Fall 2003 HOMEWORK 4

13. Show that if H has order p^2 (p prime), then $\mathrm{Aut}(H)$ has order divisible by p but not by p^2. Deduce that if G has order p^n with $n \geq 4$, then G has an abelian normal subgroup of order p^3.

14. Show that if G is a finite group, then its Frattini subgroup $\Phi(G)$ is nilpotent.

HINT: Let $P \in Syl_p(\Phi(G))$ and use the Frattini argument.

15. Show that if $G = H \times K$, then $\Phi(G) = \Phi(H) \times \Phi(K)$. Show that if H and K are isomorphic nonabelian simple groups, then G has a maximal subgroup isomorphic to H.

16. Find an infinite family of groups G with subgroups H such that H char G, $|H| > 2$, and such that the restriction map $\mathrm{Aut}(G) \to \mathrm{Aut}(H)$ has trivial image.