MATH 844: HOMEWORK 6, DUE MAR 9.

6. Call a smooth projective curve C over $\mathbf{F}_q$ maximal if $|C(\mathbf{F}_q)| = q + 1 + 2g\sqrt{q}$ (g being its genus).

(a) Show that there are no maximal curves of positive genus over $\mathbf{F}_2$.

(b) Show that the Hermitian curve $x^{q+1} + y^{q+1} + z^{q+1} = 0$ over $\mathbf{F}_{q^2}$ is maximal.

(c) If a curve is maximal, what are the α_i that appear in the numerator of its zeta function?

(d) If C is a maximal curve over $\mathbf{F}_q$, compute $|C(\mathbf{F}_{q^2})|$. Deduce an upper bound for its genus in terms of q. [Remark: maximal curves arise in coding theory and finance applications.]