

MATH 844: HOMEWORK 2, DUE SEP 27.

1. Recovering a function from its Radon transform is an important technique in applied statistics. The question of invertibility of a particular kind of discrete Radon transform amounts to finding solutions of the Diophantine equation

$$3(y^2 - 1) = 2x^2(x^2 - 1) \quad (*)$$

(a) Find a birational transformation that turns this equation into Weierstrass form.

(b) Use the tangent-chord method to find at least 17 solutions in rational numbers.

(c) Look up Mazur's theorem on the torsion subgroup of an elliptic curve over \mathbf{Q} and use this to say how many solutions there are to $(*)$ with $x, y \in \mathbf{Q}$.

(d) It has been shown that all solutions in integers to $(*)$ have $|x|$ in a set S of size 6. Find S . What is interesting about $\{(2x - 1)^2 : x \in S\}$?