MATH 844: HOMEWORK 6, DUE NOV 1.

Let E be the elliptic curve $y^2 + y = x^3 - x^2$ defined over $\mathbf{Q}$. Let $\rho_p : G_{\mathbf{Q}} \to GL_2(\mathbf{F}_p)$ denote the associated Galois action on $E[p]$.

(a) Find an equation for the x-coordinates of the points in $E[2]$. Find the image of ρ_2.

(b) Find an equation for the x-coordinates of the points in $E[3]$. Show that the only subgroup of $GL_2(\mathbf{F}_3)$ that surjects onto $PGL_2(\mathbf{F}_3)$ is $GL_2(\mathbf{F}_3)$. Find the image of ρ_3. Does E have complex multiplication?

(c) Find a point in $E(\mathbf{Q})$ of order 5. What does this tell us about the image of ρ_5?