MATH 641: HOMEWORK 4 DUE FRIDAY, MAR 2

Please solve the following problems.

1. Show that the polynomial $x^4 + x^3 + x^2 + x + 1$, with coefficients in $\mathbf{F}_2$, is irreducible. Using this polynomial, describe $\mathbf{F}_{16}$ as a set, including how addition and multiplication are defined.

2. Find an explicit element β of order 15 in the above $\mathbf{F}_{16}$, meaning that the smallest positive integer n such that $\beta^n = 1$ is 15. With a suitable choice of k, will the $[15, k]$ Reed-Solomon code produced using the powers of β ever have the same parameters as the Hamming or simplex code of that length?

3. Let C be the generalized Hamming code for $m = 4$. Using MacWilliams' Theorem, find its weight enumerator. From this, find the weight enumerator of C_{ext} and of its dual $(C_{ext})^{\perp}$ (a Reed-Muller code).