A NEW CLASSIFICATION OF SEMANTIC STRUCTURES OF ONE-STEP WORD PROBLEM SITUATIONS

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A new classification for semantic structures of one-step word problems is proposed in this paper. The classification is based on illustrations of word problem situations in Common Core State Standards (CCSSM, 2010) and related historical studies (e.g. Weaver, 1973, 1979, 1982), as well as conceptual elaborations of embodied and grounded nature in Lakoff et al. (2000). The classification identifies two main classes: action on/change of an initial quantity and coordination/comparison of two quantities, providing a unifying characteristic of basic operations of quantities. This classification is more comprehensive and differentiated than the classification of CCSSM (2010) and Polotskaia et al. (2021), as it emphasizes conceptual demands of children's mathematics, coherence and continuity of progressions, and consistency with thinking modes and/or problem-solving strategies.

Introduction

Common Core State standards describe the situation types or the categories of word problems using two tables (CCSSM, 2010). The first table shows the distinct types of addition and subtraction situations: add to, take from, put together/take apart, and compare. The second table shows the distinct types of multiplication and division situations: equal groups of objects, arrays of objects, and comparison.

Having a thorough grasp of various problem types is of utmost importance for teachers as it enables them to discern the different approaches and methods employed by students when tackling problems. On occasion, the positioning of the unknown element within a problem significantly influences the strategies utilized and the level of complexity experienced. Moreover, there is a widely held belief that students should comprehend problems and devise solutions that align with their own understanding, rather than strictly adhering to a predetermined approach based on problem types. However, an alternative viewpoint can also suggest that if feasible, aligning problem types with thinking modes and strategies could enhance comprehension and empower students to effectively employ appropriate representations and modeling techniques.

There is an expected progression in comprehending different types of situations. Initially, students learn and solve problems related to situations involving whole numbers, which later advances to word problems incorporating fractions, integers, and eventually all rational and real numbers. Additionally, teachers typically introduce addition and subtraction situations in earlier grades, while multiplication and division situations are introduced in later grades. Despite research studies, such as Carpenter et al. (1999) and others, confirming that young children are capable of solving multiplication, division, and multistep problems by directly modeling the action or structure, some teachers and researchers still tend to believe that additive reasoning always precedes multiplicative reasoning. Our argument revolves around the notion that in order

to gain a comprehensive understanding of how the situation types in word problems contribute to students' comprehension, problem-solving methods, and thinking strategies associated with these situation types, it is imperative to illuminate the continuity in progressions and the shared characteristics of *operations involving quantities* grounded in human activity. These aspects have played a crucial role in enabling students to develop skills like direct modeling, while also guiding teachers in implementing suitable teaching approaches for their students.

This study, therefore, introduces a new classification of word problem situations based on the shared characteristics of operations involving quantities and the continuity of concepts across different levels of mathematics. To develop this classification, we analyze relevant studies on quantitative operations, word problem situation types, and embodied cognition. By examining various representations and the conversions utilized during operations involving quantities, we identify commonalities and ensure continuity in progression. The findings shed light on the current presentation of word problem situation types and propose a more consistent and continuous approach enriched by grounded and embodied cognition. This research opens avenues for future experiments, discussions, and reflections in this area.

The necessity of a new classification

The types of word problems are closely connected to the semantic structures of quantity operations. In his work, Schwartz (1996) extensively examined the semantic aspects of quantities and their operations in mathematics. He proposed that the operations of quantities consist of two parts: numerical operations and operations on units of measurement, which extended the four basic operations of pure numbers. This idea highlighted the fact that multiplication and division operations of quantities could create new units of measurement, and were not merely repeated addition and subtraction. However, the addition and subtraction operations of quantities remained under the same umbrella category, similar to the traditional illustration of pure number operations or word problem types, whereas multiplication and division operations were classified under a different category. Schwartz did not specify any shared characteristics between these two categories in terms of quantity operations.

In the recent study of multiplicative structures, Polotskaia et al. (2021) have proposed a relational paradigm for understanding multiplicative structures, which includes three multiplicative relationships: multiplicative comparison, multiplicative composition, and cartesian product, and their corresponding visual models. Their view challenges traditional approaches to teaching word problem solving, which emphasize mastering elementary arithmetic operations before applying them to problem-solving. They highlighted an increasing number of studies that investigate the relationship between mathematical structures and word problem solving, such as the works of Cai et al. (2005), Ng & Lee (2009), and Verschaffel et al. (2010). Additionally, they contrasted two paradigms: the operational paradigm, which views arithmetic operations as the foundation for comprehending real-world scenarios that involve actions like adding, subtracting, comparing, and sharing, and the relational paradigm, which views relationships between three elements as the fundamental mathematical concepts, where two elements determine a unique third element as a function. The relational paradigm focuses on the idea of an operation as a function (Carraher et al., 2005) and enables various modes of thinking about arithmetic operations. Polotskaia et al. attempt to align multiplicative and additive structures using the relational paradigm and visual models. However, they still maintain a distinction between the two structures and propose three unique classes for multiplicative structure and its associated reasoning that do not apply to additive structure and its related reasoning.

