Math 475: Introduction to Combinatorics

Lecture 003, MWF 9:55–10:45, online

Syllabus for Fall 2020

Instructor: Mikhail Ivanov (office: remotely; email: `mivanov@wisc.edu`)

Office hours: Thursday 15:00–15:50 (remotely), or by apt.

Canvas: `https://canvas.wisc.edu/courses/212578`

Piazza: `https://piazza.com/class/kdyyyp2jwrb6sl`

Grader: TBD

Course Assistant: Theodore Montalbano tmontalbano@wisc.edu, Wednesday 7 - 9 pm, Friday 1 - 3 pm.

Text: Introductory Combinatorics, 5th edition, by Richard A. Brualdi. Errata for the textbook (10 pages) is located at `https://www.math.wisc.edu/~brualdi`

Prerequisites: Math 320 or Math 340 or Math 341 or Math 375 or graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International) Program.

Instructional mode: online only. All exams will be take-home style. Completed exams and homework should be uploaded to the Canvas website.

Course Designations: Level (Advanced), Breadth (Natural Sciences), L&S Credit type (C).

Course Content: We will cover roughly Chapters 2–8 and 11, 12 in the text. The main topics include permutations and combinations; pigeon-hole principle; partial orders; the inclusion-exclusion principle; recurrence relations and generating functions; Catalan numbers; Stirling numbers; partition numbers; graphs, paths, cycles, trees, graph colourings.

Credit hour expectations: The 3 credit hours are met by three 50 minute online lectures per week throughout the semester, and carries the expectation that students will work on course learning activities (reading, writing, problem sets, studying, etc.) for at least 2 hours out of classroom for every class period.

Exams and Grades: The course grade is based on two midterms and final exam, as well as homework. Each midterm is worth 25%, and the final exam is worth 35%. The graded homework is worth 15% points. Here is the exams schedule:

- Exam I: Wednesday, October 14, take-home style, 2 hours, open book,

- Exam II: Monday, November 23, take-home style, 2 hours, open book,
- Final exam: Saturday, December 12, take-home style, 3 hours, open book.

Rough Schedule:

Week	Mon		Wed		Fri	
1			Sept 2	Ch 2	Sept 4	Ch 2
2	Sept 7	Labor day	Sept 9	Ch 2	Sept 11	Ch 2
3	Sept 14	Ch 3	Sept 16	Ch 3	Sept 18	Ch 3
4	Sept 21	Ch 4	Sept 23	Ch 4	Sept 25	Ch 4
5	Sept 28	Ch 5	Sept 30	Ch 5	Oct 2	Ch 5
6	Oct 5	Ch 5	Oct 7	Ch 5	Oct 9	Ch 6
7	Oct 12	Ch 6	Oct 14	Exam I	Oct 16	Ch 6
8	Oct 29	Ch 6	Oct 21	Ch 7	Oct 23	Ch 7
9	Oct 26	Ch 7	Oct 28	Ch 7	Oct 30	Ch 7
10	Nov 2	Ch 8	Nov 4	Ch 8	Nov 6	Ch 8
11	Nov 9	Ch 8	Nov 11	Ch 8	Nov 13	Ch 11
12	Nov 16	Ch 11	Nov 18	Ch 11	Nov 20	Ch 11
13	Nov 23	Exam II	Nov 25	Review	Nov 27	Thanksgiving
14	Nov 30	Ch 12	Dec 2	Ch 12	Dec 4	Ch 12
15	Dec 7	Ch 12	Dec 9	Ch 12		

Homework assignments: Later in the syllabus there is a preliminary list of exercises to be handed in. These will be marked by a graduate student assigned to me. Your work on these exercises should be well presented, in good English. A clear explanation is just as important as the correct answer. Collaborating with other students on the homework is encouraged, but you must write up all reasoning and solutions on your own (in other words, no copying). Failure to abide by this guideline could be construed as a form of academic dishonesty (see the section on Academic Integrity below). Late homework will generally not be accepted, though I will drop the lowest homework grade at the end of the course. Each student should upload their completed homework to the Canvas website. To do this, scan it or take a photo or create a pdf file, and upload the file to the Assignments tab on Canvas.

