HW solutions Section 2.3

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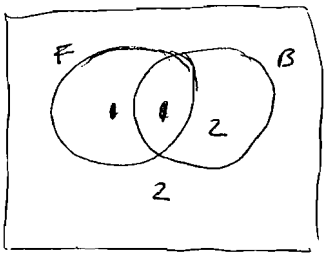
Problem	Ans	Reason
1	$C(5,3) = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1\cdot 2\cdot 1} = 10$	
	$C(7,4) = C(7,3) = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$	
	$C(8,5) = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56$	
	$C(9,2)\times C(9,7) = C(9,2)^2 = \left(\frac{9\cdot 8}{2}\right)^2 = (9\cdot 4)^2 = 36^2 = 1296$	
2	$C(9,2) = 36$	
	$C(9,3) = \frac{9\cdot 8\cdot 7}{3\cdot 2\cdot 1} = 3\cdot 4\cdot 7 = 84$	
	$C(9,4) = \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1} = 9\cdot 2\cdot 7 = 126$	
3	$C(10,5) = 252$	
4 (a)	$84 = C(9,3) > C(10,2) = 45$	
(b)	$C(12,4) < C(18,3)$	
	$=\frac{\cancel{12}\cdot 11\cdot \cancel{10}^{5}\cdot \cancel{9}^{3}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 165$	$\frac{\cancel{18}^{3}\cdot 17\cdot 16}{\cancel{3}\cdot\cancel{2}\cdot 1} = 272$
(c)	$84 = C(9,3) < 6! = 720$	
(d)	$243 = 3^5 > 5! = 120$	

Problem	Ans	Reason
5	$C(6,3) = 20$	subsets have exactly 3 elements
	$C(6,2) + C(6,4) = 15 + 15 = 30$	have 2 or 4 elements
6	$C(9,0) + C(9,1) + C(9,2) + C(9,3) = 1 + 9 + 36 + 84 = 130$	
7	(a) $C(8,2) = 28$	
	(b) $C(8,2) - C(3,2) = 28 - 3 = 25$	
8	$C(9,3) = 84$	
9	(a) $C(5,2) = 10$	
	(b) $C(3,2) = 3$	
	(c) $C(3,1)C(2,1) = 3 \cdot 2 = 6$	
	(d) $C(5,2) - 1 = 10 - 1 = 9$	
10	$4C(13,3) = \frac{4 \cdot 13 \cdot 12 \cdot 11}{3 \cdot 2 \cdot 1} = 4 \cdot 13 \cdot 2 \cdot 11 = 1144$	
	$13C(4,3) = 13 \cdot 4 = 52$	
11	$C(6,3) - C(3,1)C(2,2)$	Can't have d, n, n
	$= 20 - 3 \cdot 1$	
	$= 17$	
12	$C(4,1)C(3,1)C(10,3)$	
	$= 4 \cdot 3 \cdot 120$	
	$= 1440$	

Problem	Ans	Reason
13	(a) $C(9,3) = 84$ (b) $C(9,3) - C(2,3) - C(3,3) - C(4,3)$ $= 84 - 0 - 1 - 4$ $= 79$	
14	$C(6,1)\,C(5,1)\,C(4,1)$ $= 6 \cdot 5 \cdot 4$ $= 120$	
15	(a) $C(3,2)\,C(5,2) = 3 \cdot 10 = 30$ (b) $C(3,2) + C(5,2) = 3 + 10 = 13$ (c) interp 1 veg and 1 fruit : #ways is 30 interp 13 kinds of salad : $C(13,2) = 78$	
16	$C(3,1)\,C(3,1)\,C(5,1)\,C(5,2)$ $= 3 \cdot 3 \cdot 5 \cdot 10$ $= 450$	
17	$C(5,2)\,C(4,2)$ $= 10 \cdot 6$ $= 60$	
18	$C(7,3)\,C(5,2)$ $= 35 \cdot 10$ $= 350$	

Problem	Ans	Reason
19	(a) $C(6,3)\,C(4,2) = 20 \cdot 6 = 120$ (b) $120 + C(\underbrace{6,4}_{15})\,C(\underbrace{4,1}_{4}) = 120 + 60 = 180$	
20	$C(6,2)\,C(3,2) = 15 \cdot 3 = 45$	
21	$3 \times C(6,4) = 3 \cdot 15 = 45$	
22	$9\times9\times9 + 9\times9\times C(9,2)$ $= 9\times9\times9 + 9\times9\times9\times4$ $= 9\times9\times9\times5 = 3645$	
23	(a) $C(14,5)$ (b) $C(14,5) - C(8,5) - C(6,5)$	
24	(a) $C(5,3) = 10$ (b) $C(3,1) = 3$ (c) $10 - 3 = 7$	
25	24 bookwrong	View as $C(6,1)\,C(4,1)$ or $C(10,2) - C(6,2) - C(4,2)$
26	$C(14,7) - C(6,6)\,C(8,1)$ $= C(14,7) - 8$	

Problem	Ans	Reason
27	(a) $C(13,3)C(39,2)$ (b) $4C(13,3)C(39,2)$ (c) $4C(13,2)C(13,1)C(13,1)C(13,1)$	
28	(a) 48 (b) $4C(13,5)$	
29	$6\cdot5\cdot4 - 4\cdot3\cdot2 - 3\cdot2\cdot1$ $= 90$	
30	$9\cdot8\cdot7 - 4\cdot3\cdot2 - 5\cdot4\cdot3 = 420$	
31	$35 = C(7,4)$	
32	$n=2$	$\frac{2n(2n-1)(2n-2)(2n-3)}{4\cdot3\cdot2\cdot1} = \frac{n(n-1)}{2}$ cancel $\Rightarrow n(n-2)=0$
33	$\underbrace{\frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{3\cdot2}} \overset{?}{=} \frac{(n+1)n(n-1)}{3\cdot2}$ $\underbrace{\frac{n(n-1)3}{3\cdot2} + \frac{n(n-1)(n-2)}{3\cdot2}}$ $\frac{n(n-1)(n-2+3)}{3\cdot2}$	
34	$C(n,r) + C(n,r-1) = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$ $= \frac{n!(n-r+1)}{r!(n-r+1)!} + \frac{n!\,r}{r!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$	

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35	$n=1,2,3,4,5$	