FREE UNIT GROUPS IN ALGEBRAS

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Abstract. Let R be an algebra over a field K, and let G be a finite group of units in R. Suppose that either char $K = 0$ and G is nonabelian, or K is a nonabsolute field of characteristic $\pi > 0$ and $G/\mathbb{O}_\pi(G)$ is nonabelian. Then we show that there are two cyclic subgroups X and Y of G of prime power order, and two special units $u_X \in KX \subseteq R$ and $u_Y \in KY \subseteq R$, such that $\langle u_X, u_Y\rangle$ contains a nonabelian free group. Indeed, we obtain a rather precise description of these units, generalizing an earlier result where $R = K[G]$ was the group algebra of G over K.

1. Introduction

This is a continuation of [GP], and we use the notation of that work. In particular, a group $\mathfrak{G}$ is said to be 2-related if it contains no nonabelian free subgroup. Thus $\mathfrak{G}$ is 2-related if and only if every homomorphism from the 2-generator free group $\mathfrak{F}_2$ into $\mathfrak{G}$ has a nontrivial kernel and hence if and only if every two elements of $\mathfrak{G}$ are related, that is satisfy a nontrivial word in $\mathfrak{F}_2$. Obviously, the property of being 2-related is closed under taking subgroups and homomorphic images.

Now let R be an algebra over a field K, and let G be a finite group of units of R. The goal here is to find a free subgroup in the unit group $\mathrm{U}(R)$. In some sense, the canonical situation occurs when R is the group algebra $K[G]$. In this case, one knows that $\mathrm{U}(R)$ contains a nonabelian free group if and only if either char $K = 0$ and G is nonabelian, or K is a nonabsolute field of characteristic $\pi > 0$ and $G/\mathbb{O}_\pi(G)$ is nonabelian. Here, $\mathbb{O}_\pi(G)$ is the largest normal π-subgroup of G, and K is a nonabsolute field if and only if it is not algebraic over a finite field. For arbitrary algebras R, we know at least that R contains KG, the K-linear span of G, and that KG is a homomorphic image of $K[G]$. As in [GP], we are concerned in this paper with certain rather concrete units. Specifically, these are given by

Definition 1. *Let R be an algebra over a nonabsolute field K, let G be a finite group of units in R, and let $X = \langle x\rangle$ be a cyclic subgroup of G of prime power order. Then we say that $u_X \in \mathrm{U}(KX)$ is a special unit, depending upon the generator x, if one of the following three conditions is satisfied.*

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i. $\operatorname{char} K = 0$ *and* $u_X = (x-r)/(x-s)$ *for suitable integers* $r, s \in \mathbf{Z} \subseteq K$ *with* $r, s \geq 2$.

ii. $\operatorname{char} K = \pi > 0$, $|X|$ *is prime to* π, *and* $u_X = (x-r)/(x-s)$ *for suitable* $r, s \in K$ *that are positive powers of a fixed element* $t \in K$ *transcendental over the prime subfield* $K_0 = \mathrm{GF}(\pi)$.

iii. $\operatorname{char} K = \pi > 0$, X *is a* π*-group,* $t \in K$ *is transcendental over* K_0, *and* $u_X = 1 + t(1 + x + \cdots + x^{\pi-1})$.

In parts (ii) and (iii) above, we say more precisely that u_X is special, based on t. Using this notation, our main result is

Theorem 2. *Let* R *be a* K*-algebra and let* G *be a finite group of units of* R. *Assume that either* K *has characteristic* 0 *and* G *is nonabelian, or that* K *is a nonabsolute field of characteristic* $\pi > 0$ *and* $G/\mathbb{O}_\pi(G)$ *is nonabelian. Then there are two cyclic subgroups* X *and* Y *of* G *of prime power order, and two special units* $u_X \in \mathrm{U}(KX)$ *and* $u_Y \in \mathrm{U}(KY)$ (*based on the same preselected transcendental element if* $\operatorname{char} K > 0$), *such that* $\langle u_X, u_Y \rangle$ *is not* 2*-related.*

