MATH 844: HOMEWORK 5, DUE MAR 2.

5. (a) Let $g = x^3y + y^3z + z^3x \in \mathbf{F}_q[x, y, z]$. Let V be the projective curve given by $g = 0$. Find the zeta function of V for $q = 2$ and $q = 3$.

(b) Let $f = x^{q+1} - y^q - y \in \mathbf{F}_{q^2}[x, y]$ (q a prime power). Let C be the projective curve over $\mathbf{F}_{q^2}$ defined by f. Show that $t^q + t + 1, t^{q+1} + 1$ have roots $\alpha, \beta \in \mathbf{F}_{q^2}$ respectively. Show that if $x^{q+1} + y^{q+1} + 1 = 0$, then $\beta x \neq y$ and setting $u = \beta/(y - \beta x), v = ux - \alpha$, then $f(u, v) = 0$. How large is $\#C(\mathbf{F}_{q^2})$?