MATH 844: HOMEWORK 2, DUE OCT 13.

1. (i) For the elliptic curve $E : y^2 = 4x^3 - g_2x - g_3$, let $g_2, g_3 \to 0$. Show that the same geometric procedure for finding $P_1 + P_2$ on E makes the smooth points of the curve $y^2 = 4x^3$ into an abelian group isomorphic to the additive group of $\mathbf{C}$. Interpret this in terms of what happens to the lattice and a fundamental parallelogram.

(ii) For the same elliptic curve E, let $g_2 \to 4/3$ and $g_3 \to 8/27$. Show that this yields a curve with a nodal singularity. Show that the same geometric procedure for finding P_1+P_2 on E makes the smooth points of the curve $y^2 = 4x^3-(4/3)x-(8/27)$ into an abelian group isomorphic to the multiplicative group $\mathbf{C}^*$. Show that this is also isomorphic to the infinite cylinder $\mathbf{C}/\mathbf{Z}$ and interpret this in terms of what happens to the lattice and a fundamental parallelogram.

2. (i) Let K be a field of characteristic two. Let E be the curve $y^2 + xy = x^3 + ax^2 + b$, where $a, b \in K, b \neq 0$. Show that E is an elliptic curve. If P is the point (u, v) on E, find a formula for $2P$.

(ii) For K and E as in (i), find $E[2], K(E[2]), E[4]$, and $K(E[4])$. What is $[K(E[4]) : K]$ and is it always a separable extension?