Furthermore, we can then obtain

Corollary 3. *Let* R *be a* K*-algebra and let* G *be a finite group of units of* R. *Assume that either* K *has characteristic* 0 *and* G *is nonabelian, or that* K *is a nonabsolute field of characteristic* $\pi > 0$ *and* $G/\mathbb{O}_\pi(G)$ *is nonabelian. Then the subgroup of* $\mathrm{U}(R)$ *generated by units of the form* $x - r$ *with* $x \in G$ *and* $r \in K$ *has a nonabelian free subgroup.*

As we indicated, the previous theorem generalizes the main result of [GP], which concerns the case of group algebras $K[G]$. Not surprisingly, the proof here follows the same basic plan, but some key differences do occur. To start with, if $N \triangleleft G$, then we have an algebra homomorphism $K[G] \to K[G/N]$. But such homomorphisms do not necessarily exist when G is merely assumed to be a subgroup of $\mathrm{U}(R)$. In particular, when we proceed by induction on $|G|$, we can assume properties of proper subgroups of G, but not of homomorphic images. As a consequence, there are more critical groups to be considered here. Second, when the group algebras of these critical groups G are studied, one is free to choose any absolutely irreducible representation of $K[G]$. Here we are forced to consider only those representations that factor through R. Fortunately, all faithful nonlinear irreducible representations of these critical groups are equivalent from our point of view.

As will be apparent, the critical groups that need to be considered here are reasonably close to the ones studied in [GP]. Because of this, it turns out that almost all the necessary hard work has already been done. Thus, for the most part, our proof here will merely describe how the old arguments can be applied to this new context. The key groups of interest are described below. Here, if X is any abelian group, then we let X^p denote its p-power subgroup $\{x^p \mid x \in X\}$.

Lemma 4. *Let G be a finite group, let π be a fixed prime, and suppose that $H/\mathbb{O}_\pi(H)$ is abelian for every proper subgroup H of G. If G is nonabelian, $\mathbb{O}_\pi(G) = 1$, and the center of G is cyclic, then we have the following two possibilities.*

i. (The p-group case) *G is a p-group with $p \neq \pi$, $|G'| = p$, and either $|G| = p^3$, or $G = X \rtimes Y$ where X is cyclic and $|Y| = p$.*

ii. (The pseudo-Frobenius case) *$G = A \rtimes X$, where A is an elementary abelian q-group with $q \neq \pi$, X is cyclic of prime power order p^n with $p \neq q$, and X/X^p acts faithfully and irreducibly on A.*

Proof. Let G be a finite group, let π be a fixed prime, and assume that $H/\mathbb{O}_\pi(H)$ is abelian for every proper subgroup H of G. We first show that such a group G must be solvable and, since this property is inherited by subgroups and homomorphic images, it suffices to show that G cannot be nonabelian simple. Suppose, by way of contradiction that G is nonabelian simple. Then G is not a π-group, so we can choose a prime $p \neq \pi$ dividing $|G|$. If P is any nonidentity p-subgroup of G, then $\mathbb{N}_G(P)$ is proper in G and hence, by assumption, this group has a normal p-complement. But then, Frobenius' theorem implies that G has a normal p-complement, certainly a contradiction. Thus G is solvable, as required.

Now assume in addition that G is nonabelian, $\mathbb{O}_\pi(G) = 1$, and the center of G is cyclic. If G is nilpotent, then $\mathbb{O}_\pi(G) = 1$ implies that G is a π'-group, and hence every proper subgroup of G is abelian. In particular, since G is nonabelian, we see that G must be a p-group for some prime $p \neq \pi$. Furthermore, since all maximal subgroups of G are abelian, it follows easily that $|G : \mathbb{Z}(G)| = p^2$ and that $|G'| = p$. Hence, since $\mathbb{Z}(G)$ is assumed to be cyclic, we see that G' is the unique minimal normal subgroup of G. In other words, every proper homomorphic image of G is abelian, and the result follows from [GP, Lemma 1.4].

