**25.** Let $G$ be a finite group and $A$ a subgroup of its automorphism group such that $(|G|, |A|) = 1$. Suppose that $(G_i : 0 \leq i \leq n)$ is an $A$-invariant normal series for $G$ such that $A$ centralizes (i.e. acts trivially on) $G_{i+1}/G_i$ for $0 \leq i < n$. Show that $A$ centralizes $G$ (so $A = 1$). Produce a counterexample when $(|G|, |A|) \neq 1$.

HINT: Reduce to the case when $A$ is a $p$-group and look at orbit sizes modulo $p$.

**26.** Let $G$ act transitively on set $X$, $x \in X$, $H = G_x$, and $K \leq H$. Let $S$ be the subset of $X$ of elements fixed by $K$. Show that $N_G(K)$ is transitive on $S$ if and only if $K^G \cap H = K^H$.

**27.** Show that every group of order 30 is isomorphic to one and only one of the following groups: $C_{30}, C_5 \times D_6, C_3 \times D_{10}, D_{30}$.

HINT: Show that $G$ is an extension of $C_{15}$ by $C_2$.

**28.** Show that for every integer $n \geq 1$ there exists a solvable finite group of derived length $n$ (i.e. $G^{(n-1)} \neq G^{(n)} = 1$).

HINT: Show that if $G$ is a finite group of derived length $n$, then the wreath product of $G$ with $C_2$, $G \; wr \; C_2$, has derived length $n + 1$.