)(n+2)(n+1)} < \frac{1}{10}$?
	$n \geq 40$	$< \frac{1}{100}$?

Problem	Ans	Reason
37	$1 \leq n \leq 9$	$n(S) = C(3n, 3)$ $n(E) = C(2n, 3)$

when is

$$\frac{C(2n,3)}{C(3n,3)} < \frac{28}{100} \quad ?$$

$$\frac{2n(2n-1)(2n-2)}{3n(3n-1)(3n-2)} < \frac{28}{100} \quad ?$$

$$\frac{2(2n-1)(2n-2)}{3(3n-1)(3n-2)} < \frac{7}{25}$$

$$50(2n-1)(2n-2) < 21(3n-1)(3n-2)$$

$$11n^2 - 111n + 58 < 0$$