Next, suppose that G is not nilpotent and let $F = \mathrm{Fit}(G)$ be the Fitting subgroup of G, namely the largest normal nilpotent subgroup of G. Then $\mathbb{O}_\pi(G) = 1$ implies that F is a π'-group. Furthermore, since G is not nilpotent, F is a proper subgroup of G and hence it is abelian. In particular, since G is solvable, Fitting's theorem implies that $\mathbb{C}_G(F) = F$. Let $G \supseteq G_0 \supseteq F$ with $|G_0/F| = p$ for some prime p. Then G_0 is nonabelian and $G_0' \subseteq F$ is a π'-group, so the minimality of G implies that $G = G_0$. In other words, there exists a cyclic p-group X with $G = FX$ and $X^p \subseteq F$. Now X acts nontrivially on F, so it acts nontrivially on some Sylow q-subgroup Q of F, and then minimality again implies that $G = QX$. Certainly, $q \neq p$ since G is not nilpotent, and hence $G = Q \rtimes X$. We now know that Q is abelian and that X centralizes every proper X-stable subgroup of Q. Since $q \neq p$, this clearly implies first that $Q = A$ is elementary abelian, and second that X/X^p acts faithfully and irreducibly on A. □

In order to handle the pseudo-Frobenius case, we will need

Lemma 5. *Let A be an elementary abelian q-group, and let X be a group of prime order $p \neq q$ which acts faithfully and irreducibly on A. If $\mathrm{Aut}_X(A)$ denotes the group of automorphisms of A that commute with the action of X, then $\mathrm{Aut}_X(A)$ is transitive on the nonidentity elements of A and hence on the nonprincipal linear characters of $K[A]$, when the field K contains a primitive qth root of unity.*

Proof. Since X is cyclic and acts faithfully and irreducibly on the elementary abelian q-group A, the $\mathrm{GF}(q)$-subalgebra of $\mathrm{End}(A)$ generated by X is a finite field $\mathcal{F} = \mathrm{GF}(q^n)$. It follows that $A \cong \mathcal{F}^+$, the additive group of $\mathcal{F}$, and that every $x \in X \setminus 1$ acts on A like multiplication by an element of order p in the multiplicative group $\mathcal{F}^\bullet$. In particular, multiplication by $\mathcal{F}^\bullet$ is contained in $\mathrm{Aut}_X(A)$, and note that $\mathcal{F}^\bullet$ acts regularly on the nonzero elements of $\mathcal{F}^+$. Furthermore, $\mathcal{F}^\bullet$ acts regularly on the nonprincipal linear characters of $K[\mathcal{F}^+]$, when K contains a primitive qth root of unity. □

2. Proof of the Main Theorem

This section is entirely devoted to the proof of Theorem 2 and Corollary 3, and we let K, R and G satisfy the given hypothesis. In particular, R is a K-algebra, G is a finite subgroup of $\mathrm{U}(R)$, and K is a nonabsolute field. Furthermore, either $\mathrm{char}\, K = 0$ and G is nonabelian, or $\mathrm{char}\, K = \pi > 0$ and $G/\mathbb{O}_\pi(G)$ is nonabelian. For convenience, we write $\mathrm{char}\, K = \pi$ in all cases, and if $\pi = 0$, then we define $\mathbb{O}_\pi(G) = 1$. In addition, we let $K' \subseteq K$ denote either the field of rationals in the characteristic 0 case, or the field $\mathrm{GF}(\pi)(t)$ in case $\pi > 0$. In the latter, t is of course a fixed element of K transcendental over $\mathrm{GF}(\pi)$. Since the proof of Corollary 3 follows precisely as in [GP, Proof of Corollary 1.3], we need only prove Theorem 2. For this, we proceed in a series of four steps.

Step 1. *We can assume that*

i. *$H/\mathbb{O}_\pi(H)$ is abelian for every proper subgroup H of G.*
ii. *$R = FG$, where F is any field of our choosing that contains K' and all $|G|$ th roots of unity.*
iii. *R is a full matrix ring over F corresponding to some nonlinear absolutely irreducible representation of the group algebra $F[G]$.*
iv. *$\mathbb{O}_\pi(G) = 1$ and $\mathbb{Z}(G)$ is cyclic.*

Proof. (i) We proceed by induction on $|G|$. If G has a proper subgroup H with $H/\mathbb{O}_\pi(H)$ nonabelian then, by induction, $KH \subseteq KG$ contains special units u_X and u_Y with $\langle u_X, u_Y\rangle$ not 2-related. Thus, it clearly suffices to assume that, for all proper subgroups H of G, we have $H/\mathbb{O}_\pi(H)$ abelian.

