REVIEW OF **THE WORLD'S MOST FAMOUS MATH PROBLEM**

Written by Marilyn vos Savant

Reviewed by Nigel Boston and Andrew Granville

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Is all publicity good publicity? Are popular books about mathematics, chock-full of inaccuracies and misunderstandings, nonetheless beneficial for the subject as a whole? Probably the answer is 'yes', and this is just about the only good thing that can be said about this book, written by Marilyn vos Savant. You probably know of Marilyn vos Savant. She became famous because the *Guinness Book of World Records* presented her as the person with the highest IQ ever recorded, as a child and as an adult. (As we shall discuss, her intelligence shines through in many places in the book, making it that much sadder that she could have written such drivel.) In her "Ask Marilyn" 'problem-analysis column' each Sunday in *Parade* magazine, she popularises mathematical puzzles and conundrums, reaching out to a potential audience of seventy million people. And now she has challenged the orthodoxy of the mathematics world by refuting Wiles's purported proof of Fermat's Last Theorem, by claiming that it is wrong because it is illogical, relying, she believes, on ridiculous inconsistencies accepted by mathematicians. For example, the concept of a non-Euclidean geometry. For another example, proofs using induction. Would you expect that such a controversial opinion, from the person with the world's highest IQ, is based on a careful analysis of the ideas involved in these concepts, by studying the appropriate literature at hand, and by justifying her substantial findings with an irrefutable argument? Well don't expect such an argument in *this* book, because you'll be disappointed. In fact, she even boasts that she wrote the book in just three weeks! Not that she began as an expert; no, she just dived in, read a few popular mathematics books, and then proceeded to her startling conclusions.

Before getting into some specific criticisms, we first mention her disingenuous use of four eminent mathematicians in the publicity for this book. She gives 'a personal thank you to Barry Mazur, Kenneth Ribet and Karl Rubin for being such good sports and for putting up with my faxes'; in addition she includes an endorsement from Martin Gardner on the back cover. Surprisingly she did not raise any of her doubts about Wiles's work with them before going to press; which is a bit rich since these acknowledgements are certainly presented so as to make them look like endorsements. We took a brief survey of the four:

- Gardner says that he saw an early draft of the book and praised it for its good presentation of difficult concepts, so as to be accessible to non-mathematicians (we agree); but her contentious views were not included in the draft that he received.
- Mazur does not remember having any encounter with her whatsoever! However, he says that his secretary did mail reprints of his article *Number Theory as Gadfly* to inquirers. Mazur asks what would warrant an 'impersonal thanks'. Quite!
- Ribet has no memory of dealing with her at all, ever.
- Rubin never talked to her, but did give permission to her assistant, Richard Romano, to include his now famous 'email message to the world' in her book.

Extraordinary acknowledgements for such tenuous contacts.

On the other hand both authors of this review *were* contacted by the author's husband, Dr. Jarvik (of artificial heart fame), to talk about the proof. He never mentioned the forthcoming book, in fact indicating that he was simply a 'curious scientist'. He did, however, express surprise that the second reviewer believed the proof to be correct, given that he (Granville) admitted to understanding only the basic overview of the work. Jarvik wondered how one could believe Mazur, Ribet, and Rubin since they had the most to gain if the proof were accepted. In the book, vos Savant writes, 'when a proof is supported by a small group of people,... when ... virtually no one outside this group is capable of understanding it, the cautionary bell ... rings'. Funny how, when it came down to it, the gap in the proof (now apparently fixed) was found by that same 'small group of people' and that Wiles acknowledged the existence of the gap in the loudest forum of them all, the ultimate plenary talk at the 1994 International Congress of Mathematicians.

This book was written, start-to-finish, in three weeks. Judging from the lengthy bibliography, vos Savant did a lot of valuable reading in that time and culled some interesting ideas. It seems, however, that there were a number of things which she came across that did not make much sense to her. Rather than allow an expert to explain these things to her, she just decided that they were wrong. After all, she seems to reason, if these things are not self-evident then how could they possibly be correct? From Mazur's article she seems to have gleaned the fact that the Taniyama–Shimura conjecture may be re-phrased in part in terms of hyperbolic geometry. Thus she concluded that Wiles gave a 'hyperbolic method of proving F.L.T.' In fact, her central theme is that non-Euclidean geometry, and indeed any mathematics related to non-Euclidean geometry, is nonsense. Her thesis seems to be that, since it was proved in 1882 that 'squaring the circle' is impossible in a Euclidean setting, and since Bolyai managed to 'square the circle' in an appropriate non-Euclidean geometry, thus non-Euclidean geometry is inconsistent with Euclidean geometry. However since Fermat's Last Theorem is a statement consistent with regular geometry, it cannot be proved by arguments that involve any non-Euclidean geometry. After all, 'one of the founders of hyperbolic geometry [J. Bolyai] managed to square the circle?! Then why is it known as such a famous impossibility?' Therefore, she concludes, '*if we reject a hyperbolic method of squaring the circle, we should also reject a hyperbolic proof of Fermat's Last Theorem!*'. This is typical of the inane reasoning (and hyperbole) that pervades this book.

