HOMEWORK 1, DUE FEB 15.

1. (a) Let L/K be a finite Galois extension of local fields (both finite extensions of $\mathbf{Q}_p$). Let $x \in L$ have conjugates $x_1(=x), x_2, ..., x_n$ over K. Suppose $y \in L$ satisfies $|y-x| < |y-x_i|$ for $i \geq 2$. Show that $x \in K(y)$.

(b) Let K be a finite extension of $\mathbf{Q}_p$ and $f \in K[X]$ be a separable, irreducible polynomial of degree n, defining extension L of K (i.e. $L \cong K[X]/(f)$). Show that every polynomial $h \in K[X]$ of degree n that is close enough to f, is irreducible and that the extension $K[X]/(h)$ of K is isomorphic to L.

2. (a) Construct an abelian extension of $\mathbf{Q}(\sqrt{2})$ that is not cyclotomic.

(b) Suppose $K/\mathbf{Q}$ is a quadratic extension. How are its discriminant and conductor related?

(c) Suppose p is prime and 1 (mod 3). Show that there is a unique Galois cubic extension of $\mathbf{Q}$ ramified only at p. What are its discriminant and conductor?

(d) Using the Jones-Roberts database, find the four Galois cubic fields of smallest discriminant that are not produced by the construction in part (c). What are their discriminants and conductors?

(e) There is a simple formula relating discriminant and conductor for all the fields in (b),(c),(d) above. Find two number fields for which this formula fails.

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