	Ch	Exercises	Due date
	1	3, 4, 7, 14, 17, 30, 31, 36, 43	Not to hand in
HW1	2	4abc, 5b, 6, 7, 9, 13ab, 14, 15ab, 16 (the reference should be 2.3.1), 19b, 20 (you may use or may not use hint), 26ab	Fri, Sept 11
HW2	2	11(1 and 20 are included), 30, 32, 35, 38, 39abc, 42, 45abc(replace "at least twenty" with "at most twenty"), 55b, 63abcde	Fri, Sept 18
HW3	3	4, 5, 9(without last question), 10, 14, 17, 18, 27, 28	Fri, Sept 25
HW4	4	1, 6b, 7b, 8, 15abcd, 16abcd, 19, 20, 29, 33, 36	Fri, Oct 2
HW5	5	5, 6, 7, 12, 13, 16, 18, 23, 24, 25, 27, 29	Fri, Oct 9
	5	8, 9, 15, 28, 30, 31, 34, 37, 39, 40, 43, 48, 50	Not to hand in
HW6	6	2, 3, 9, 12, 13, 16, 17, 21, 24ac, 26, 27	Fri, Oct 23
HW7	7	1ac, 8, 11, 12, 13ab, 18, 19, 23, 26	Fri, Oct 30
HW8	7	28, 32, 34, 36, 37, 40, 43, 47, 48d, 50	Fri, Nov 6
HW9	8	1, 2, 3,4ab, 12abcd, 15, 16	Fri, Nov 13
	8	19, 20, 26, 27, 29, 30	Not to hand in
HW10	11	1, 2, 3, 9, 10, 13, 16, 20, 29, 39	Fri, Dec 4

How to prepare for the exams: The list of exercises later in this syllabus is the minimal homework requirement; it is recommended that you do many more exercises on your own. For each exam, including the final, the exam problems will be based on the exercises that appear at the end of the relevant chapters in the text. These exercises might not appear in the table above in the syllabus. Generally speaking, the more exercises from the text that you work out and understand, the easier the exam problems will seem.

Course Goals:

- The student will master the basic counting strategies, such as staged thought-experiments, inclusion/exclusion, generating functions, and recurrence relations, and use these strategies to solve a wide variety of counting problems.
- The student will become familiar with the basic objects that are used in combinatorics, such as permutations and combinations of sets and multisets, binomial and multinomial coefficients, the Catalan numbers, the Stirling numbers, and the partition numbers.
- The student will be able to analyze a given combinatorial problem using the standard theorems of combinatorics, such as the pigeonhole principle, the Newton binomial theorem, the multinomial theorem, the Ramsey theorem, the Dilworth theorem, the Euler theorem.
- After the student solves a combinatorial problem, the student can explain how the solution was obtained, and why it is correct. This explanation typically justifies the

counting strategy used, identifies the appropriate concepts, and identifies the relevant theorem to invoke. The student can also support their argument by displaying examples or counterexamples.

- The student can convey his or her arguments in oral and written form in English, using appropriate mathematical terminology, notation, and grammar.

Course learning outcomes: By the end of the course, the student should be able to solve the following 20 problems, and other problems of a similar nature.

- Each day a student walks from her home to school, which is located 10 blocks east and 14 blocks north from home. She always takes a shortest walk of 24 blocks. How many different walks are possible?
- Consider the set of integers from 1 to 20 inclusive. This set has how many 3-element subsets, such that no two consecutive integers are in the subset?
- Two red rooks and four blue rooks are placed on a 6-by-6 chessboard, so that no two rooks can attack each other. In how many ways can this be done?
- A bagel store sells six different kinds of bagels. Suppose you choose 16 bagels at random. What is the probability that your choice contains at least one bagel of each kind?
- Construct a permutation of 12345678 whose inversion sequence is 66142100.
- Compute the sum of the cubes of the first million positive integers.
- Find the number of integers between 1 and a million inclusive, that are not divisible by 4, 5, or 6.
- Find the number of integers between 1 and a million inclusive, that are neither perfect squares nor perfect cubes.
- How many ways are there to put 14 indistinguishable marbles into 4 distinguishable boxes?
- For each row of Pascal's triangle, find the sum of the squares of the entries in that row.
- Find the number of permutations of 12345678 for which exactly three integers are in their natural position.
- At a party, seven gentlemen check their hats. In how many ways can their hats be returned, so that no gentleman receives his own hat?
- A subway has six stops on its route from its base location. There are 10 people on the subway as it departs its base location. Each person exits the subway at one of its six stops, and at each stop at least one person exits. In how many ways can this happen?
- Find the number of 1-by-100 chessboards for which the squares of the chessboard can be colored red, white, and blue so that no two squares colored red are adjacent.
- Find the number of 1-by-100 chessboards for which the squares of the chessboard can be colored red, blue, green, and orange such that an even number of squares are colored red and an even number are colored green.

