MATH 844: HOMEWORK 8, DUE NOV 22.

(a) Show that $ab(a-b)(a+b)$ is a congruent number if $a > b$ are positive integers.

(b) Find the density of squarefree odd positive integers among all odd positive integers.

(c) Deduce that there are infinitely many triples of consecutive odd positive integers that are squarefree.

(d) Deduce that there are infinitely many squarefree congruent numbers.

(e) If E is the elliptic curve over $\mathbf{Q}$ given by the equation $y^2 = f(x)$, its quadratic twists are the curves $E_d : \ dy^2 = f(x)$. Show that E and E_d have the same j-invariant. Find an E of rank 0, which has infinitely many non-isomorphic quadratic twists of rank > 0.