Considering the fundamental but distinct roles of additive and multiplicative structures and their associated reasoning in teaching and learning mathematics, it is reasonable to investigate whether these structures share commonalities or similarities. Such an exploration could yield significant insights and may justify the development of a new classification scheme that highlights their shared features. This classification could pave the way for innovative discoveries and advancements in teaching and learning of mathematics.

Development of a new classification

Expanding on the previous discussion, it is crucial to investigate the feasibility of uncovering a common theme that is applicable to both additive and multiplicative structures in the context of word problem situations or operations involving quantities. To identify such a theme, we are specifically focusing on the following two frameworks.

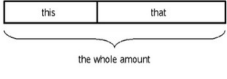**Grounded and Embodied Nature of Operations of Quantities**

In their work, Lakoff and Núñez (2000) put forth a novel perspective regarding numerical operations, proposing that addition, subtraction, multiplication, and division are not separate entities but are instead rooted in our embodied experiences and perceptions of the physical world. They achieved this by utilizing semantic primitives and conceptual metaphors to map arithmetic operations onto source domains such as object collection, object construction, measuring stick, and motion along a path. These metaphors serve to illustrate the commonality across the four operations and challenge the traditional notion that addition and subtraction are fundamentally distinct from multiplication and division. Furthermore, the authors emphasize the strong connection between operations involving quantities and the same metaphoric source domains.

To exemplify the grounded and embodied nature of quantitative operations, let's examine the addition of two quantities. Imagine having two containers with different capacities, each holding varying amounts of water, and a third empty container large enough to hold the combined water. The action of pouring water between containers represents the source domain, while the addition operation signifies the target domain within this conceptual metaphor. Depending on the context and affordances present (as described by Gibson, 1979), we can pour smaller amounts into the larger container or merge the water from both containers to determine the total sum. Pouring smaller amounts into larger containers is typically easier for humans due to the enhanced visibility of changes and reduced risk of spillage. Through these everyday activities, we can deduce the fundamental principles that underlie quantitative operations.
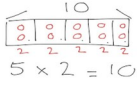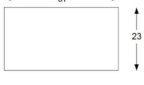
Unary(ish) vs. Binary Operations of Quantities

In his lesser-known but significant studies, J.F. Weaver (1973, 1979, 1982) emphasized the importance of students comprehending both binary-operation and unary(ish)-operator meanings of symbolic number sentences in the context of addition and subtraction. However, there is a lack of research on how young children interpret these types of sentences. It is uncertain whether binary and unary interpretations can develop simultaneously or if one needs to be learned before the other. Moreover, it is unclear when interference between the two interpretations may arise.

Although some questions remain unanswered, his studies have provided valuable insights into the operations of quantities, including not just addition and subtraction, but also multiplication and division. Based on this knowledge, we propose a new approach that is more logical and intuitive for understanding these operations. This approach is illustrated in Figure 3, and the symbolic representation shows that the operations of quantities can be viewed as functions. This shared semantic structure leads to the following features of the four basic operations of quantities, as demonstrated in Figure 1 and Figure 2.

	Addition (A+B=C)	
	Action/Change-focused way	Comparison/Coordination-focused way
Linguistic description	"Augend" + "Addend" = Result **Add** (Addend) **to** (Augend)	"Summand" + "Summand" = "Sum" **Add** (Summand) **and** (Summand)
Key Model	3 / 4 5 6 7 8 9	this / that / the whole amount

	Subtraction (A-B=C)	
	Action/Change-focused way	Comparison/Coordination-focused way
Linguistic description	"Minuend" - "Subtrahend" = Result **Subtract** (Subtrahend) **from** (Minuend)	Quantity - Quantity = "Difference" **Find the difference between** (Quantity 1) **and** (Quantity 2)
Key Model	1 2 3	26 / Tameka 19 ? / Jackson 19

