Fall 2003 HOMEWORK 9

33. Let $G =< x, y \mid x^3 = y^3 = (xy)^3 = 1 >$. Show that G is the semidirect product of A by $< t >$, where $A =< a > \times < b >$ is the direct product of two infinite cyclic groups, t has order 3, and the action of t is given by $a^t = b, b^t = a^{-1}b^{-1}$.

HINT: Show that $< xyx, x^2y >$ is a normal abelian subgroup.

34. Let G be a nonabelian simple group and $\tilde{G}$ its universal covering group. Show that $\mathrm{Aut}(G) \cong \mathrm{Aut}(\tilde{G})$.

35. Let $(G_i : i \in I)$ be perfect groups and $\tilde{G}_i$ the universal covering group of G_i. Show that the universal covering group of the direct product G of the groups G_i is the direct product of the covering groups $\tilde{G}_i$ and hence that the Schur multiplier of G is the direct product of the Schur multipliers of the groups G_i.

36. Show that a cyclic group has no nontrivial central extensions. Find all central extensions of the Klein 4-group $C_2 \times C_2$.