For those readers unfamiliar with the peaceful co-existence possible between non-Euclidean and Euclidean geometry, let us now give a simple (not unrelated) example of such co-existence between concepts that seem inconsistent to the very naive:

In decimal arithmetic, 1+1=2. In binary arithmetic, $1 + 1 = 10$. This, by vos Savant's way of thinking, is not logical. Binary presents a different formal system of logic from the normal one, so we cannot accept any mathematics done on computers (since they work in binary). So let's unplug all computers! Likewise, clock arithmetic tells us $7 + 6 = 1$, so clocks are out too.

Different systems of logic *can* peacefully co-exist. One might visualise two giant computers hooked up, one called "Euclidean geometry", the other "non-Euclidean geometry". There are some things one can do accurately on one computer that would produce error messages galore with the other. However they can still work

precisely together, to solve problems, by translating information between them.

Actually much important science has evolved from initiating new and different techniques for understanding old problems. We go furthest in science precisely when we stop being frightened to embrace new ideas, just because they don't fit in easily with our pre-conceived notions.

There are other misunderstandings in this book. The author mistakenly believes that the Pythagorean theorem yields infinitely many right-angled triangles which necessarily have integer sides. She confuses inductive logic with mathematical induction (something that was sorted out even by Fermat's time!). She mangles the notion of proof by contradiction. Since this book is in a popular vein, let's borrow an idea from late night talk shows and give a list of five quotable quotes from the book:

5) vos Savant challenges the reader to 'use hyperbolic geometry to prove that F.L.T. is impossible to prove. A contradiction of this magnitude ... could cause the entire field to collapse'.

4) 'Using inductive logic, F.L.T. is proved after enough examples have been found'.

3) 'No system of geometry is found in nature, including the Euclidean system. The earth is not a perfect sphere ...' Hmmm, good point!

2) 'The square root of $+1$ is a real number because $+1 \times +1 = +1$; however, the square root of -1 is imaginary because $-1 \times -1 = +1$.'

1) 'it is logically inconsistent to *reject* a hyperbolic method of squaring the circle and *accept* a hyperbolic method of proving F.L.T.'.

Vos Savant's nastiest contention is that most research mathematics is basically nonsense, that we mathematicians are only protecting our territory; moreover that mathematics derived in the last thirty years is too new to have been adequately checked for mistakes (amazing how she has spotted so many fundamental flaws in the reasoning in just three weeks). There certainly *are* significant philosophical difficulties in accepting many of the most abstract techniques — after all there could be a weak link in the chain of reasoning that may have been overlooked — and indeed some important papers have been found to have a 'gap' only after years of intense scrutiny. However vos Savant has entirely failed to get into the psychology of research mathematicians, and their 'quest for the truth'; or to understand how one can, in good faith, believe something to be true, even if one is not familiar with every detail. She fails to understand how arguments 'hang together', how Wiles's contemporaries were sure that his ideas would yield big new results, even if the original approach to the biggest result didn't entirely work out. Vos Savant could never have imagined that Wiles would *publicly admit* that there is a gap in the reasoning, work hard to isolate exactly what the problem is, and then fix that. She would never have predicted this eventuality (a plausible scenario to most mathematicians) because she never took the time and trouble to seriously investigate what she was writing about. That success in mathematics relies on questioning every step in one's arguments, is completely lost on Marilyn vos Savant.

The book does have some good explanations of some tricky mathematical concepts. It also has some pithy anecdotes. However it is mostly an expression of the author's confusion with ideas that require serious, intensive study. Indeed many

experts, including Mazur, have written to vos Savant since the book appeared in order to rebut her arguments. Unfortunately she has seen fit to reply only with a supercilious form-letter, which shows that she is prepared neither to think further about these issues, nor to concede that she does not understand what she is writing about.

Without doubt, mathematicians need to make their subject more accessible. If fewer people had 'math-phobia' then there might be a higher level of numeracy and general scientific awareness amongst the general public. Martin Gardner, Ian Stewart and Keith Devlin, to name but a few, have written accessible, but accurate, books which succeed in capturing the imaginations of inquisitive minds. Marilyn vos Savant's weekly newspaper column frequently presents useful insights that can invigorate people's thinking about mathematics. We need to make people think about mathematics if they are going to appreciate the beauty that is there to be found. In the end, Marilyn vos Savant's book will probably do that as well as any other. It will stir up many young minds, making them realize that there is something exciting, worthwhile, even controversial, in the study of mathematics; and we all know that when students are so persuaded, then learning is so much easier. What a pity though that vos Savant could not have given these students an adequate foretaste of the intellectual delights that are to come.

We'd like to thank Sharon Stone, Boris Yeltsin and the Vienna Boys Choir for being such good sports with all of our faxes, and for (presumably) putting them to the appropriate use.