HOMEWORK 4, DUE APR 12.

1. Complete the proof of the local reciprocity law. In other words, let K be a local field and prove the following.

(a) Show that if M/K is a finite Galois extension and L/K a finite Galois subextension, then the diagram

$$\begin{CD} K^*/Nm(M^*) @>>> \mathrm{Gal}(M/K)^{ab} \\ @VVV @VVV \\ K^*/Nm(L^*) @>>> \mathrm{Gal}(L/K)^{ab} \end{CD}$$

commutes.

(b) Show that if π is a uniformizer of K and L/K a finite, unramified extension, then the image of π acts as the Frobenius automorphism on L.

(c) How do (a) and (b) yield a unique $\phi_K : K^* \to \mathrm{Gal}(K^{ab}/K)$?

2. (a) For exactly which primes p does there exist a finite Galois extension $K/\mathbf{Q}_p$ with Galois group S_3? [You might like to explore this question using the tables at https://math.la.asu.edu/~jj/localfields/, make a guess, and then prove it.]

(b) For exactly which primes p does there exist a finite Galois extension $K/\mathbf{Q}_p$ with Galois group D_4, the dihedral group of order 8?

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