Group Theory **N. Boston**

Fall 2003 HOMEWORK 10

37. Let $G =< x, y \mid xyx = y, yxy = x >$. Show that $x^4 = y^4 = 1$ and deduce the structure of G.

38. Let $F(2,6)$ be the Fibonacci group $< a, b, c, d, e, f \mid ab = c, bc = d, cd = e, de = f, ef = a, fa = b >$. By constructing a homomorphism from $F(2,6)$ to $GL(2, \mathbf{Z})$, show that $F(2,6)$ is infinite.

39. Let $G =< x, y \mid x^5 = y^3 = (xy)^2 >$. Show that G/G' is trivial and that $Z(G)$ has finite index. Deduce that G is finite. In fact $G \cong SL(2,5)$. Assuming this, what is $M(Alt(5))$? Why?

HINT: Identify $G/ < x^5 >$. You may quote that $Alt(5) \cong PSL(2,5)$.

40. Let x and y be two involutions in group G. Show that either x and y are conjugate in G or there is an involution u in G that commutes with both x and y.