(ii) Since $K' \subseteq K$, we see that $K'G$ is a K'-subalgebra of R, and it clearly suffices to replace R by $R' = K'G$. Furthermore, for any field $F \supseteq K'$, we can then consider the F-algebra $\widetilde{R} = F \otimes_{K'} K'G = FG$. Indeed, since the definition of special units involves using field elements only from K', there is no harm in replacing R by $\widetilde{R} = FG$. In particular, $R = FG$ is now a homomorphic image of the group algebra $F[G]$ and, if we let F contain all $|G|$ th roots of unity, then it follows that all irreducible representations of R are absolutely irreducible.

(iii)(iv) Let J be the Jacobson radical of $R = FG$, and suppose that R/J is commutative. If G' denotes the commutator subgroup of G, then we must have $\{1 - g \mid g \in G'\}$ contained in the nilpotent ideal J. It then follows that G' is a normal π-subgroup of G, contradicting the fact that $G/\mathbb{O}_\pi(G)$ is assumed to be nonabelian. Note that if $\pi = 0$, then $F[G]$ is a Wedderburn ring, and hence so is R.

It follows that R/J is noncommutative, and hence R has an irreducible representation of degree $m > 1$. Thus, since all irreducible representations of R are absolutely irreducible, we have an epimorphism $^-\colon R \to \mathrm{M}_m(F)$. Of course, $\overline{G} \subseteq \mathrm{U}(\overline{R})$ and $\overline{R} = F\overline{G}$. Consequently, since $\mathbb{O}_\pi(\overline{G})$ is contained in the kernel of all absolutely irreducible representations of $F[\overline{G}]$, we have $\mathbb{O}_\pi(\overline{G}) = 1$ and also $\mathbb{Z}(\overline{G})$ is cyclic. Now, as in [GP, Proof of Theorem 1.2], any special unit $u_{\bar{X}}$ of $\overline{R}$ lifts to a special unit u_X of R. In particular, if $\overline{R}$ contains special units $u_{\bar{X}}$ and $u_{\bar{Y}}$ which generate a group that is not 2-related, then R must also contain two special units u_X and u_Y which generate a group that is not 2-related. With this and induction on $|G|$, it suffices to assume that $R = \overline{R}$ and that $G = \overline{G}$. □

For the remainder of the argument, we let F be the field given by [GP, Lemma 3.2] based on $n = |G|$, and we assume that R and G enjoy all the properties given above. In particular, $R = \mathrm{M}_m(F)$, G is a subgroup of $\mathrm{U}(R)$, and $R = FG$. Furthermore, by Lemma 4, there are two possible structures for G, the p-group case and the pseudo-Frobenius case. We use the description of G and the notation as given by that lemma. In addition, we let $\theta\colon F[G] \to R$ denote the given absolutely irreducible representation of $F[G]$. Of course, θ is faithful on G.

Step 2. *The result holds in the p-group case.*

Proof. Note that the groups G given here are precisely the same as the minimal nonabelian p-groups studied in [GP, Lemma 1.4]. Thus, since θ is faithful on G, it follows that $\theta(G' \setminus 1)$ consists of nonidentity scalar matrices. In particular, if $G = X \rtimes Y$, where X is a cyclic subgroup of index p, then Frobenius Reciprocity implies that $\theta = \rho^G$ is induced from a faithful linear character ρ of X. With this, [GP, Lemma 3.3, Case 1] yields the result. On the other hand, since θ is certainly a nonlinear representation, the result for the remaining nonabelian groups of order p^3 follows immediately from [GP, Lemma 3.3, Cases 2 and 3]. □

Finally, we consider the pseudo-Frobenius case. Here G is the semi-direct product $A \rtimes X$, where A is an elementary abelian q-group with $q \neq \pi$, and where X is a cyclic p-group with $p \neq q$. Furthermore, X/X^p acts faithfully and irreducibly on A.

