

**MATH 641: HOMEWORK 5 DUE FRIDAY, MAR 16**

Please solve the following problems.

1. Let  $C$  be a binary  $[16, 8, 6]$  code with weight enumerator  $A(x, y) = x^{16} + 112x^{10}y^6 + 30x^8y^8 + 112x^6y^{10} + y^{16}$ . Suppose that  $C$  is used on a  $BSC(p)$ .

(a) How many errors can  $C$  correct?

(b) What is the probability of decoding failure for  $p = 0.005$  if the code is used to correct 2 errors?

2. Let  $C$  be a  $[10, 6]$  Reed-Solomon code over  $\mathbf{F}_{11}$ .

(a) Prove that 2 is a primitive element of  $\mathbf{F}_{11}$ . Is it true that all of the elements 2, 3, 4, ..., 10 are primitive mod 11?

(b) Using the powers of 2, write out a parity-check matrix  $H$  for  $C$ . How many codewords does  $C$  contain?

(c) Let  $r = (3, 0, 0, 10, 5, 4, 0, 6, 10, 0)$  be a received vector. Decode the vector to codeword  $c$  and find  $f$  such that  $c = \text{eval}(f)$ .