Figure 1. Addition and Subtraction of Two Quantities

	Multiplication (AxB=C)	
	Action/Change-focused way	Comparison/Coordination-focused way
Linguistic description	"Multiplier" x "Multiplicand" = Result (or "Multiplicand" x "Multiplier" = Result in some countries) **Multiply** (Multiplicand) **by** (Multiplier), (Multiplier) **of** (Multiplicand), (Multiplier) **groups of** (Multiplicand)	"Factor" x "Factor" = "Product" **Multiply** (Factor 1) **and** (Factor 2)
Key Model	10 / $5 \times 2 = 10$	37 / 23

	Division (A ÷ B=C)	
	Action/Change-focused way	Comparison/Coordination-focused way
Linguistic description	"Dividend" ÷ "Divisor" = Result **Divide** (Dividend) **by** (Divisor), **Divide/Partition** (Quantity) **into** parts or portions.	Quantity ÷ Quantity = "Quotient" **Find the ratio/rate between** (Quantity 1) **and** (Quantity 2). **Distribute** (Quantity 1) **into** (Quantity 2). **How many times** (Quantity 1) **goes into** (Quantity 2)?
Key Model	12 / child / ? / $12 / Pack ...? / $4	$18 \div 6 = 3$ / Car 1 Car 2 Car 3 Car 4 Car 5 Car 6 / *There will be 3 people in each car.*

Figure 2. Multiplication and Division of Two Quantities

Even though we juxtapose addition and subtraction together, and multiplication and division together, the main classes of action on/change of an initial quantity and comparison/coordination of two quantities are consistence throughout any operations of quantities.

	Operation # : **A # B = C**	
Modes of Thinking	Action/Change-focused mode	Comparison/Coordination-focused mode
Structure	An operand with a numerical operator	Two operands with an operator sign
Symbolic description	$A \xrightarrow{\#B} C$	$\#(A,B) \longrightarrow C$
Problem Situation	Initial quantity with Change in the quantity results in Changed quantity	Initial quantity #1 together with Initial quantity #2 produce/result in New quantity
Number of involved quantities for input	Binary (Unary-ish)	Binary
Number of involved units of measurement/counting	• One unit (initial quantity and final quantity have the same unit. Change does not have any units, or has the same unit with the initial and final quantities. e.g. a location vector + translation vector = a new location vector, % + p.p. = % etc.)	• One unit (addition, subtraction of two quantities of the same type) • Two units (multiplication, division of two quantities of the same type) e.g. 2m x 4m = 8 m^2 • Three units (all operations of two quantities of different types) e.g. 2 apples and 3 pears all together 5 fruits. Time x velocity = distance, etc.
Main Idea	• Modification of an initial quantity	• Amalgamation of two quantities to produce a single quantity, or • Synthesis of two quantities to figure out the relationship between two.
Related Advanced Math Idea	• "Reverse Engineering", Inverse Functional Thinking	• Commutative Property, Associative Property • Operations in MAGMAS

Figure 3. Two Operations of Quantities

Proposition of New Semantic Structure and Word Problem Situations

The proposed semantic structure and word problem situations in Figure 4 aim to provide a framework for classifying arithmetic word problems based on the types of actions or relationships involved. While some researchers, such as Carpenter et al. (1999), suggest that these classifications correspond to students' thinking about the problems, others, like Mulligan & Mitchelmore (1997), argue that they are somewhat arbitrary and can be extended, collapsed, or refined depending on the investigation's purpose. Multiple versions of these classifications exist, reflecting their ongoing development and refinement. Although the semantic structures of word problems may not always accurately reflect students' thinking or solution strategies, they can still be valuable tools for researchers seeking to understand and predict these thought processes, as well as for developing explicit models of knowledge structures and solution processes.

The common theme among the past and current classifications of semantic structure of the word problems is grouping of addition/subtraction, and multiplication/division based on the distinction between additive nature of thinking and multiplicative nature of thinking. However, those classifications hardly noticed a unifying nature of arithmetic operations regardless of the obvious types of operations or distinction between additive and multiplicative nature of thinking.