Step 3. *The result holds in the pseudo-Frobenius case with $p \neq \pi$.*

Proof. Let $X = \langle x\rangle$ and observe that x^p is central in G and hence in $R = FG$. Thus $x^p = \kappa_0 I$, where I is the identity matrix and κ_0 is a primitive $|X^p|$ th root of unity. Since F contains all $|X|$ th roots of units, there exists $\kappa \in F$ with $\kappa^p = \kappa_0$. In particular, if we set $\bar{x} = x\kappa^{-1}$, then $\bar{x}^p = 1$ and $\overline{X} = \langle\bar{x}\rangle$ is a group of order p. Furthermore, $\overline{X}$ acts on A in the same manner as X/X^p, so $\overline{G} = A \rtimes \overline{X}$ is a Frobenius group contained in $\mathrm{U}(R)$. Of course, the natural map $\bar{\theta}\colon F[\overline{G}] \to R$ is an absolutely irreducible representation of $F[\overline{G}]$. Hence, by [GP, Lemma 1.5], $\deg\bar{\theta} = p$ and $\bar{\theta}_{\bar{X}}$ is the regular representation of $F[\overline{X}]$, so all the p distinct linear characters $\bar{\lambda}_i\colon F[\overline{X}] \to F$, with $i = 1, 2, \ldots, p$, occur as constituents of $\bar{\theta}_{\bar{X}}$, each with multiplicity 1. Thus, since $x = \kappa\bar{x}$, the p linear characters of $F[X]$ given by $\lambda_i = \kappa\bar{\lambda}_i$, where $\lambda_i(x) = \kappa{\cdot}\bar{\lambda}_i(\bar{x})$, are the constituents of the restriction θ_X. In other words, there is a one-to-one correspondence between the p distinct eigenfunctions $\bar{\mu}$ of $\overline{X}$ and the p distinct eigenfunctions μ of X given by $\mu = \kappa\bar{\mu}$. Furthermore, the idempotents $e_\mu \in FX$ and $\bar{e}_{\bar{\mu}} \in F\overline{X}$ associated with these corresponding eigenfunctions are identical.

Suppose first that $\operatorname{char} F = 0$. Then, by the work of [GP, Lemma 3.4 and Proposition 3.5], there exists $1 \neq a \in A$ and an eigenfunction $\bar{1} \neq \bar{\lambda}$ of $\overline{X}$ such that $\bar{e}_{\bar{\mu}} a \bar{e}_{\bar{\eta}} \neq 0$ and $\bar{e}_{\bar{\mu}} a^{-1} \bar{e}_{\bar{\eta}} \neq 0$ in R, for all $\bar{\mu}, \bar{\eta} \in \{\bar{1}, \bar{\lambda}\}$. Consequently, $e_\mu a e_\eta \neq 0$ and $e_\mu a^{-1} e_\eta \neq 0$ in R, for all $\mu, \eta \in \{\kappa\bar{1}, \kappa\bar{\lambda}\}$. Now observe that $f_\eta = a^{-1} e_\eta a$ is an idempotent in $FY \subseteq R$, where $Y = a^{-1}Xa$, and that the preceding inequations imply that $f_\eta e_\mu \neq 0$ and $e_\mu f_\eta \neq 0$ for all $\mu, \eta \in \{\kappa\bar{1}, \kappa\bar{\lambda}\}$. In other words, we have verified the necessary idempotent condition, and the argument of [GP, Lemma 3.4] now yields the result.

On the other hand, suppose $\operatorname{char} F = \pi > 0$ and let $t \in F$ be the given transcendental element. We work in the group algebra $F[\overline{G}]$ and follow the argument of [GP, Proposition 3.7]. In particular, using the trace notation of [GP], there exists a nonprincipal linear character $\bar{\lambda}\colon F[\overline{X}] \to F$ and an element $1 \neq a \in A$ with

$$\tau = (\operatorname{tr}_{\bar{1}} a)(\operatorname{tr}_{\bar{\lambda}} a)(\operatorname{tr}_{\bar{\lambda}^{-1}} a^{-1}) \neq 0$$

in $F[A]$. Thus, by the semisimplicity of the group algebra $F[A]$, there exists an irreducible representation $\rho\colon F[A] \to F$ with $\rho(\tau) \neq 0$. Note that $\rho \neq 1$ since $\operatorname{tr}_{\bar{\lambda}} a$ is contained in the augmentation ideal of $F[A]$. Furthermore, by Lemma 5, the group of all automorphisms of A that commute with the action of $\overline{X}$ is transitive on $A \setminus 1$ and therefore also on the nonprincipal linear characters of $F[A]$. Hence, by replacing a by another element of $A \setminus 1$, if necessary, we can assume that ρ is a constituent of the restriction $\bar{\theta}_A$ of $\bar{\theta}$ to $F[A]$. In other words, we can now assume that $\bar{\theta}$ is the representation of $F[\overline{G}]$ induced from ρ. As a consequence, when τ is viewed as an element of R, it is nonzero and hence so are its three factors $\operatorname{tr}_{\bar{1}} a$, $\operatorname{tr}_{\bar{\lambda}} a$ and $\operatorname{tr}_{\bar{\lambda}^{-1}} a^{-1}$.

