

## HOMEWORK 4, DUE OCT 5.

1. The following is an English sentence encrypted by means of a Vigenère cipher.

PPKVF AZUNG SSHUN KZLQY MUNMH FOWKI ZYNAW HGSAL UB-  
HWK VVBOY TVJOH APLJR LGIGY TLUGB YAUAP LCRZY CQULO UL-  
MAO CSANU VLWAT SWHHC LFOVA QZMIK YAFLA LLVLX VKJBV LYVPE  
TSQCY RWION HBZNA TZOTK JYCDN YTJLM UCGTU TKJLP XILLD TVOIU  
WOIMK EMKNH LUTTP KZQIA BTJYM HNIUU NGUKV ONAYR VOIAQ  
AZWOO WMACT PPETH BOYRA BAPJW TJESF IPNEV HTOYB CABS  
Y OMNHG ZBYCK LLSYP BOFIC YRRVW SMFLL NCULV NOYLE UAWTU  
KLXEE PAPPE JINVY QIOTP INUHV KMEAO PEOMS MEHMMW KU

Find the Vigenère keyword (which need not be an English word) and find the plaintext. For full credit you must explain your working.

2. Let  $H(x) = -x \log_2(x) - (1-x) \log_2(1-x)$  ( $0 < x < 1$ ).

(a) Show that  $H(x)$  attains a maximum when  $x = 1/2$ .

(b) Interpret part (a) in terms of the entropy/uncertainty in tossing a (possibly biased) coin.

3. The accuracy of a certain radio station's meteorologist at predicting rain is given by the following chart.

|                  | Actual rain | Actual no rain |
|------------------|-------------|----------------|
| Predicts rain    | 1/12        | 1/6            |
| Predicts no rain | 1/12        | 2/3            |

For example, 1/12 of the time the meteorologist predicts rain when in fact it does rain. Calculate the entropy of the above distribution.

Notice that the meteorologist is correct 3/4 of the time. An uninformed listener observes that he could be correct 5/6 of the time by simply always predicting no rain. He applies for the meteorologist's job. However the station manager declines to hire the listener, on the basis that the listener provides less information. Explain this decision from the point of view of entropy as information content.