

Lec 10 Appendix

More comments about the Onsager Lie algebra $\mathcal{O}$

Given any Lie algebra $\mathcal{L}$ over $\mathbb{F}$ and any
assoc algebra A over $\mathbb{F}$, the corresponding

Loop algebra is the Lie algebra over $\mathbb{F}$

consisting of the vector space

$$\mathcal{L} \otimes A$$

$$\otimes = \otimes_{\mathbb{F}}$$

and Lie bracket

$$[u \otimes a, v \otimes b] = [u, v] \otimes ab$$

$$u, v \in \mathcal{L} \quad a, b \in A$$

We take

$$\mathcal{L} = \mathfrak{sl}_2(\mathbb{F})$$

$$\mathbb{F} \text{ char } 0$$

$$A = \mathbb{F}[t]$$

$$t = \text{indet}$$

We view the elements of $sl_2(\mathbb{F}) \otimes \mathbb{F}[t]$

as 2×2 matrices with entries in $\mathbb{F}[t]$ and trace 0

obs

$sl_2(\mathbb{F}) \otimes \mathbb{F}[t]$ has basis

$$\begin{array}{cccc}
 e \otimes t^i & f \otimes t^i & h \otimes t^i & i \in \mathbb{N} \\
 \parallel & \parallel & \parallel & \\
 \begin{pmatrix} 0 & t^i \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ t^i & 0 \end{pmatrix} & \begin{pmatrix} t^i & 0 \\ 0 & -t^i \end{pmatrix} &
 \end{array}$$

Ex $\exists$ Lie algebra hom

$$\gamma: \mathfrak{g} \rightarrow sl_2(\mathbb{F}) \otimes \mathbb{F}[t]$$

that sends

$$A \rightarrow \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$A^* \rightarrow \begin{pmatrix} -1 & 0 \\ -2t-2 & 1 \end{pmatrix}$$

Moreover

γ is injective.

The map ζ acts on the Θ -basis

$$A_i, \quad B_i, \quad C_{in} \quad i \in \mathbb{N}$$

as follows:

ζ sends

$$A_i \longrightarrow \begin{pmatrix} -t^i & 2t^i \\ 0 & t^i \end{pmatrix}$$

$$B_i \longrightarrow \begin{pmatrix} -t^i & 0 \\ -2t^{in} - 2t^i & t^i \end{pmatrix}$$

$$C_{in} \longrightarrow \begin{pmatrix} t^{in} & 0 \\ 0 & -t^{in} \end{pmatrix}$$

for $i \in \mathbb{N}$