- Find the exponential generating function for the sequence of cubes $0, 1, 8, 27, \ldots$
- Find the number of 100-digit integers with all digits odd, such that 1 and 3 each occur a nonzero, even number of times.
- Start with a set S with 100 elements. Consider the partially ordered set consisting of all subsets of S, with partial order by inclusion. What is the cardinality of the largest antichain in this partially ordered set?
- Choose 100 equally spaced points around a circle. Find the number of ways to join the 100 points in pairs, so that the resulting 50 line segments do not intersect.
- Find the number of different necklaces that contain eight red and ten blue beads.

Official Course Description from MyCourseGuide: Problems of enumeration, distribution, and arrangement. Inclusion-exclusion principle. Generating functions and linear recurrence relations. Combinatorial identities. Graph coloring problems. Finite designs. Systems of distinct representatives and matching problems in graphs. Potential applications in the social, biological, and physical sciences. Puzzles. Problem solving.

Rules, Rights & Responsibilities: During the global COVID-19 pandemic, we must prioritize our collective health and safety to keep ourselves, our campus, and our community safe. As a university community, we must work together to prevent the spread of the virus and to promote the collective health and welfare of our campus and surrounding community.

Academic Integrity: By enrolling in this course, each student assumes the responsibilities of an active participant in the UW-Madison community of scholars, in which everyone's academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct and Community Standards for additional review. For more information, refer to

`https://studentconduct.wiscweb.wisc.edu/academic-integrity/`

Accommodations for students with disabilities: The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform the instructor of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. The instructor will work either directly with you or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA. For more information, refer to

`https://mcburney.wisc.edu/facstaffother/faculty/syllabus.php`

Diversity and Inclusion: Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals. The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world. For more information, refer to

`https://diversity.wisc.edu/`

UW-Madison Badger Pledge: See

`https://smartrestart.wisc.edu/badgerpledge`

UW-Madison Face Covering Guidelines: While on campus all employees and students are required to wear appropriate and properly fitting face coverings while present in any campus building unless working alone in a laboratory or office space.

Face Coverings During In-person Instruction Statement (COVID-19): Individuals are expected to wear a face covering while inside any university building. Face coverings must

be worn correctly (i.e., covering both your mouth and nose) in the building if you are attending class in person. If any student is unable to wear a face-covering, an accommodation may be provided due to disability, medical condition, or other legitimate reason. Students with disabilities or medical conditions who are unable to wear a face covering should contact the McBurney Disability Resource Center or their Access Consultant if they are already affiliated. Students requesting an accommodation unrelated to disability or medical condition, should contact the Dean of Students Office. Students who choose not to wear a face covering may not attend in-person classes, unless they are approved for an accommodation or exemption. All other students not wearing a face covering will be asked to put one on or leave the classroom. Students who refuse to wear face coverings appropriately or adhere to other stated requirements will be reported to the Office of Student Conduct and Community Standards and will not be allowed to return to the classroom until they agree to comply with the face covering policy. An instructor may cancel or suspend a course in-person meeting if a person is in the classroom without an approved face covering in position over their nose and mouth and refuses to immediately comply. Quarantine or Isolation Due to COVID-19: Students should continually monitor them- selves for COVID-19 symptoms and get tested for the virus if they have symptoms or have been in close contact with someone with COVID-19. Students should reach out to instruc- tors as soon as possible if they become ill or need to isolate or quarantine, in order to make alternate plans for how to proceed with the course. Students are strongly encouraged to communicate with their instructor concerning their illness and the anticipated extent of their absence from the course (either in-person or remote). The instructor will work with the student to provide alternative ways to complete the course work.