Arithmetic Operation	Addition 2+3		Subtraction 5-2		Multiplication 3×2		Division 6÷3	
Quantitative Operation (Mode of Thinking)	Semantic Structure / Word Problem Type / Problem Situation							
Action on / Change of initial quantity (operand and operator)	Add to	Initial quantity unknown	Take from	Initial quantity unknown	Multiply by / Scale up and down 3 groups of 2 are 6, Multiplication of 2 by 3 is 6.	Initial quantity unknown, **Classical PD**	Partition and select / Measure by cutting out	Initial quantity unknown
		Change unknown		Change unknown		Action unknown, **Classical MD**		Action unknown
		Result unknown		Result unknown		Result unknown **Classical Product 1**		Result unknown
Coordination of two quantities (operand and operand)	Put together	One quantity unknown	Take apart / Compare	One quantity unknown	Product / Times / Multiply and 3 times 2 is 6, Product of 3 and 2 is 6.	One quantity unknown	Distribute / Assign / Combine / Coordinate	One quantity unknown
		Result unknown		Result unknown		Result unknown **Classical Product 2**		Result unknown

Figure 4. New Semantic Structure of Word Problem Situations

Discussion

In this section, we hope to show the usefulness of this new classification in several aspects of research studies by exploring some key signifying examples. These are parts of a larger on-going research project, and open to further studies and discussions.

Conceptual demands of children's mathematics

Our new classification provides a more explicit explanation of the conceptual demand in children's mathematics that was previously considered as their direct modeling or various strategies without understanding the reasons why children adopt those strategies or where they come from. Direct modeling involves modeling the action or relationships described in word problems, making the action or relationships depicted in word problems important clues to understanding why children take particular approaches. As a result, our new classification is

particularly useful for analyzing these approaches. For instance, Carpenter et al. (1999) presented the following problem situation to observe children's use of the partitive strategy, a form of direct modeling of partitive division.

Mr. Franke baked 20 cookies. He gave all the cookies to 4 friends, being careful to give the same number of cookies to each friend. How many cookies did each friend get?

Three different types of strategies were introduced as variations of Partitive strategy.

Ellen counts out 20 counters. She placed the counters into 4 separate places one at a time. After she puts one counter in each spot, she starts over and adds another counter to each set, continuing this process until she has used up all the counters. Then she counts the counters in one pile and says, "5...each would get 5 cookies."

Based on our new classification, Ellen's problem can be categorized as a division problem that involves the coordination of two quantities. One of the quantities is clearly the number of counters, while the other quantity is the number of places or spots, even though they are not explicitly visible. The coordination of these two quantities is presented through actions such as distributing, assigning, combining, coordinating two objects, and observing the relationship between them. This coordination is highlighted further in Rita's problem, as shown below.

First Rita counts out 20 counters. Then she selects 4 additional counters that are not part of the 20 to represent the 4 friends and puts them in separate places on the table. She deals the counters one by one to each of the 4 separate "friends" places on the table. When she has used up all 20 counters, she counts the number of counters in one of the groups, not counting the single counter that she first put out to identify the group and answers, "5."

Teacher: Good. I see how you got the 5, but can you tell me why you didn't count this [indicates the counter that represented the group]?

Rita: That's one of the friends.

Rita's case makes it clear that the division operation involves two quantities as inputs, represented by counters for cookies and friends. This is an example of coordinated division with two operands. The other cases are different in that they do not require two quantities to start with.

Kang counts out 20 counters. He places 4 counters in one group, 4 in another group, 4 in another, and 4 in another until he sees that there are 4 groups. At this point he sees that he had not used up all the counters, so he adds 1 counter to each group. Then he counts the counters in one of the groups and answers, "5."

Kang's strategy appears to be a combination of the measurement strategy, which involves repeated subtraction, and the partitive strategy, which involves partitioning, as described by Carpenter et al. (1999). It is possible that Kang initially selected four counters out of 20 to distribute among his friends later, but kept track of them mentally rather than physically. This approach does not involve coordination, assignment, or distribution of two quantities. However, Kang eventually realized that he had created four equal groups or partitions, which he identified as another type of object or quantity, and then attempted to coordinate the remaining counters with these four groups. This case demonstrates Kang's shift in thinking from an action or change of an initial quantity mode with 20 counters to a coordination of two quantities mode involving the leftover counters and the four groups he created.

When dealing with questions that involve changing an initial quantity, division of a single quantity can be demonstrated through repeated subtraction or partitioning. For instance, starting with 20 counters, students can divide them into two equal groups of 10 counters each through partitioning. They can then further partition each of the 10-counter groups into two equal groups of 5 counters each. This is an example of division through repeated partitioning with only one quantity. No distribution to other objects/people is involved. To assume that the strategies employed by children to solve Partitive Division problems are variations of the Partitive strategy (Carpenter et al., 1999) would overlook important distinctions.

Based on this discussion, it can be inferred that the strategies devised by students are grounded in clear and intuitive principles, warranting their inclusion in a new classification. The conventional classification of partitive division and measurement division (quotative) primarily distinguishes these strategies based on the placement of unknowns within the multiplicative structure of mathematics, where the operator times the operand equals the product.

