

Your Name (please print) \_\_\_\_\_

Open book exam. No collaboration allowed

The expectation is that you spend 1 hours and 30 minutes on the exam, and then you have 30 minutes to scan and upload your work as a single PDF file.

Please present your solutions in a clear manner. Justify your steps. A numerical answer without explanation cannot get credit. Cross out the writing that you do not wish to be graded on.

You do not have to carry out complicated numerical computations, but you should simplify your answer if it is possible with reasonable effort.

Problem	Value	Score
1	25	
2	20	
3	10	
4	15	
5	10	
6	10	
7	10	
Total	100	

1. Consider a word

$$\mathbb{T} = \text{CORONAVIRUSDISEASE}$$

- (a) (2 points) Is the RECURSION a 9-permutation of  $\mathbb{T}$ ?
- (b) (4 points) How many 2-permutations of  $\mathbb{T}$  are there (simplify your answer)?
- (c) (4 points) How many 3-combinations of  $\mathbb{T}$  are there (simplify your answer)?
- (d) (3 points) How many permutations of  $\mathbb{T}$  are there?
- (e) (7 points) How many permutations are there if no two of letters "S" can be consecutive?
- (f) (5 points) 3 letters were randomly chosen from  $\mathbb{T}$ . Determine the probability that all three letters are different.

**2.** Consider the following partial order on the set  $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ :  
 $x \leq_R y$  if and only if  $x$  is a divisor of  $y$ .

- (a) (5 points) Draw the Hasse diagram for this poset.
- (b) (2 points) How many maximal elements are there?
- (c) (5 points) Find a largest chain (with proof).
- (d) (8 points) Find a largest antichain (with proof).

### 3.

- (a) (5 points) Construct permutations of  $\{1, 2, 3, 4, 5, 6, 7, 8\}$  whose inversion sequence is 6, 6, 1, 4, 2, 1, 0, 0.
- (b) (5 points) Construct the inversion sequence of the permutation 7, 3, 6, 5, 8, 4, 1, 2.

4. Prove that for any positive integers  $n, m, k$  with  $n > m > k$

$$\binom{n}{k} \binom{n-k}{m-k} = \binom{m}{k} \binom{n}{m}.$$

(a) (5 points) algebraically,

(b) (10 points) using combinatorial reasoning.

**5.** (10 points) Provide the Gray code of order 3 starting from 010 and ending at 110.

**6.** (10 points) Solve the equation for positive integer  $n$ :

$$\binom{n}{9} = \binom{n}{4}.$$

**7.** (10 points) There are several rocks with a total weight 10 tons. It is known that the weight of each rock is less than 1 ton. We need to deliver all these rocks to the nearest construction site. The delivery company we use has only the trucks with a weight capacity 3 tons. What is the minimum number of trucks they need to deliver all the rocks in one ride regardless of the size of the rocks?