Now set $\alpha = u_{\langle a\rangle} = (a - t^q)/(a - t) \in \mathrm{U}(F\langle a\rangle)$ and observe, by [GP, Lemma 3.6], that $\operatorname{tr}_{\bar{\sigma}} \alpha \neq 0$ and $\operatorname{tr}_{\bar{\sigma}} \alpha^{-1} \neq 0$ in R, for all $\bar{\sigma} \in \{\bar{1}, \bar{\lambda}, \overline{\lambda^{-1}}\}$. Thus, by [GP, Lemma 3.4] again, we have $\bar{e}_{\bar{\mu}} \alpha \bar{e}_{\bar{\eta}} \neq 0$ and $\bar{e}_{\bar{\mu}} \alpha^{-1} \bar{e}_{\bar{\eta}} \neq 0$ in R, for all $\overline{\mu}, \overline{\eta} \in \{\overline{1}, \overline{\lambda}\}$. Consequently, $e_\mu \alpha e_\eta \neq 0$ and $e_\mu \alpha^{-1} e_\eta \neq 0$ in R, for all $\mu, \eta \in \{\kappa\overline{1}, \kappa\overline{\lambda}\}$. Since this verifies the idempotent condition, the argument of [GP, Lemma 3.4] now implies that there exists a special unit $u = u_X$ based on t, such that $\langle u, \alpha^{-1} u \alpha\rangle$ is not 2-related. In particular, the larger group $\langle u_{\langle a\rangle}, u_X\rangle = \langle \alpha, u\rangle$ has a nonabelian free subgroup. □

Step 4. *The result holds in the pseudo-Frobenius case with $p = \pi$, and the theorem is proved.*

Proof. Since $p = \pi$ and $\mathbb{O}_\pi(G) = 1$, we must have $G = A \rtimes X$ where $X = \langle x\rangle$ has prime order p. We first work in $F[G]$, and set $\alpha = 1 + x + \cdots x^{p-1}$ and $\beta = \alpha^a$ for some $1 \neq a \in A$. Then, by the argument of [GP, Lemma 4.2], we know that $\alpha^2 = 0 = \beta^2$ and that $\alpha\beta$ is not nilpotent. In particular, $\alpha\beta$ is not nilpotent modulo the radical of $F[G]$, so there exists an irreducible representation φ of the group algebra with $\varphi(\alpha\beta)$ not nilpotent. This, of course, implies that $\varphi(\alpha\beta) \neq 0$ and hence, since $\operatorname{char} F = p$, we see that φ cannot be the principal representation of G. As a consequence, φ must be the irreducible representation of $F[G]$ induced from some nonprincipal linear character ρ of $F[A]$. Furthermore, θ is the irreducible representation of $F[G]$ induced from some nonprincipal linear character ρ' of $F[A]$. Thus, since $\operatorname{Aut}_X(A)$ is transitive on these nonprincipal linear characters, by Lemma 5, we can replace a by another element of $A \setminus 1$ if necessary, to assume that $\varphi = \theta$. In other words, we now know that $\alpha\beta$ is not nilpotent in R, and therefore as in [GP, Lemma 4.1], $u = 1 + t\alpha$ and $v = 1 + t\beta$ generate the free product $C_p * C_p$ of cyclic groups of order p. In particular, $\langle u, v\rangle$ contains a nonabelian free subgroup when $p > 2$.

Finally, suppose $p = 2$. Then, as in [GP, Lemma 4.4], $G = A \rtimes X$ is a dihedral group with A cyclic of order q. Furthermore, $\theta = \mu^G$ for some nonprincipal linear character μ of $F[A]$, so the argument of [GP, Lemma 4.4] now yields the result. □

References

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