Coherence and continuity of progressions

The new classification places significant importance on the coherence and continuity of progressions in semantic structures of operations. Throughout the progressions from additive reasoning to multiplicative reasoning in both numerical and quantitative senses, two main classes of actions are maintained: actions that involve changing an initial quantity, and actions that involve coordinating or comparing two quantities. These two classes have also been identified in visual representations of multiplication operation (Kwon et al., 2017, 2019). Specifically, the multiplication area model was reconstructed into two types: the length-to-area model and the area-to-area model, which are illustrated in Figure 5 below.
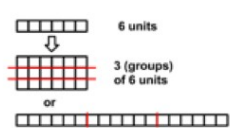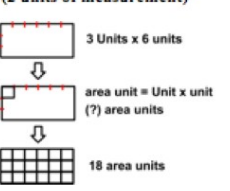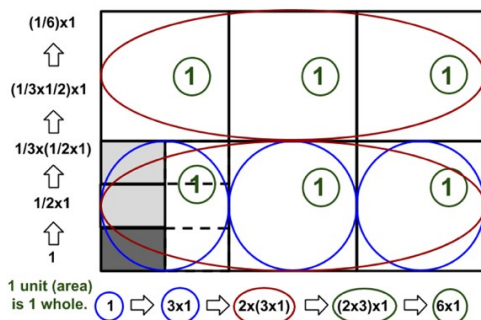
	Problem Types	Equal Groups	Arrays / Areas	Compare
	Word Problems	A. There are 3 bags with 6 plums in each bag. How many plums are there in all? B. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	A. There are 3 rows of apples with 6 apples in each row. How many apples are there? B. What is the area of a 3 cm by 6 cm rectangle?	A. A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? B. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?
Area-to-Area Model (1 unit of measurement) 6 units 3 (groups) of 6 units or	Action on/Change of an initial quantity	A. 3 groups of 6 plums B. 3 (groups) of 6 inches	A. 3 groups of 6 apples	A. 3 groups of $6 B. 3 of 6cm
Lengths-to-Area Model (2 units of measurement) 3 Units x 6 units area unit = Unit x unit (?) area units 18 area units	Operation/coordination of two quantities	A. 3 bags times 6 plums/bag B. 3 lengths times 6 inches/length	A. 3 rows times 6 apples/row B. 3 cm times 6 cm	A. 6 $/blue hat times 3 blue hat/red hat

Figure 5. Multiplication Area Models with Multiplicative Word Problem Situations (Kwon et al., 2019)

The semantic structures of multiplication rely on the differentiation between action on/change of an initial quantity and coordination of two quantities. This differentiation allows for the extension of the multiplication operation and its area model to fractions and beyond, without any confusion. As shown in Figure 4 below, the area-to-area model illustrates the connection between whole number multiplication and fraction multiplication, ensuring the coherence and continuity of progressions in semantic structures of multiplication.

Figure 6. Whole Number Multiplication and Fraction Multiplication in Area-to-Area Model (Kwon et al., 2017, 2019)

Coherence and continuity in operations extend to early algebra and algebra, encompassing both discrete and continuous models. Recent studies, such as those on the Davydov curriculum (e.g. Freiman, 2021; Mellone et al., 2021; Polotskaia et al., 2021; Coles, 2021, etc.), highlight the importance of maintaining coherence and continuity in the progressions of quantitative operations across various representations. A rapid transition from discrete to continuous objects and associated quantitative operations can aid early understanding of algebra, where variables are typically seen as continuously varying.

According to the development of the new classification, the way two quantities are processed is an essential component of quantity operations that extends beyond grade levels and into everyday human activities. This implies that the continuity between early and higher grades is not only related to objects, quantities, or raw materials, but also to the underlying concept that is deeply ingrained in the physical experiences of the body on how quantity operations are performed.

Conclusion and Suggestion

The primary objective of this paper is to introduce a new and unifying classification of semantic structures in word problems, departing from the conventional focus on differentiating additive and multiplicative structures. This inclusive classification offers a more comprehensive and nuanced approach compared to previous classifications such as CCSSM (2010) and Polotskaia et al. (2021). It places emphasis on the conceptual foundation of children's mathematics, prioritizing coherence and continuity in the progressions of arithmetic operations and aligning with various thinking modes and problem-solving strategies. This innovative classification paves the way for further research, including exploring the use of technology incorporating hand and touch in problem-solving, investigating discrepancies between word problem structure/context and models/representations, examining different modes of thinking in mathematical modeling, and delving into inventive teaching and learning approaches for word problem solving.

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